THE GROVND OF ARTES, TEACHING the perfect VVorke and practise of *Arithmeticke, both in whole numbers* and Fractions, after a more easie and exact sort, than hitherto hath beene set foorth.

Made by M. ROBERT RECORD, D. in Physicke.

And now lately diligently corrected and beautified with sundry new Rules and necessary Additions: And further endowed with a third part of Rules of practise abridged into a briefer method than hitherto hath bene published: with diuerse such necessarie Rules as are incident to the trade of Merchandise.

Whereunto are also added diuerse Tables and Instructions that will bring great profite and delight vnto Merchants, Gentlemen, and others, as by the Contents of this Treatise shall appeare.

By IOHN MELLIS.

AT LONDON.

Imprinted by Richard Field for John Harison, dwelling in Pater noster row, at the signe of the Grey-hound.

1596.

To the Reader.

THat which my friend hath well begun
For verie loue to commonweale,
Of good will hath bene ouerrunne
Correcting each part euery deale.
And beautifull for thy behoofe
with briefer wayes for practise lore,
As by the triall and the proofe
Was neuer yet in print before.
Of numbers use the endlesse might
No Wit nor language can expresse:
Apply, and trie, both day and night,
And then this trueth thou Wilt confesse,

The Books verdict.

TO please or displease sure I am,
But not of one sort to euery man.
To please the best sort would I faine,
The froward displease shall I certaine.
Yet wish I will, though not with hope
All eares and mouethes to please or stop.

TO THE RIGHT VVORshipful M. Robert Forth Doctor of *law, and one of the Masters of the Queens* Maiesties High Court of Chauncerie.

WHereas heretofore (Right worshipfull Sir) I trauelled to the enlarging of this Author, wherein I abbreuiated some Rules to a more plainer method, as also enlarged him with sundry rules & questions for the better vnderstanding of such as delight in this excellent Art. And seeing that within these 8. *yeares, two impressions of these my labors dedicated to your Worship are already worne out. And for asmuch as my former labors to your worship dedicated, were right curteously accepted of you: I am still bould, being my first patron, to present you with this a third Impression: the rather that it may appeare both to your Worship and other that fauour knowledge, how this noble Art now florisheth in this noble Realme by the imparting of so many of these bookes in so short a time. Which simple talent of like pure affection as afore time, (as one of the greatest Iewels that God hath bestowed vpon me) I againe dedicate to your Worship as a testimonie of the dutifull zeale & good will I beare you: not doubting but that you will accept the giuers mind more than the gift, which I confesse is small. But being throughly ac-* 1. Rule.

 quainted

quainted with the great fauour your Worship beareth to such as delight in any good exercise, emboldneth me to put forth this simple Addition vnder your worships defence, The entire loue & exercise of this excellent Art, with drawing of Proportions, Maps, Cards, Buildings, Plats. &c. *were the only studies whereunto I euermore haue bin enclined. Touching Drawing, it was only* Dei beneficio, *naturally giuen me from my youth without instruction of any man more than* Loue *thereof, delectation, desire and practise. In this Art also hauing great delight, I had no other instruction at my first beginning, but onely this good Authors booke, but afterwardes I greatly increased the same during the time I serued your Worship in* Cambridge, *in going to the* Arithmeticke *Lecture at the common Schole: And more furthered since the time I left your worships seruice, which is about eight and twentie yeares past, by continuall exercise therein (the mother & nurse of Science) during which time my onely vocation hath bin (thinking it a meet exercise for a common wealth) in trayning vp of youth to vvrite and dravv, with teaching of them the infallible principles and briefe practises of this worthie Science, hauing (I praise God for it) brought vp a number to become faithfull and seruiceable to their maisters in great affaires, and many of them good mẽbers of a common wealth, which is no small comfort to me in Christ. Amongst which number a*

countri-

countriman of mine, a Norwich man, a Booke-seller here in London, was the first that became an earnest suter vnto me, to doe a deed of charitie vpon the ground of Artes, viz. to peruse and amend it of the imperfections and faults that haue crept into it through negligence of often printing. Which earnest request of his & others, bred two strifes in me: The one was, I was loth to do it; knowing my selfe inferior to a great number that might better do it than I. And on the other side, considering it is a booke hath done many a thousand good, which when a young beginner commeth to a confused or mistaken figure, bringeth him into a wonderfull discouragement and maze: which thing considered, for meere loue to a common wealth, and to the booke, being my first Author, I willingly graunted to do my good will. And passing vnder the file of correction, I here and there increast it with such necessarie Additions as I knew might encourage a yonger learner, and more would haue done, but for feare the booke would rise too thicke or grow too deere. And being thus entred into the veine thereof, and knowing that this Author was the onely light and the chiefe Lodestone vnto the vulgar sort of English men in this worthy Science, that euer writ in our naturall tongue, I haue (according to my simple knowledge) yeelded againe some part of my receiued talent with aduantage: and endowed him to the further increase of his memory, with Rules of breuitie and

 practise,

Ann Cheney

practise, abridged into a briefer method than hitherto hath bene published in our English toung, with other right necessary Additions, Rules and Tables, which I trust will do my Countrie good, and be right commodious to all sortes of men: all which I cōmit to the fauourable censure of your Worship, & all such as loue knowledge, desiring their fauourable correctiō herein, if ought be amisse. The which with greater affection than I am able to vtter, I dedicate to your Worship, as a meete patrone, both for learning, godlynes, and loue of the same. Which coming from your Worship into the hands of many, shall (I doubt not) do many good, as heretofore it hath alreadie done. So shall you (as the best benefactor of these labours) be partaker of all their prayers that shall reape profite or knowledge by this worthie Art: in commendation whereof if I should write, I should rather blemish then adorne it. For the Authours Epistle vnto that famous Prince of worthie memorie King Edward *the sixt, and his preface to the Reader, are sufficient.*

Thus crauing fauorable acceptation of this my homely and duetifull present, I humbly leaue you to the conduction of the Almightie, whom I beseech long to preserue you in continuall health, with daily increase of worship, to the glory of his name, and to the ioy of all such as vnfeignedly loue you.

At my house in Southwarke this 12 of August,
Your Worships most bounden to cōmaund,

John Mellis.

TO THE MOST MIGHTY Prince Edward the sixt, by the grace of God king of England France, and Ireland &c.

THE excellencie of mans nature, being such, as it is by Gods diuine fauour (most mightie Prince) not onely created in highnesse of degree farre aboue all other corporall things, but by perfection of reason, and search of witte, much approching toward the image of God, as not onely the holy Scriptures do testifie, but also those naturall Philosophers, which exactly did consider the nature of man, and namely the farre reach and infinite compasse of the workes of the mind, were inforced to confesse, that man scarcely was able to know him selfe. And if he would duely ponder the nature of him selfe, he would find it so strange, that it might seeme vnto him a verie miracle. And thereof sprang that saying; *Magnum miraculum est homo, maximum miraculum sapiens homo.* For vndoubtedly as man is one of the greatest miracles that euer God wrought, so a wiseman is plainely the greatest.

And therefore was it that some did account the head of a man the greatest miracle in the world, because not only of the strange workemanship that is in it, but much more of the efficacie of reason, wit, memory, imagination, and such other powers and workes of the mind, which can more easily conceiue any thing in a maner, than vnderstand it selfe. And amongst all the creatures of God it findeth none more difficult to be perceiued than the same powers it selfe, whereby it doth conceaue & iudge: as it may be well coniectured by the diuersitie of opinions, that the wisest philosophers did vtter touching the spirit of man & the substance of it: whereof at this present I intend to make no rehearsall, but who so listeth to read therof, may find it largely set forth, not onely in Aristotle his bookes *de Anima*, but also in Galene his booke called *Historia Philosophica*, and againe in Plutarch his worke *De Philosophorum placitis*, whose wordes are also repeated of Eusebius in the 15. booke [illegible], vnto whom I remit them that haue desired to vnderstand the intricate difficultie of knowing our owne selues, as touching our best parte, and that parte whereby we deserue to beare the name of men.

This matter seemeth so obscure and difficult in knowledge, that Gelene, who for his excellent wisedome and iudgement in naturall workes, is called of many men a Miracle in nature, yet in searching the nature & substance of the spirite of man, he not onely confesseth him selfe ignorant, but counteth it plaine temeritie to attempt to finde it: so farre aboue the hope of mans knowledge is that part, whereby man doth know and iudge of things. And although the ignorant sort (which hate all thinges that they know not) do litle esteeme the profoundnesse of mans spirit and of reason, the chiefe power and facultie of it, yet as here is a kind of feare and obedience of vnreasonable beasts vnto man by the working power of God

so is there in those small reasoned persons, a certaine kind of reuerence toward wisedome & reasons, which they do shew oftentimes, and by power of perswasion are enforced to obey reason, will they nill they.

And hereby came it to passe, that the rudenesse of the first age of man was brought vnto some more ciuill trade, as it is well declared by Cicero in the beginning of his first booke *De inuentione Rhetorica*, where he saith thus.

Nam fuit quoddam tempus quod in agris homines passim bestiarum more vagabantur, & sibi victu ferino vitam propagabant, nec ratione animi quicquam, sed pleraque viribus corporis administrabant. Nondum diuinæ religionis, non humani officij ratio colebatur: Nemo legitimas viderat nuptias, non certos quisquam inspexerat liberos, non ius æquabile quid vtilitatis haberet, acceperat: ita propter errorem atque inscitiam cæca ac temeraria dominatrix animi cupiditas, ad se explendam viribus corporis abutebatur, perniciosissimis satellitibus.

Quo tempore quidam, magnus vide licet vir & sapiens, cognouit quæ materia esset, & quanta ad maximas res opportunitas in animis inesset hominum, si quis eam posset elicere, & præcipiendo meliorem reddere: Qui dispersos homines in agris, in tectis syluestribus additos ratione quodam compulit in vnum locum & congregauit: eos in vnamquamque rem inducens vtilem atque honestam primo propter insolentiam reclamantes, deinde propter rationem atque orationem studiosius audientes, ex feris & immanibus mites reddidit, & mansuetos.

This long repetition of Tullies wordes will seeme tediouse to them which fone but litle, and care much lesse for the knowldege of reason, but vnto your Maiestie (I dare say) it is a delectable remembrance, & vnto me it seemeth so pleasaunt, that I could scarce stay my penne from writing all that mine eyes did so greedily reade.

This sentence of Cicero am I loath to translate into English, partly for that vnto your Maiestie it nee-

deth no translation. But especially knowing how farre that grace of Tullies eloquence doth excell any English mans tongue, and much more exceedeth the basenesse of my barbarous stile, yet for the fruict of the sentence, I had rather vnto my meere English Countrymen vtter the rudenesse of my translation, then to defraude them of the benefite of so good a lesson, trusting they will also gladly and greedily imbrace all good sciences that may help to the iust furniture of the same, when they consider that informed reason was the only instrument, or at least the chiefe meane to bring men to Ciuill gouernemente, from barbarous maners and beastly conditions.

For the time was (saith Tullie) that man wandered abrode in the fieldes vp and downe like beasts, & vsed no better order in feeding than they, so that by reasons rule they wrought nothing, but most of their doings did they atchieue by force of strength. At this time there was no iust regard of religion toward God, nor of dutie toward man. No man had seene right vse of mariage, neither did any man know their owne children from other, nor no man had felt the cōmoditie of iust Lawes: so that through errour & ignorance wilfull lust, like a blinde & headie ruler, abused bodily strength, as a most mortall minister for the satisfying of his desire. At that time was there one, which not onely in power, but also in wisedome was great: & he considered, how that in the mindes of men was both apt instrumentes, and great occasion to the due accomplishment of most waightie affaires, if a man could applie them to vse, and by teaching of rules frame them to better trade. This man with perswasion of reason gathered into one place the people that were wandring about the fieldes, and lay lurking in wild cottages and woodes: And bringing them into one common societie, did trade them to al such things as eyther were profitable or honest, although not without repining at the first, by reason that they had

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not bene so accustomed before, yet at legth through reason and perswasion of wordes, they obeyed him more diligently, and so of a wild and cruell people, he made them courteous and gentle.

Thus did Tullie set forth the efficacie of reason and perswasion, how it was able to conuert wild people to a mildnes, and to change their furious cruelnes into gentle courtesie. Were it not now a great reproch in this our time (when knowledge raigneth so large) that men should shew themselues lesse obsequious to reason? vnlesse it may be thought; that now euery man hauing sufficient knowledge of him selfe, needeth not to hearken to the perswasion of others.

In deede he that thinketh him selfe wise, will not esteeme the reason of any other, be he neuer so wiser so that of such one it may be well sayd, He that thinketh himselfe wiser then he is, may iustly be counted a double foole: wherefore such men are not to be permitted in open audience to talke, but must be put to silence, and made to giue eare to reason, which reason consisteth not in a multitude of wordes, heaped rashly togeather, and applied for one purpose, but reason is the expressing of a iust matter with witie perswasion, furnished with learned knowledge. Such knowledge had *Moses*, being expert in all learning of the *Assyrians*, as the Scriptures declare, and therefore was able to perswade the stubburne people of the Iewes, although not without great payne. Such knowledge and such reason did *Druys* shewe, which was the first Law maker of all the West partes of Europe. Like reason and wisedome did *Hamolius* vse amongst the *Gothes*: *Lycurgus* vnto the *Lacedemonians*, *Zeleucus* to the *Locrians*, *Solon* to the *Athenians*, and *Donuallo* *Malmutius* two thousand yeares past, amongst the old *Brittaines* of this realme. And thereby it came to passe, that their Lawes continued long, till more perfect reason altered many of them, and wilfull power oppressed most of them.

Druys was son to king Sarron, and succeeded him in his kingdome.

At the beginning when these wise men perceiued how harde it was to bring the rude people to vnderstand reason, they judged the best means to attaine this honest purpose, to depend of learning in euerie kinde: for by learning, as Ouid saith, *pectora mollescunt asperitasque fugit*: Stout stomaches do waxe mild, and sharpe fiercenesse is exiled. Therefore as Berosus doth testifie, Sarron that was the thirde King ouer all this West parte of Europe; for to bring the people from beastly rage to manly reason, did erect schooles of liberall Artes, which tooke so good successe, that his name continued in that sort famous aboue two thousand yeares after: for Diodorus Siculus which was in the time of Iulius Cæsar, maketh mention of the learned men of the Gothes, and named them Sarronides, that is to say, Sarron his scholers and followers.

Among these Artes that then were taught, some did informe the tongue, and make men able both to vtter aptly their mind, and also to perswade, as Grammer, Logicke, and Rhetoricke, although not so curiously as in this time: some other did appertaine to the iust order of partition of Landes, the true vsing of waights, mesures and reckenings in all sorts of bargaines, and for order of building, and sundry other vses, those were Arithmeticke and Geometrie. Againe, to encourage men to the honor of God, they taught Astronomie, whereby the wonderfull workes of God were so manifestly set foorth, that no mans tongue nor penne can in like sort expresse his infinite power, his vnspeakable wisedome, and his exceeding goodnesse toward man, whereby he doeth bountifully prouide for men all necessaries, not onely to liue, but also to liue pleasauntly. And so was their confidence in Gods prouidence strongly stayed, knowing his goodnesse to be such, that he would helpe man as he could, and his power to be so great, that he could doe what he would: and thirdly, his wisdome to be so pure, that he would doe nothing but that was best. Beside these

Sciences, they taught also Musicke, which most commonly they did applie partly to religious Sciences, to draw men to delight therein, and partly to songs made of the maners of men in praise of vertue and discommendation of Vice, whereby it came to passe that no man would displease them, nor do any thing euill that might come to their hearing: for their Songs did make euill men more abhorred in that time, than any excommunication doth in this time. The posteritie of these Musitians continue yet both in Wales & Ireland called *Bardes* vnto this day, by the auncient name of *Bardus*, their first founder.

And as these Sciences did increase, so did vertue increase therby. Againe, as these Sciences did decay, so vertue lost her estimation, and consequently was litle in vse: Whereof to make a full declaration, were a thing meete for a Prince to heare, but it would require a peculiar treatise. Wherefore at this present I count it sufficient, lightly to haue touched this matter in generall words, and to say no more of the particularitie thereof, but onely touching one of those sciences, that is Arithmeticke, by which not onely iust partition of lands was made, but also touching buying and selling, all assises, weights, and measure were deuised, and all reckenings and accounts driuen: yea by proportion of it were the true orders of Iustice limited, (as Aristotle in his Ethicks doth declare) and the degrees of estates in the common wealth established. Although that proportion be called Geometricall, and not Arithmeticall, yet doth that proportion appertaine to the Art of Arithmeticke: & in Arithmeticke is taught the progression of such proportions, and all things thereto belonging. Wherefore I may well say, that seing Arithmeticke is so many wayes needfull vnto the first planting of a common wealth, it must needes be as much required to the preseruation of it also: for by the same meanes is any common wealth continued, by which it was erected

This Bardus Druidus the fifth king of the Celtes, raigned 69. yeares, and 1832 yeares before Christ

and established. And if I shall in small matters in appearance, but in deede verie waightie, put one example or two, what shall wee say for the Statutes of this Realme, which be the onely stay of good order in manner nowe? As touching the measuring of ground by length and breadth, there is a good and auncient Statute made by the Arte of Arithmeticke, and now it shall be to little vse, if by the same Arte it be not practised and tried. For the assise of bread and drinke, the two most common and most necessarie things for the sustentation of man, there was a goodly ordinance in the Lawe made, which by ignorance hath so growen out of knowledge and vse, that fewe men doe vnderstand it, and therefore the Statute bookes are wonderfully corrupted, and the Commons cruelly oppressed: notwithstanding some men haue written, that it is too doubtfull a matter to execute those Assises by those statutes, by reason they depend of the standard of the Coine, which is much changed from the state of that time, when those statutes were made. This shall euery man reade that listeth, in the Abridgement of Statutes, in the title of waightes & Measures, in the seuenth number of the English booke, where he should haue translated a good ordinaunce, which is set forth in the French booke: but no maruell if the Abridgement doth omit it, seeing the great booke of statutes doth omit the same statute, as it hath done diuerse other very good Lawes. And this is the fruite of ignorance, to reiect all that it vnderstandeth not, although they vse some clokes for it: but such clokes, as being allowed, might serue to repell all good lawes: which God forbid.

Againe there is an ancient order for Assise of firewood, and coales, which was renued not many yeares past, and now how Auarice and ignorance doth canuas the statute, it is too pitifull to talke of, and more miserable to feele.

Furthermore, for the statute of Coynage, and the

the Standerd thereof, if the people vnderstood rightly the Statute, they should not, nor would not (as they often doe) gather an excuse for their follie thereby: but as I sayd, these Statutes by wisedome and good knowledge of Arithmeticke, were made, and by the same must they bee continued. And let ignoraunce no more meddle with the vse of them, then it did with the making of them. Oh in how miserable case is that Realme, where the ministers and Interpreters of the Lawes are destitute of all good Sciences, which be the keyes of the Lawes? How can they eyther make good Lawes, or mainetaine them, that lacke that true knowledge wherby to iudge them? And happie may that Realme be accounted, where the Prince him selfe is studious of learning, & desireth to vnderstand equitie in all Lawes. Therefore most happie are wee the louing subiects of your Maiestie, which may see in your Highnesse, not onely such towardnesse, but also such knowledge of diuerse Artes, as seldome hath bene seene in any Prince of such yeares, whereby we are enforced to cõceiue this hope: Certainely, that he which in those yeares seeketh knowledge when knowledge is least esteemed, and of such an age can discerne them to be enemies both to his royall person, and to his Realme, which labour to withdraw him from knowledge to excessiue pastime, and from reasonable studie, to idle or noisome pleasures, hee must needes when he commeth to more mature yeares, be a most prudent prince, most iust Gouernour, and a right Iudge, not onely of his subiectes commonly, but also of the ministers of his Lawes, yea, and of the Lawes thẽselues: and to be able to conceiue the true equity and exact vnderstanding of all his Lawes & statutes, to the cõfort of his good subiectes, and the confusion and reproch of them which labour to obscure or peruert the equitie of the same Lawes & statutes. How some of those Statutes may bee applied to vse, as well in

our time, as in any other time, I haue particularly declared in this booke, & some other I haue omitted for iust considerations, till I may offer them first vnto your Maiesty, to weigh them, as to your Highnesse shall seeme good: for many thinges in them are not to be published without your Highnesse knowledge and approbation, namely because in them is declared all the rates of alloyes for all standerds from one ownce vpward, with other mysteries of Mynt matters, and also most part of the varieties of coynes that haue bene currant in this your Maiesties Realme by the space of sixe hundred yeares last past, and many of them that were currant in the time that the Romanes ruled here.

All which with the ancient description of England & Ireland, and my simple censure of the same I haue almost completed to be exhibited to your Highnesse.

In the meane season most humbly beseeching your maiestie to accept this simple treatise, not worthy to be presented to so high a prince, but that my lowly request to your Maiestie is, that this amongst other of my bookes may passe vnder the protection of your Highnesse, whom I beseech God most earnestly and daily, according to my duetie, to aduaunce in all honour and princely regalitie, and to increase in all knowledge, iustice, and godly pollicie.

Amen.

Your maiesties most obedient
Subiect and Seruant,

ROBERT RECORD.

TO THE LOVING READER.

The Præface of Maister R. Record.

SORE oftentimes haue I lamented with my selfe the infortunate condition of *Englande*, seeing so many great Clerkes to arise in sundrie other parts of the world, and so few to appeare in this our nation: whereas for pregnancie of naturall wit (I thinke) few nations do exceede Englishmen: but I cannot impute the cause to any other thing then to the contempt and misregard of learning. For as Englishmen are inferior to no men in mother wit, so they passe all in vaine pleasures, to which they may attaine without great paine or labour: and are as slacke to anie neuer so great commoditie, if there hang on it any painefull studie or trauellsome labour.

Howbeit, yet all men are not of that sort, though the most part be, the more pitie it is: but of them that are so glad, not onely with painefull studie, and studious paine to attaine learning, but also with as great studie and paines

to communicate their learning to others, and make all England, if it might be, partakers of the same, the most part are such, that vnneth they can support their owne necessarie charges, so that they are not able to beare anie charge in doing of that good, that else they desire to doe.

But a greater cause of lamentation is this: that when learned men haue taken paines to do things for the ayde of the vnlearned, scarce they shall be allowed for their well doing, but derided and scorned, and so vtterly discouraged to take in hand anie like enterprise againe. So that if anie be found (as there are some) that do fauour learning and learned wits, and can be content to further knowledge, yea onely with their word, such persons, though they be rare, yet shall they incourage learned men to enterprise some things, at the least, that England may reioyce of. And I haue good hope that England will (after she hath taken sure taste of learning) not onely bring foorth more fauourers of it, but also such learned men, that she shall be able to compare with anie Realme in the world. But in the meane season, where so fewe regarders of learning are, how greatly they are to be esteemed that do fauour and further it, my pen will not suffice at full to declare.

Therefore gentle Reader, whereas I do vpon most iust occasion iudge, yea and knowe assuredly, that there bee some men in this Realme, which both loue, and also much desire to further good learning, and yet am not well able to write their condigne praise for the same, I thinke it better with silence to ouerpasse it, than either to say too litle of it, or to prouoke against them the malice of such other, which do nothing themselues that is praise worthy, and therefore can not abide to heare the praise of anie others mans good deed.

And considering their great fauour vnto learning, although I my selfe bee not worthie to bee reckoned in the number of great learned men, yet am I bold to put my selfe in preace with such abilitie as God hath lent mee,

 though

though not with ſo great cunning as manie men, yet with as great affection as any man, to helpe my countriemen, and will not ceaſe dailie (as much as my ſmall abilitie will ſuffer me) to indite ſome ſuch thing, that ſhall be to the inſtruction, though not of learned men, yet at the leaſt to the vulgar ſort, whoſe argument alwaies ſhall be ſuch, that it ſhall delight all learned wits, though they do not learne any great things out of it.

But to ſpeake of this preſent booke of Arithmeticke, I dare not, nor will not ſet it foorth with any wordes, but remit it to the iudgement of all gentle Readers, & namely ſuch as loue good learning, beſeeching them ſo to eſteeme it, as it doth ſeeme worthie. And ſo either to accept the thing for it ſelfe, either at the leaſt to allow my good endeuour. But I perceiue I need not to vſe any perſwaſions vnto them, whoſe gentle nature and fauourable minde is readie to receiue thankfully, and interpret to the beſt, of all ſuch enterpriſes attempted for ſo good an ende, though the thing do not alwayes ſatisfie mens expectation.

This conſidered, did bolden me to publiſh abroad this litle booke of the Arte of Numbring, which if you ſhall receiue fauourably, you ſhall encourage me to gratifie you hereafter with ſome greater thing.

And as I iudge ſome men of ſo louing a minde to their natiue Countrey, that they would much reioyce to ſee it to proſper in good learning & wittie Arts, ſo I hope well of all the reſt of Engliſhmen, that they will not be vnmindfull of his due praiſe, by whoſe meanes they are helped & furthered in any thing. Neither ought they to eſteeme this thing of ſo litle value (as many men of litle diſcretion oftentimes do:) For who ſo ſetteth ſmall price by the wittie deuiſe and knowledge of numbring, he litle conſidereth it to be the chiefe point (in manner) whereby men differ from all brute beaſtes: for as in all other things, (almoſt) beaſtes are partakers with vs, ſo in numbring wee differ cleane from them, and in manner particularly, ſith that in many things they excell vs againe.

The Foxe in craftie wit exceedeth most men
A Dogge in smelling hath no man his Peere,
To foresight of weather if you looke then,
Many beastes excell man, this is cleare.
The wittinesse of Elephants doth letters attaine,
But what cũning doth there in the Bee remaine?
The Emmet foreseeing the hardnesse of winter,
Prouideth victuals in the time of Summer.
The Nightingale, the Linet, the thrush, the Lark
In musicall harmony passe many a Clarke.
The Hedgehog of Astronomie seemeth to know,
And stoppeth his caue where the wind doth blow.
The spider in weauing such Art doth show,
No man can him mend, nor follow I trow.
When a house will fall, the Mice full quicke.
Flee thence before. Can man do the like?

Many things else of the wittinesse of beastes and birds might I here say, saue that another time I intend to write wherein they excell in manner all men, as it is daily seene: but in number was there neuer beast found so cunning, that could know or discerne one thing from many, as by daylie experience you may well consider: when a bitch hath many Whelpes, or a Henne many Chickens: and likewise of other whatsoeuer they be, take from them al their young sauing onely one, and ye shall perceiue plainly that they misse none, though they will resist you in taking them away, and will seeke them againe if they may know where they be, but else they will neuer misse them truely, but take away that one that is left, and then will they crie and complaine: and restore to them that one, then they are pleased againe: so that of number this may I iustly say: It is the onely thing (almost) that separateth man from beastes. He therfore that shall contemne number,

ber, he

ber he declareth himselfe as brutish as a beast, and vnworthie to be counted in fellowship of men. But I trust there is no man so fowlie ouerseene, though manie right smally do it regard.

Therefore will I nowe stay to write against such, and returne againe to this booke, which I haue written in the forme of a Dialogue, because I iudge that to be the easiest way of instruction, when the Scholer may aske euerie doubt orderly, and the Maister may answere to his question plainely.

Howbeit I thinke not the contrarie, but as it is easier to blame another mans worke than to make the like, so there will be some that will find fault, because I write in a Dialogue: but as I coniecture, those shall be such, as do not, cannot, either will not perceiue the reason of right teaching, and therefore are vnmeet to be answered vnto: for such men with no reason will be satisfied.

And if any man obiect, that other bookes haue bene written of Arithmetick alreadie so sufficiently, that I needed not now to put pen to booke, except I will condemne other mens writings: to them I answer, That as I condemne no mans diligence, so I know that no one man can satisfie euerie man, and therfore like as many do esteeme greatly other bookes, so I doubt not but some will like this my booke aboue any other English Arithmeticke hitherto written, and namely such as lacke instructors, for whose sake I haue so plainely set forth the examples, as no booke (that I haue seene) hath done hitherto: which thing shall be great ease to the rude Readers.

Therefore (gentle Reader) though this booke can be small aide to the learned sort, yet vnto the simple ignorant (which needeth most helpe) it may be a good furtherance and meane vnto knowledge. And though vnto the King his Maiestie priuately I do it dedicate, yet I doubt not (such is his clemencie) but that he can be content, yea and much desirous, that all his louing subiectes shall take the vse of it, & applie the same to their owne profit.

Which thing if I perceiue that they thankfullie do, and receiue with as good will as it was written, then will I shortly with no lesse kindnesse set foorth such introductions into Geometrie and Cosmographie, as I haue at other times promised, and as hitherto in English hath not bene enterprised, wherewith I dare say all honest heartes will be pleased, and all studious wits greatly delighted.

I will say no more, but let euerie man iudge as he shall see cause. And thus for this time I will stay my penne, committing you all to that true fountaine of perfect number, which wrought the whole worlde by number and measure: he is Trinitie in Vnitie, and Vnitie in Trinitie: to whom be all praise, honour, and glorie.

Amen.

HERE FOLLOVV-weth a Table of all the Contents of this Booke.

The Contents of the first Dialogue containe the Declaration of the profite of Arithmeticke.

The Contents of the ſecond part touching Fractions.

Againe

The Contents.

The Contents of the third Addition to this Booke. 405

FINIS.

A COLLECTION of such Tables as are contained in this Treatise.

Lord God Bill

Lar
A Large

A Large Table of Numeration,
A Table of Multiplication.

A Table of all the Golde and Silver Coynes in this Realme, with the most usuall gold Coines throughout Christendome, with their severall weights of pence and graines, and what they are worth in currant money English.

Certaine Tables or Notes of the Contentes of Ale, Beere, Wine, Butter, Sope, Salmon, Eeles, &c. both what such vesselles ought to containe by the Statute, and what those vessels emptie ought to weigh.

A Table of the quantity of dry measures, as Peckes, Bushels, Quarters, Weyes, &c.

A Table of the proportion of measure, touching Lengths or breadths: to wit, from the inch to the foote, and so to the yard, the Elle, with their partes, the pearch, the rod, the furlong, the mile, &c.

A Table made by Progression Arithmeticall, which containeth a double Table of Multiplication.

A Table or demonstration of a figure or measure for the perfect understanding of Frac-

Lastly,

Lastly, a Table demonstrating the true solution of 3 diuerse things hidden of thrée seuerall persons in pastime.

Note that this △ Caracter representeth the crowne by exchange.

Item whereas I haue augmented diuerse necessary questions and rules vnto the Author. the beginning of which additions are marked with this ¶ note. And where they end with this * note.

FINIS.

BEFORE THE IN-troduction of Arithmeticke, it were very good to haue ſome vnder-ſtanding and knowledge of theſe figures and Notes.

i.	1	one.	xx.	20	twenty.
ii.	2	two.	xl.	40	forty.
iii.	3	thrée.	l.	50	fifty.
iiii.	4	foure.	lx.	60	ſixty.
v.	5	fiue.	lxx.	70	ſeuenty.
vi.	6	ſixe.	xc.	90	ninety.
vii.	7	ſeuen.	C.	100	a hundred.
viii.	8	eight.	CC.	200	two hundred.
ix.	9	nine.	D.	500	fiue hundred.
x.	10	ten.	DC.	600	ſixe hundred.
xi.	11	eleuen.	M.	1000	a thouſand.
xii.	12	twelue.	MD.	1500	a thouſ.5.hund.
&c.	&c.	&c.	&c.	&c.	&c.

112 l' make a C. waight
56 halfe a C. waight.
28 a quarter of a hundred waight.
16 ounces make a pou͂d.
A pound Troy which is the ſtandard of England, whereby is onely waied gold, ſiluer, & bread, is 12. ounces.
12 inches make a foote.

3 foote make a yard.
The yard is parted into 4 quarters.
And ech quarter into 4 nailes.
5 quarters of a yarde make an Elle. Which Elle is alſo parted in 4 quarters.
And ech quarter into foure nayles.

A DIALOGVE BETVVEENE THE MASTER and the Scholer, teaching the Art *and vse of Arithmeticke* with Penne.

The Scholer speaketh.

SYR, such is your authoritie in mine estimation, that I am content to consent to your saying, and to receiue it as truth, though I see none other reason that doeth lead me therunto: wheras else in mine owne conceipt it appeareth but vaine, to bestow any time priuatly in learning of that thing, that euerie childe may and doth learne at all times and houres, when he doth any thing himselfe alone, and much more when he talketh or reasoneth with other.

Master. Loe, this is the fashion and chaunce of all them that seeke to defend their blind ignorance: that when they thinke they haue made strong reason for themselues, then haue they proued quite contrarie. For if numbring be so common (as you grant it to be) that no man can do any thing alone, and much lesse talke or

bargaine with other, but he shall still haue to do with number: this proueth not number to be so contemptible and vile, but rather right excellent & of high reputation, sith it is the ground of all mens affaires, so that without it, no tale can be tolde, no communication without it can be long continued, no bargayning without it can duely be ended, or no buisinesse that man hath, iustly completed. These commodities (if there were none other) are sufficient to approue the worthinesse of Number. But there are other innumerable farre passing all these, which declare Number to excede all praise. Wherefore in all great workes are Clerkes so much desired? Wherefore are Auditors so richly feed? What causeth Geometritians so highly to be enhaunced? Why are Astronomers so greatly aduaunced? Because that by Number such things they find, which else should farre excell mans minde.

Scholer. Verily Syr if it be so, that these men by numbring their cunning do attaine, at whose great workes most men doo wonder, then I see well I was much deceiued, and numbring is a more cunning thing than I toke it to be.

Master. If Number were so vile a thing as you did esteeme it, then neede it not to be vsed so much in mens communication. Exclude Number, and answer to this question. How many yeares old are you?

Scholer. Mum.

Master. Howe manie dayes in a weeke? howe manie weekes in a yeare? what landes hath your Father? howe manie men doth hee keepe? how long is it since you came from him to me?

Scholer. Mum.

Master. So that if number want, you answere all by Mummes: How many myle to London?

Scholer. A poake full of Plummes.

Master. Why, thus you may see, what rule number beareth, and that if number be lacking it maketh men dumbe, so that to most questions they most answer Mum.

Scholer. This is the cause sir, that I iudged it so vile, because it is so common in talking euerie while: For plentie is not deintie, as the common saying is.

Master. No, nor store is no sore: perceiue you this? The more common that a thing is, being néedfully required, the better is the thing, and the more to be desired. But in numbring, as some of it is light and plaine, so the most part is difficult, and not easie to attaine. The easier part serueth all men in common, and the other parte requireth some learning. Wherefore as without numbring a man can do almost nothing, so with the help of it, you may attaine to all things.

Scholer. Yea sir: why then it were best to

learne the Art of Numbring, first of all other learning, and then a man néed learne no more, if all other come with it.

Maister. Nay not so: but if it be first learned, then shal a man be able (I meane) to learn, perceiue, and attaine to other sciences, which without it, he should neuer get.

Scholer. I perceiue by your former words, that Astronomy and Geometry depend much on the helpe of Numbring: but that other sciences, as Musicke, Phisicke, Law, Grammar, and such like, haue any helpe of Arithmetike, I perceiue not.

Maister. I may perceiue your great Clerklinesse by the ordering of your sciences: but I will let that passe now, because it toucheth not the matter that I entend, and will shew you how Arithmeticke doth profit in all these, somewhat grossely, according to your small vnderstanding, omitting other reasons more substantiall.

First (as you recken them) Musicke hath not onely great helpe of Arithmeticke, but is made, and hath his perfectnesse of it: for all Musicke standeth by number and proportion.

And in Phisicke, beside the calculation of Criticall daies, with other things which I omit, how can any man iudge the pulse rightly, that is ignorant of the proportion of Numbers?

And as for the Lawe, it is playne, that the

mans

man that is ignorаunt of Arithmeticke, is neither meete to be a Juge, neither an Aduocate, nor yet a Proctour. For how can he well vnderstand another mans cause appertaining to distribution of goods, or other debts, or of sums of money, if he be ignoraunt of Arithmeticke? This oftentimes causeth right to be hindred, when the Judge either delighteth not to heare of a matter that he perceiueth not, or cannot iudge it for lacke of vnderstanding: This commeth by the ignorance of Arithmeticke.

Now as for Grammar, mee thinketh you should not doubt in what it needeth number, sith you haue learned that Nounes of all sorts, Pronounes, Verbes, and Participles, are distinct diuersely by numbers: besides the variety of Nounes of number, and Aduerbes. And if you take away number from Grammar, then is all the quantity of Sillables lost.

And many other waies doth number helpe Grammar. Whereby were all kindes of meeters found and made? Was it not by Number?

But howe needefull Arithmeticke is to all partes of Philosophy, they may soone see, that doe read either Aristotle, Plato, or any other Philosophers writing. For all their examples almost, and their probations, depend of Arithmeticke. It is the saying of Aristotle, that hee that is ignoraunt of Arithmeticke, is meete to no Science. And Plato his maister wrote a like

sentence ouer his Schoolehouse doore. Let none enter in hither (quoth hee) that is ignorant of Geometrie. Seeing he would haue all his scholers expert in Geometrie, much rather he wold the same in Arithmeticke, without which Geometrie cannot stand.

And how needfull Arithmeticke is to Diuinitie, it appeareth, seeing so manie Doctors gather so great mysteries out of number, and so much do write of it. And if I should goe about to write all the commodities of Arithmeticke in ciuill actes, as in gouernaunce of common weales in time of peace, and in due prouision and order of armies in time of warre: For numbring of the host, summing of their wages, prouisions of victuals, viewing of artillerie, with other armour: Beside the conningest poynt of all, for casting of ground, for encamping of men, with such other like. And how many wayes also Arithmeticke is conducible for all priuate weales of Lordes, of all possessioners, of Merchants, & all other occupiers, and generally for all estates of men, besides Auditors, Treasurers, Receiuers, Stewards, Bailiffes and such like, whose offices without Arithmeticke are nothing: If I should (I say) particularly repeate all such commodities of this noble science of Arithmeticke, it were ynough to make a very great booke.

Scholer. No, no sir, you shall not neede: For I doubt not, but this that you haue sayd, were inough

enough to perswade any man to thinke this Art to be right excellent and good, and so necessary for man, that (as I thinke now) so much as a man lacketh of it, so much he lacketh of his sense and wit.

Master. What, are you so farre chaunged since, by hearing these fewe commodities in generall? By likelihoode you would be farre chaunged, if you knewe all the commodities particular.

Scholer. I beseech you Syr, reserue those commodities that rest yet behinde unto their place more conuenient. And if yee will be so good as to utter at this time this excellent treasure, so that I may be somewhat enriched therby, and if euer I shalbe able, I will requite your paine.

Master. I am uerie glad of your request, and will do it speedily, sith that to learne it you be so readie.

Scholer. And I to your authoritie my wit doe subdue, whatsoeuer you say, I take it for true.

Master. That is too much, and meete for no man to be beleeued in all things, without shewing of reason. Though I might of my scholer som credence require, yet except I shew reason, I do it not desire. But now sith you are so earnestly set this Arte to attaine, best it is to omit no time, least some other passion coole this great heate, and then you leaue off be- Perseuerance in studie.

fore you see the end.

Scholer. Though many there be so vnconstant of minde, that flitter and turne with euerie winde, which often begin, and neuer come to the end, I am none of this sort, as I trust you partly know. For by my good will, what I once begin, till I haue it fully ended, I would neuer blin.

Master. So haue I found you hitherto indeed, and I trust you will encrease rather then go backe. For better it were neuer to assay, than to shrinke and flie in the middle way. But I trust you will not so do, therfore tel me briefly. What call you the Science that you desire so greatly?

Scholer. Why sir, you know.

Master. That maketh no matter: I would heare whether you knowe, and therefore I aske you. For great rebuke it were to haue studied a Science, and yet cannot tell howe it is named

Scholer. Some call it Arsemetrike, and some Augrime.

Master. And what doo these names betoken?

Scholer. That, if it please you, of you would I learne.

Master. Both names are corruptly written, Arsemetrike for Arithmetike, as the Greekes call it, and Augrime for Algorisme, as the Arabians sound it, which both betoken the

the Science of numbring. For Arithmos in Greeke, is called number; and of it commeth Arithmeticke, the Arte of numbring. So that Arithmetike is a Science or Arte teaching the manner and vse of Numbring. This Art may be wrought diuersly, with Penne or with Counters. But I will first shewe you the working with the Penne, and then the other in order.

Scholer. This I will remember. But how many things are to be learned, to attaine this Arte fullie?

Master. There are reckoned commonly seuen partes or workes of it.

Numeration, Addition, Subtraction, Multiplication, Diuision, Progression, and Extraction of rootes: to these some men adde Duplation, Triplation, and Mediation. But as for these last three, they are contained vnder the other seuen. For Duplation and Triplation, are contained vnder Multiplication, as it shall appeare in their place. And Mediation is contained vnder Diuision, as I will declare in his place also.

Scholer. Yet then there remaine the first seuen kindes of Numbring.

Master. So there doeth: Howbeit, if I shall speake exactly of partes of Numbring, I must make but fiue of them: For Progression is a compound operation of Addition, Multiplication and Diuision. And so is the extrac-

ction of rootes. But it is no harme to name them as kindes severall, seeing they appeare to haue some severall working. For it forceth not so much to contend for the number of them, as for the due knowledge and practising of them.

Scholer. Then you will that I shall name them as seauen kindes distinct. But nowe I desire you to instruct me in the vse of eache of them.

Master. So will I, but it must be done in order: for you must not learne the last as soone as the first, but you must learne them in that order, as I did rehearse them, if you wil learne them speedily and well.

Scholer. Euen as you please. Then to begin, Numeration is the first order: what shall I do with it?

Master. Firste you must knowe what the thing is, and then after learne the vse of the same.

Numeration.

NUmeration is that Arithmeticall skill, whereby we may duly value, expresse and read any number or summe propounded: or else in apt figures and places, set downe any number knowne or named.

Scholer.

Scholer. Why? then me thinketh you put a difference betwæne the value and the Figures.

Master. Yea so doe I: For the value is one thing, and the figures are another thing: and that commeth partly by the diuersitie of figures, but chiefly of the places wherein they be set.

Scholer. Then I must knowe here thrées thinges; the Value, the Figure, and the Place.

Master. Euen so: but yet adde Order to them as the fourth. And first marke, that there are but ten figures, that are vsed in Arithmeticke, and of those ten, one doth signifie nothing, which is made like an o, and is priuately called a Cypher, though all the other sometime be likewise named. The other nine are called signifying figures, and be thus figured. A Cipher.

1 2 3 4 5 6 7 8 9
And this is their value.
i. ij. iij. iiij. v. vi. vij. viij. ix.

But here you must marke, that euerie Figure hath two values: One alwayes certaine that it signifieth properly, which it hath of his forme: and the other vncertaine, which he taketh of his place.

A place is called the seate or roume that a Figure standeth in. And looke how manie Fi- A place.

gures are written in one summe, so many places hath that whole number. And the first place must bee called that that is next to the right hand, and so reckoning by order towards the left hand, so that that place is last, that is next to the left hand. As for example: If there stood before you sixe men in a row, side by side, and you shall tell them as they stand in order, beginning with the mã that were next to your right hand: then he that were next him should be called the second, and so forth to the farthest from your right hand, which is the sixt and the last.

Scholer. Sir, I perceive you well: so might I recken letters or any other thing. As if I should write eight letters after this order, a,b, c,d,e,f,g,h. then must I say, h is the first, g the ii, f the iii. e the iiii, d the v, c the vi, b the vii, and a the viii.

Maister. That is well done. And after the same sort use hereafter, that what I declare by one example, do you expresse by another, and so I shall perceive whether you understand it or no. And so passe over nothing, till you perceive it well, and be expert therein.

Scholer. Sir, I pray you how many of these places be there in all?

Maister. There is no certaine number of them, but they are sometimes more & somtimes fewer, according to the summe that is expressed. For so many as the figures are, so many are

the

the places: and the last place is so called, not because it is last of al other, but it is the last of that present summe, and it may be the middle place in another summe.

Scholer. Me séemeth I perceiue this very well: as touching the order of reckoning of the places: But as for the number of them, you say there is no certainety. Now there resteth to declare the value of the figures by the diuersity of places, which you called the Value vncertaine. Value vncertaine.

Maister. But first let me heare whether you know perfectly the certaine value.

Scholer. Yes sir, as you wrote them, so I marked them.

Maister. How wrote you then fiue?

Scholer. By this figure 5.

Maister. And how sixe?

Scholer. Thus, 6.

Maister. Write these thrée numbers each by it selfe, as I speake them vii. iiii. iii.

Scholer. 7. 4. 3.

Maister. How write you these foure other, ii. i. ix. viii.

Scholer. Thus (I trow) 2. 1. 6. 8.

Maister. Nay, there you misse: Looke on mine example againe.

Scholer. Sir, truth it is, I was to blame, I tooke 6 for 9, but I will bée ware hereafter.

Maister. Nowe then take héede, those cer-

taine values euerie figure representeth, when it is alone written without other figures ioyned to him. And also when it is in the first place, though manie other doe follow: as for example: This figure 9 is ix, standing nowe alone.

Scholer. How? is he alone and standeth in the middle of so many letters?

Master. The letters are none of his fellows. For if you were in Fraunce in the middle of a M. Frenchmen, if there were no English man with you, you would recken your selfe to bee alone.

Scholer. So it is. Then 9 without more figures of Arithmeticke, betokeneth ix, whatsoeuer other letters be about it.

Master. Euen so, and so doth it, if it be in the first place ioyned with other, how many soeuer do follow, as in this example, 3679. you see 9 in the first place, and doth betoken nine, as if he were alone.

Scholer. I perceiue that. And doeth not 7 that standeth in the second place, betoken vii? and 6 in the third place betoken vi? And so 3 in the fourth place, betoken three?

Master. Their places be as you haue sayd, but their values are not so. For as in the first place, euerie figure betokeneth his owne value certaine onely, so in the second place euerie figure betokeneth his owne value certaine ten times: as in the example, 7 in the second place

place in seuen times x, that is, lxx. And in the thirde place, euerie figure betokeneth his owne value a hundreth times, so that 6 in that place betokeneth vi. C. And in the fourth place, euerie figure betokeneth his own value a thousand times, as in the aforesaide number, 3 in the fourth place, standeth for 3 M. And in the fifth place euerie figure standeth for his owne value x M. times. And in the vi. place a C. M times: and in the vij. place a M. M. times: and in the viij. place x. M. M. so that euerie place exceedeth the former x. times

Scholer. As thus: if I make this number at all aduentures, 91359684, here are eight places. In the first place is 4, and betokeneth but foure: in the second place is 8, and betokeneth ten times 8, that is 80, in the third place is 6, and betokeneth sixe hundred. In the fourth place 9, is nine thousande. And 5 in the fifth place is ten M. times fiue, that is fiftie M. So 3 in the sixt place, is a C. M. times three, that is CCC. M. Then 1 in the seuenth place, a M. M. And 9 in the eight place ten thousand thousand times 9, that is, xc. M. M. But now I cannot easily nor quickly read it in order.

Master. That shall you practise by this meanes. First if you pricke ouer the fourth figure, and so ouer the seuenth. And (if you haue so many) ouer the tenth, thirteenth, sixteenth, and so forth, still leauing two figures betweene eache two prickes. And those two

Ternaries. rowmes betwéene the prickes, are called Ternaries.

Then begin at the last pricke, and sée how many figures are betwéene him and the ende, which cannot passe thrée: reckoning himselfe for one: then pronounce them as if they were written alone from the rest, and adde at the ende of their value so many times thousande as your number hath prickes.

After that come to the next thrée figures, and sound them as if they were apart from the rest, & adde to their value so many times thousands, as there are pricks betwéene them and the first place of your whole number. And so do by euerie other thrée figures following, if you haue mo: as in example, 9135968 4, this was your number.

Put a pricke ouer 9 in the fourth place, and ouer 1 in the seuenth place, and then no more, (for your places come not to tenne) as thus: 9135968 4.

Now go to the last pricke ouer 1, and take it and the figure 9 that followeth it, & value them alone.

Scholer. 91, that is xci.

Master. So it is: but then adde for the number of your prickes twice M.

Scholer. That is xci thousand thousand.

Master. So it is. Then take the thrée other figures from one to the next pricke, and value them.

Scholer

Scholer. 359. that is CCC. lix.

Maister. Now adde for the one pricke, that is betweene them and the first place, M.

Scholer. CCC. lix. thousand.

Master. Then come to the other thrée figures that remaine.

Scholer. 684, that is, vi. C lxxxiiii.

Maister. Now haue you valued all. And at the end of the last number you shall adde nothing, because there remaineth no pricke nor number after it: yet proue in an other number, as thus 23086408910534 0.

Scholer. 23086408910534 0. I haue pricked them as you taught me: but I am in doubt, whether I haue done well or no, because of the Ciphers: for I remember, you told mee that they doe signifie nothing, and therefore I doubt whether I should recken them for a figure in setting of the prickes: and againe, I know not wherefore they serue.

Master. That will I tell you now. In deed they are of no value themselues, but they serue to make vp number of places, and so make the figure following them to be in a further place, and therefore to signifie the more value: as in this example 90, the Cypher is of no value, but yet he occupieth the first place, and causeth 9 to be in the second place, and so to signifie tenne times 9, that is, xc. so that two Ciphers thrusteth the figure following them, into the third place, and so forth.

Scholer. Then I perceiue in the example aboue, I haue pricked well inough: for though that Cipher that is pricked signifie nothing, yet must he haue the pricke, because hee came in the xiii. place. Then will I prooue to number that summe. First there is 230. M. M. M. M. and then followeth 864. M. M. M. And what shall I now doe? There is a Cipher in the third place, and no figure after him, but they that I haue reckoned.

Maister. Hee did serue for them that you haue already reckoned, to make them in a place further than they should be, if he were away: and therefore now you shall let him go. And so doe alwaies when he occupieth that place next before any pricke, which is the last of that Ternary, and a Cipher in the last place doth nothing.

Scholer. Then shall I say but 89. M M.

Maister. So, but go forth.

Scholer. 105 thousand. Now are all my prickes spent, and yet remaine 340, so that I must value them CCC.xl onely.

Maister. Now can you recken after this sort: and remember, that euery such roume so parted, is called a Ternarie or Trinity: for you haue numbered or valued the summe most truely, and by the ayde of the prickes each Denomination is distinct most plainely.

Scholer. What call you Denominations?

ons?

Master. It is the laste value or name added to any summe. As when I say: a hundreth two and twentie pounds: Pounds is the Denomination. And likewise in saying 25 men, men is the Denomination, and so of other. But in this place (that I spake of before) the last number of euerie Ternarie, is the denomination of it. As for the first Ternarie, the Denomination is vnites; and of the second Ternarie, the Denomination is thousandes: And of the third Ternarie, thousand thousands or Millions: of the fourth, thousand thousand, thousandes, or thousand Millions: and so forth.

Scholer. And what shall I call the value of the thrée figures that may be pronounced before ẏ Denominators? as in saying: 203000000, that is, two hundreth thrée millions, I perceiue by your wordes, that millions is the denomination: but what shall I call the CC ioyned before the millions?

Master. That is called the Numerator or valuer, and the whole summe that resulteth of them both, is called the summe, value or number. Numerator. Summe.

Scholer. Nowe is there any thing else to be learned in Numeration: or else haue I learned it fully?

Master. I might shew you here who were the first inuentors of this Art, and the reason of

all these things that I haue taught you, but that I will reserue till yée haue learned ouer all the practise of this Art, least I should trouble your wit with ouer many thinges at the first.

Three kinds of number. But yet this must you marke, that there are thrée kindes of number: one called Digits: an other Articles, and the thirde mixte numbers.

Digit. A Digit is any number vnder tenne, as this: 1, 2, 3, 4, 5, 6, 7, 8, 9.

Article. And 10 with all other that may be deuided into ten partes iust, and nothing remaine, are called articles: such as are 10, 20, 30, 40 50, &c. 100, 200, &c. 1000, &c.

Mixt or compoūd. And that number is called mixt, that containeth articles, or at the least one article, and a digit: as 12, 16, 19, 21, 38, 107, 1005 and so forth. And for the more ease of vnderstanding and remembrance, marke this: The digit number is neuer written with more than one figure, but the article and the mixt number are euer written with more than one figure. And thus they differ, that the Article hath euermore this Cipher 0, in the first place: and the mixt number hath euer there some digit.

Scholer. By these last wordes, I perceiue it much better than I did before, and now (I thinke) I will neuer misse to know those thrée assunder.

Master.

Maister. If you remember now all that I haue saide, you haue learned sufficiently this first kinde of Arithmetike called Numeration. Howbeit, I will exhort you now, to remember both this that I haue saied, and all that I shall say, and to exercise your selfe in the practise of it: For rules without practise, are but a light knowledge: and practise it is that maketh men perfect and prompt in all things.

And as you haue learned to gather and expresse the value of a summe propounded, and set downe before you: so must you practise to marke, note or write downe with apt figures, and in due places, any number, onely named or recited to you, or of your selfe imagined: as for a proofe. Now note you, or write downe this summe, fiue thousand, two hundreth, fiftie and seuen.

Scholer. This troubleth me nowe, whether I should beginne at the first or at the last. For reason (me thinketh) should cause me to begin at the first: and yet if I write it as you speake it, I must begin at the last.

Master. When you know your places perfectly, you may begin where you list. But the more ease for your hand is to beginne with the last, that is to say, as I did speake them. Yet for the more suretie, a while you may beginne with the first, repeating my words backward thus: Seuen, fiftie, two hundreth, fiue thou-

sand: or else sounding them all by their digit or valuer: as thus: seuen, fiue, two fiue: for that way is easiest. But then must you looke well whether there be any Cipher in your summe, that he may be set in his place. As if your last valuer of your summe (as you call it) be aboue 9, then is there a Cipher in the first place. And if it be a hundred or aboue, then is there two Ciphers, one in the first place, and an other in the second, and so forth.

But because this thing is such that cannot be set foorth without many wordes, I thinke best here now at the end of Numeration to adde a Table easie and ready for the first exercise of it.

Loe this is the Table.

The left side or hand.

X. M. of milliōs.	M. of millions.	C. of millions.	X. of millions.	Millions.	C. of thousands.	X. Thousands.	Thousands.	Hundreds.	Tennes.	Unites.	The denominatours of the place or value vncertaine.
9	9	9	9	9	9	9	9	9	9	9	Nine.
8	8	8	8	8	8	8	8	8	8	8	Eight.
7	7	7	7	7	7	7	7	7	7	7	Seuen.
6	6	6	6	6	6	6	6	6	6	6	Sixe.
5	5	5	5	5	5	5	5	5	5	5	Fiue.
4	4	4	4	4	4	4	4	4	4	4	Foure.
3	3	3	3	3	3	3	3	3	3	3	Three.
2	2	2	2	2	2	2	2	2	2	2	Two.
1	1	1	1	1	1	1	1	1	1	1	One.
0	0	0	0	0	0	0	0	0	0	0	Cipher.
Eleuenth	Tenth.	Ninth.	Eight	Seuenth	Sixt.	Fift.	Fourth.	Third.	Second.	First.	The order of the places.

The right side or hand.
The names of digits, values certaine, or valuers.

This Table (as you may see) hath eleuen places, and in each of them are set all the Digits, whose certaine value is written on the right hand of the Table, and the value vncertaine on the left hand. So that by this Table you may learne both how to expresse any number that you list, (if that excede not

eleuen places) that is to say, X C. thousand millions, and so may you by the helpe of it, value all summes proposed vnder the sayd number.

For example: take the summe that I proposed before, which was fiue thousand, two hundred fifty and seuen. And if you will expresse it, take the first number (as I speake it) which is fiue M. whose valuer or certaine value is v, and his vncertaine value or denomination is M. First you shall seeke at the right hand of the valuer 5. Then seeke along vnder the title of denomination toward the left hand, till you finde thousandes, and vnder it right at the foote of the Table, is the number of the place, that is the fourth, wherin you must write your digit or valuer 5.

Afterward come to the second part of the number, two hundred, whose valuer is 2, and his denomination C. Seeke two at the right hand of the Table, and go along vnder the denomination toward the left hand, till you come vnder C: then looke to the Table, and there you shall see the number of the place, that is to say, three, wherein you must set your digit 2.

Then doe so by your other two numbers that remaine, and you shall finde 5 in the second place for your fifty, and 7 in the first place for your seuen. And thus may you doe with other numbers.

Scholer.

Scholer. Maiſter, I thanke you heartily, I perceiue you ſeeke to inſtruct me moſt plainely and briefly, and not to hide your knowledge with ſubtill wordes as many doe. For this rule is ſo plaine, that I can deſire it no plainer. And though it ſeeme ſomewhat long, yet I perceiue it to be a ſure way.

Maiſter. So is it, and though it be long, yet it is neither too long, neither too plaine for young learners that lacke practiſe: for this Table is in ſtead of a teacher, to them that lacke one. But now I truſt I haue ſayde inough of Numeration: which after you haue well practiſed, then may you learne forth.

Scholer. Yet I pray you in one thing to tell me your iudgement. Why doe men recken the order of the places backeward, from the right hand to the left?

Maiſter. In that thing all men doe agree, that the Chaldees, which firſt inuented this Art, did ſet theſe figures as they ſet all their letters: for they write backward, as you terme it, and ſo doe they reade. And that may appeare in all Hebrue, Chaldee, and Arabike bookes, for they be not onely written from the right hand to the left, and ſo muſt bee read, but alſo the right end of the booke is the beginning of it: whereas the Greekes, Latines, and al Nations of Europe, doe write and read from the left hand toward the right: and all their bookes begin at the left ſide.

Scholer. That reason doth satisfie me.

Maister. It neither satisfieth mee, neyther liketh me well, because I see that the Chaldees and Hebrues doe not so vse their owne numbers, as at an other time I will declare. But this plaine reason may best satisfie you presently: That seeing in pronouncing of numbers we keepe the order of our owne reading, from the left hand to the right: and againe, we doe euer name the greater numbers before the smaller: it was reason, that the lesser places containing the lesser numbers, should be set on the right hand, and the greater places containing the greater numbers, to proceede toward the left hand.

Scholer. This reason is to me so plaine, that it seemeth now against reason to make a doubt of that order. So that now for Numeration I am satisfied: hoping that practise shall make me fully ready and expert in it. And in the meane season, I desire to learne the other kinds of Arithmeticke.

Maister. That is well said: but what should you next learne, can you tell?

Scholer. I remember you sayd that Addition was next.

Maister. Euen so, and what that is, must you first know.

Addition.

Addition is the gathering together and bringing of two numbers or more into one totall summe: as if I haue 160 bookes in the Latine tongue, and 136 in the Greeke tongue, and would know how many they be in all, I must write these two numbers one ouer another, writing the greatest number highest, so that the first figure of the one, be vnder the first figure of the other. And the second vnder the second, and so forth in order.

When you haue so done, draw vnder them a right line, then will they stand thus.

Now begin at the first places toward the right hand alwaies, and put together the 2 first figures of those two numbers, and looke what commeth of them, write vnder them, right vnder the line. As in saying, 6 and 0, is 6, write 6 vnder 6; as thus.

$$\begin{array}{r} 160 \\ 136 \\ \hline \end{array}$$

$$\begin{array}{r} 160 \\ 136 \\ \hline 6 \end{array}$$

And then go to the second figures, and doe likewise: as saying 3 and 6 is 9: write 9 vnder 6 and 3, as here you see.

$$\begin{array}{r} 160 \\ 136 \\ \hline 96 \end{array}$$

And likewise do you with the figures that be in the third place, saying: 1 and 1 be 2: write 2 vnder them, and then will your whole summe

$$\begin{array}{r} 160 \\ 136 \\ \hline 296 \end{array}$$

appeare thus.

So that now you see, that 160, and 136 doe make in all, 296.

Scholer. What: this is verie easie to do: me thinketh I can do it euen since.

There came through Cheapeside two droues of cattell: in the first was 848 sheepe, and in the second was 186 other beastes.

Those two summes I must write as you taught me, thus.

848
186

Then if I put the two first figures together saying: 6 and 8 they make 14. That must I write vnder 6 and 8, thus.

848
186
14

Master. Not so, and here are you twise deceiued. First in going about to adde together two summes of sundrie thinges, which you ought not to do, except you seeke only the number of them, and care not for the things. For the summe that should result of that addition, should be a summe neither of sheepe, nor of other beastes, but a confused summe of both. Howbeit sometimes yee shall haue summes of diuerse denominations to be added, of which I will tell you anon: but first I will shewe you where you were deceiued in another point, and that was in writing fourteene (which came of sixe and eight) vnder sixe and eight: which is vnpossible. For, howe can two figures of two places be written vnder one figure, and one place?

Scholer

Scholer. Truth it is: but yet I did so vnderstand you.

Master. I sayde indéede, that you should write that vnder them, that did result of them both together: which saying is alwaies true, if that summe doe not excéede a digit. But if it be a mixt number, then must you write the digit of it vnder your figures, as I haue saide before: but and if it be an article, then write o vnder them, and in both sorts you shall kéepe the article in your minde. And therefore when you haue added your second figures, which occupie the place of tennes, you shal put that one thereto, which you kepte in your minde: for though it were ten indéede, yet in that place it is but as one, because that euerie 1 of that place, is ten, for that it is the place of tennes. & in like maner: if you haue in the second place so great a number, that it amounteth aboue 9, then write the digit, and reserue the article in your mind, euer adding it to the next place following: and so of all other places, how many soeuer you haue. And if you haue a mixt number, when you haue added your last figures, then write the digit vnder the last figures, and the article in the nexte place beyond them: so shall your number resulting of Addition, haue one place more than the numbers which you should adde together.

Scholer. Now do I perceiue you, and the reason of this is, (as I vnderstand) because

that no one place can containe aboue 9, which is the greatest figure that is, and then all tens or articles must be put to the next place following: for euerie place (as I may see) exceedeth the other place next before him by 10.

Now (if it please you) I will returne to my example of cattell. But I remember you saide, I might not adde summes of sundrie things together, and that I might see by reason.

Master. Trueth it is, if you seeke the due summe of any thing, but if you onely seeke a bare summe, and haue no respect to the thing, then were it better to name the summe onely without any thing, as in saying: 848, without naming sheepe, or any thing else. And likewise 186, naming nothing.

Now let me see: how can you adde those two summes?

Scholer. I must first set them so, that the two first figures stand one ouer another, and the other each one ouer his fellowe of the same place: then shall I drawe a line vnder them both. And so likewise of other figures, setting alwaies the greatest number highest, thus as followeth.

848
186

Then must I adde 6 to 8, which maketh 14, ŷ is mixt number: therfore must I take the digit which is 4, and write it vnder 6 and 8, keeping the Article 1 in my mind thus.

848
186
4

Next that doe I come to the se-

cond

cond figures, adding them together, saying, 8 and 4, make 12; to the which I put the one reserued in my minde, and that maketh 13, of which number I write the digit 3 under 8, and 4, and keepe the article in my minde thus.

848
186
34

Then come I to the thirde figures, saying: 1 and 8, make 9, and 1 in my minde maketh 10. Sir, shall I write the Cipher under 1 and 8?

Maister. Yea.

Scholer. Then of 10 I write the Cipher under 1 and 8, and keepe the article in my minde.

Maister. What needeth that, seeing there followeth no more figures?

Scholer. Sir, I had forgotten, but I will remember better hereafter. Then seeing I am come to the last figures, I must write the Cipher under them, and the article in a further place after the Cipher, thus.

848
186
1034

Maister. So now yee see, that of 848, and 186 added together, there amounteth 1034.

Scholer. Now I thinke I am perfect in Addition.

Master. That will I prooue by this example.

There are two armies of souldiers: in the one are 106800, and in the other 9400. How many are there in both armies, say you?

Scholer. First I set them one ouer another, beginning with the first numbers of the right hand, thus. But the neather number will not match the ouer number.

106800
9400

Maister. That forceth not.

Scholer. Then doe I adde o to o, and there remaineth o, that must I write vnder the first place, thus.

106800
9400
0

Maister. Well sayd.

Scholer. Then likewise in the second place I adde o to o, and there ariseth o, which I write vnder the second place, thus.

106800
9400
00

Then I come to the third place saying: 4 and 8 make 12, of which I write the digit 2, and keepe the article 1 in my minde, thus.

106800
9400
200

Then I adde 9 to 6, which maketh 15, to that I adde the article 1 that was in my minde, and it is 16. I write 6 vnder 6 and 9, and keepe one in minde, thus.

106800
9400
6200

Maister. Why doe you not write both figures, seeing you are come to the last couple of numbers.

Scholer. Nay reason sheweth mee, that I must adde that article that is in my mind, vnto the next figure of the ouer summe, though

there

there be no moꝛe in the neather ſumme.

Maiſter. That is well conſidered: then do ſo.

Scholer. Then ſay I, 0 in the ouer ſumme, and 1 in the minde, maketh 1, that wꝛite I vnder 0: Then followeth there yet one moꝛe in the ouer ſumme, which hath none to be added to it; foꝛ there is none in the neather ſumme, noꝛ yet in my minde, therefoꝛe I thinke I muſt wꝛite that euen as it is.

Maiſter. Yea.

Scholer. Then doth my whole ſumme appeare thus.

106800
9400
116200

Maiſter. If you marke this, you haue learned perfectly the common addition of all ſummes which are of one denomination: ſo that yee obſerue this alſo, that in Addition you muſt haue two numbers at the leaſt, oꝛ elſe how can you ſay that you doe adde? And euer let the greateſt number be wꝛitten higheſt: foꝛ that is the beſt way, though it be not neceſſary.

And foꝛget not this, that if you haue many numbers to adde together, you ſhall haue oftentimes an article of a greater value than 10, ſometimes 20. ſometimes 30, ſometimes moꝛe, yea peraduenture 100. Therefoꝛe as you did with the article 10, ſo doe with them, reſeruing them in your minde, and adding to the number next following, ſo many as their

valuer or value certaine is: that is to say, 2 for 20, 3 for 30,5 for 50, 10 for 100,12 for 120. and so forth of other like. So that if the Article be 100,then must you set downe the 0,and keepe 10 in minde to be caried to the next row of figures or place, if any such happen to come, which for your better understanding take this example for all.

I would adde these xiii.sums	4889
into one, which I set after this	4599
maner:then do I begin and ga-	2299
ther the summe of the first row	3699
of figures which commeth to	2399
107, for I take 9 there x. times	4090
and that is 90, then nine and 8	1099
is 17,that is in all 107,of which	3198
summe I write the 7 under the	299
first row of figures, and then	699
for that 100 is x,tens I keepe x.	899
in mind: which ten I must adde	499
unto the next row of figures	389
which are in the second place,	

which second rowe of figues when they are added together with the x. that I had in my mind, make in all 125, of which summe, I write the digit 5 under the second row, and then for that 120 containeth xii tens. I keepe xii in minde to be added to the third place or row of figues, which being added together make in all 60, the Cipher 0, I set downe under the row of figures in the third place.

And

And the figure 6 I kepe in mind to be added to the rowe of figures in ẏ fourth place, which when they are added together, make 29: The figure or digit 9 I set downe vnder the fourth place, and because it is my last worke: I set downe the 2 also that I haue in minde to the 9 in the fifth place: those sums do make in all 29057.

4889
4599
2299
3699
2399
4099
1099
3198
299
699
899
499
389
29057

¶ But for your more ease in work when you haue an addition of so many summes to be added together, you were best part that summe into two or thrée partes, and worke them seuerall, and so put their additions together, and this were the best thing you could do, when ouer many sums fall to be added.

Scholer. This séemeth somewhat hard, by the reason of so many numbers together. Howbeit I thinke if I doe often proue euen with this same example either by working of it alone, or else by parting it as you said euen now, that I shall be able to do so shortly with any other summe,

Master. So shall you. For it is often practise that maketh a man quicke and ripe in all things. But because as well in great summes as in small, there may chaunce to be some er-

rour, I will teach you howe you shall prooue whether you haue done well or no.

Scholer. That were a great helpe and ease.

The proof of Addition.

Master. Begin first with the highest number, and then to all the other orderly, and adde them together, not hauing regarde to their places; but as though they were all vnites: and still as your number encreaseth aboue 9, cast away 9. Then go forth, euer casting away 9, as often as it amounteth thereto: and so doe till you haue gone ouer all the numbers that you intended first to adde: and whatsoeuer remaineth after such addition and casting away of 9 write it in some voyde place by the end of a line for the better remembrance: and thus is the first parte of your worke prooued. Then secondly put together the Figures that result of the addition vnder the line, still casting away 9 also. And then that that remayneth, write at the other end of that line: and if those two Figures be like, then haue you well done, but if they be vnlike, then haue you missed. As for example in this present summe: The first Figure of the ouer line is 9, let him goe: then 8 and 8 is 16, take away nine, resteth 7, and adde that 7 to 4 that followeth, and it maketh 11, from which if you take nine, there resteth 2. Then come to the next rowe, whose first and second numbers are 9, therefore ouerpasse them both, and take the fiue to the

the 2 which did remaine in the first rowe, that maketh 7, put thereto the 4 following, and that maketh 11, thence take 9, and there remayneth 2. Next unto that goe to the thirde line, whose two first numbers you may lette passe because they are nines, then take the two figures of 2, which with the other two that remayned in the seconde rowe make 6. Then goe to the fourth rowe, whose two first numbers let goe, and take the 6 to the 6 that remayned, and that maketh 12, take away 9, and there resteth 3, which with the 3 that is next, maketh 6. And so goe through all the other numbers, and you shall finde that there remayneth 5, after you haue cast away 9 as often as you canne finde it: therefore write 5 at the ende of a lyne in a boyde place thus.

5————————

Then gather all the Figures of the totall summe which is under the lowest line, and caste away 9 as often as you can finde it, as thus: 7 and 5 make 12, take away 9, there resteth 3: so that if you adde the 2 that is last (for you may omit the 9) then doeth it make 5, which 5 you must write at the other ende of the line that you made in the boyde place thus.

5————————5

And then you see that those two figures be like, whereby you may know that you haue done well, and so may you proue in any other.

Scholer. If it please you, I will proue in

another summe.

Master. With a good will.

Scholer. Then will I take one of your former examples, which was this.

First in the highest line, 8 and 6 make 14, then 9 taken away, there remaine 5, to which I adde the 1 that followeth, & that maketh 6. Then come I to the second line, where I find first 4 which with 6 maketh 10, from that I take 9, and there resteth 1, the next figure is 0, and therefore I let him alone, so finde I one remaining, which I set at the end of a line thus. 1 ———

106800
9400
116200

Then I come to the totall summe, and there I finde that all the figures put together make tenne, from which I take nine, and there resteth 1 also, which I put at the other end of the line thus. 1 ——— 1

And because they be like, I know that I have well added

Master. So you knowe nowe both howe to adde two summes or more together, and also how to prove whether you have done well or no: and nowe will I teach you how to adde summes of diverse denominations together: which thinge can never be but when the one denomination is such that it containeth the other certaine times. And yet you shall adde them to the other, not after this sort as you did them that were of one denomination, but after

such

ſuch a ſort as I will now ſhew you, that is to ſay.

If you haue a ſumme of diuerſe denominations, then looke that ye ſet euery denominations by himſelfe, with ſome note or figure of his denomination, as they be woont to be written. Then write your other ſummes ſo vnder that firſt, that euery one be ſet vnder the other of the ſame denominations, as for example: if your denominations be poundes, ſhillings, and pence, write poundes vnder poundes, ſhillings vnder ſhillings, and pence vnder pence, and not ſhillings vnder pence, nor pence vnder pounds.

Scholer. Now that you haue ſpoken it, me thinketh it needeth not to warne me of it, for it were againſt reaſon ſo to confound ſummes: but yet if you had not ſpoken of it, peraduenture I ſhould haue bene deceiued in it.

Maſter. If you doe ſay it is ſo plaine, I will ſpeak no more of it, but with an example make the matter to appeare euidently.

Firſt, one man oweth mee 22 li, 6 s, 8 d. An other oweth me 45 li, 16 s, 6 d. And an other oweth me 4 li, 3 s. I would know what this is altogether. Therefore muſt I firſt ſet downe my greater ſumme, and then the other, euery one vnder his denomination agreeing to the greateſt ſumme, as here you ſee with a

li	s	d
22	6	8
45	16	6
4	3	

line vnder them.

Then muſt I beginne at ſmalleſt numbers, (which muſt alwaies be ſet next the right hand) and adde them together, and if the ſumme of them will make 1, or 2, or 3, of the next denomination, then muſt I keepe it in my minde till I come to that place, and vnder that firſt place muſt I note the reſidue, if there remaine any of the ſame denomination, but if there remaine none, then neede I to write vnder it nothing. And this is all that you muſt marke in this Addition: for all other thinges are like to the other manner of Addition before mentioned: Therefore the chiefeſt point of this addition is, to know the values of common Coines and rated ſummes. As how many ſhillings bee in a pound: how many pence in a ſhilling, of which & of other like things I will inſtruct you hereafter in teaching of reduction: But now I may not diſturbe your wit from the thing that we are about.

Therefore let vs returne to that former example which I propoſed of three debters, which ſummes when I had ſet orderly, they ſtoode thus with a line vnder them.

l	s	d
22	6	8
45	16	6
4	3	

Then to adde them vnto one ſumme, I muſt beginne at the right hand, where the ſmalleſt denomination is, and adde them together firſt, ſaying: 6 and 8 make 14. Now ſeeing theſe

14 are pence, which containe one shilling and 2 pence. The 2 pence I set downe under the line of pence. And the one shilling I keepe in my mind to cary to the next row being the place of shillings.

l	s	d
22	6	8
45	16	6
4	3	
		2

Then doe I adde the shillings together, saying, 1 in minde and 3 make 4, and 6 make 10, and 6 make 16, and 1 in the second place which standeth for 10, make 26, which is 1 pound, 6 shillings. The 6 shillings I set down under the place of shillings as appeareth in the example. And the one pound I keep to cary to the pounds.

l	s	d
22	6	8
45	16	6
4	3	
	6	2

Then come I to the poundes, adding them altogether, saying 1 that I keepe, and 4 make 5, and 5 make 10, and 2 make 12. The figure or Digit 2 I set downe right under that place or row of poundes where I gather them. And the Article 1 I keepe to cary to the next place, saying 1 in minde, and 4 is 5, and 2 is 7, which 7 I set downe directly under that row also. And then appeareth my whole summe thus.

l	s	d
22	6	8
45	16	6
4	4	
72	6	2

And thus must you doe with any such like, summes whatsoever, whether they be money

weight or measure, which if you practise your selfe well therein by setting downe of diuerse summes, you shall be wel acquainted with the feate of Addition.

But now can you tell how to proue this addition or such other like of diuerse denominations, and to try whether you haue well done or no?

Scholer. I would I could.

Maister. That shall you do by this meanes.

Proofe of addition of diuerse denominations.

You must make a crosse which shall haue so many lines as you haue sundry denominations in your Addition: As if you haue but two Denominations, then you may make it thus, that the ouer parte and the neather part may serue for one denominatiō. And if you haue thrée denominations, as poundes, shillings and pence, then must you make thrée lines thus. The vpright line may serue for pounds and the highest thwart line for shillings, & the lowest for pence: as for example by our summe which we last wrought.

l	s	d
22	6	8
45	16	6
4	3	0
72	6	2

0

6 — | — 6

2 — | — 2

0

For

For the prœfe of the which, because it containeth thrée denominations, I must make a crosse of thrée lines, as in the example before. Then I reckon first at the right hand the pence: 6 and 8 make 14, from which I take 12 for the next denomination, that is to say a shilling, & there resteth 2, which I must write at one end of the neather thwart line.

After that I gather the summe of the shillings, 3, 16, 6, which maketh 25, to them I put 1 that I tooke of the pence, and that maketh 26: from those I take 20, the quantitie of the next greater denomination, that is to saye, a pound, and there resteth 6, which I write at the end of the highest thwart line.

Thirdly, I adde together the pounds 4, 5, and 2, which make 11, to them I adde the one that came of the shillings, and they make 12, from whence I cast nine, and there resteth 3. That thrée I ioyne to the 4 in the next place, and they make 7, then the 2 more make nine, which I cast away, and so there be left 0, which 0 I set at the upper end of the crosse also. (And so shoulde there be set also any number that were left vnder 9.) And thus is my first part of my worke prœued.

That done, I come to the totall summe vnder the line, and examine it, beginning at the pence, where I finde but two, and cannot take nine from him, therefore I set him at the other end of the neather thwart line: Then I

come to the shillings, where I finde onely 6, which because it is lesse than 9, I set it at the other ende of the line of shillings, that is, the ouermost thwart line.

Last of all, of the 72 £, I take eight times 9, which is 72, and there remaineth 0, which I write vnder the vpright line, either else I may reckon them simplie without any respect of their valuation or place: in saying 2 and 7 make 9, which I take alway, and so resteth 0.

Then I consider euerie number, comparing it to the number that is against it, and because I finde them to be euerie one like his match, I know that I haue well done.

Scholer. This crosse I perceiue doth serue for these three denominations, pounds, shillings, pence. But what if I had £, s, d, ob. and q.

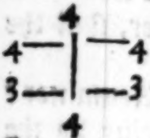
Maister. These lines as I haue sayed, doe serue for three denominations, such as they be: as here they do serue for pounds, shillings, and pence: but if you haue no pounds in your sum, then may they serue for shillings, pence and halfepenies: yea for d, ob. and q.: or in weight for C. q. and £, or in measure for Elles, quarters, and nailes, if you haue no greater denomination, so that you remember that the vpright line serueth for the greatest denomination, and the highest thwart line for the next, and the lowest for the least.

And so if you haue foure Denominations you

you must make your crosse with so many lines. And if that your summe be of more denominations, make so many lines in your crosse. And thus will I make an ende of Addition, sauing that here for the better vnderstanding of this Rule, I haue sette you downe certayne examples both of money, weight, and measures, with their workes and proofes.

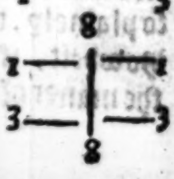
Examples of Addition.

l	s	d	l	s	d
23	10	4	130	17	10
45	6	8	28	6	8
37	2	9	13	13	4
15	13	6	120	0	0
121	13	3	292	17	10



The proofes.

C	q	l	yards	q	nailes
24	1	3	17	3	3
12	2	2	35	2	1
7	3	4	26	1	3
13	0	13	54	2	0
57	2	22	134	1	3



Subtraction.

Scholer.

Then haue I learned the two first kindes of Arithmetik: now as I remember, doth follow Subtraction, whose name me thinketh doth sound contrarie to Addition.

Master. Soe is it indeede: for as Addition encreaseth one grosse summe by bringing many into one, so contrarie wayes Subtraction diminisheth a grosse summe by withdrawing of other from it, so that Subtraction or Rebating is nothing else, but an Art to withdraw and abate one summe from an other, that the Remainer may appeare.

Scholer. What doe you call the Remainer?

Master. That you may perceiue by the name.

Scholer. So me thinketh: but yet it is good to aske the truth of all such things, least in trusting to mine owne coniecture, I be deceiued.

Master. So it is the surest way. And as I see cause, I will still declare things vnto you so plainely, that you shall not neede to doubt. Howbeit, if I do ouerpasse it sometimes (as the maner of men is to forget the small know-

ledge

ledge of them to whom they speake; then doe you put me in remembraunce your selfe, and that way is surest.

And as for this word that you last asked me, take you this description: The Remainer is a summe left after due subtraction made, which declareth the excesse or difference of the other two numbers: as if I would abate or subtract 14 out of 18, there should remaine 4, which is called the Remainer, and is the difference betwéene those two numbers 14 and 18. Remainer.

Scholer. I perceiue then what Subtraction is: Now resteth to know the order to worke it.

Maister. That shall you doe by this means. First you must consider, that if you should goe about to rebate, you must haue two sundrie summes proposed, the first which is your grosse summe or summe total: and it must be set highest) and then the rebatement or summe to be withdrawen, which must be set vnder the first (whether it be in one parcell or in many) and that in such sort that the first figures be one iust ouer another. and so the second and third, and all other following, as you did in Addition: thẽ shal you draw vnder thẽ a line, and so are your summes duely set to beginne your working.

Then begin you at the right hand (as you did in Addition) and withdraw the neather number out of the higher, and if there remaine

maine any thing, write that right vnder them beneath the line: and if there remaine nothing (by reaſon that the two figures were equall) then write vnder them a Cipher of nought. And ſo doe you with all the other figures, euermore abating the lower out of the higher, and write vnder them the Remainer ſtill, till you come to the end. And ſo will there appeare vnder the line what remaineth of your groſſe ſumme, after you haue reduced the other ſumme from it, as in this example.

I receiued of your father 48 s̃, of which I haue laid out for you 36 s̃: now would I know what doth remaine, and therefore I ſet my numbers thus in order: Firſt I write the greateſt ſumme, and vnder him the leſſer, ſo that the figures at the right ſide be euen one vnder another, and ſo the other thus.

s̃
48
36

Then do I rebate 6 out of 8, & there reſteth two, which I write vnder them right beneath the line, thus.

48
36
2

Then I go to the ſecond figures, and do rebate 3 out of 4, where there remaineth 1, which I write vnder them right, & then the whole ſum & operation appeareth thus.

48
36
12

Whereby it appeareth, that if I withdraw

36 out of 48, there remaineth 12.

Scholer. Nowe will I prooue in a greater ſumme: And I will ſubtract 2367924 out of 3468946. Thoſe ſums I ſet in order thus.

3468946
2367924

Then do I beginne at the right ſide, and deduct 4 out of 6, and there reſteth 2, which I write vnder them. Then goe I to the ſeconde figures, and withdraw 2 out of 4, and there remaine two, which I ſet vnder them alſo: then I take 9, out of 9, and there reſteth 0, which I write vnder them: for you ſay, that if the figures be equall, ſo that nothing doe remaine, I muſt write this Cypher 0 vnder them.

Maſter. It was well remembred, nowe go forth.

Scholer. Then I come to the fourth place and draw 7 out of 8, and there remaineth 1, which I write vnder them alſo. Then in the fifth place I take 6 out of 6, and there reſteth nought, for it I write vnder them a Cypher, 0: Then in the 6 place 3 rebated from 4, there remaineth 1, which I write vnder them: and likewiſe in the vii and the laſt place, 2 taken from 3, there is lefte one, which I write vnder them: ſo haue I doone my whole working, and my ſummes appeare thus. Whereby

3468946
2367924
1101022

I see, that if I do rebate 2367924, out of 3468946, there remaineth 1101022.

Master. This is well done. And that you may be sure to perceiue fully the Arte of Subtraction, let me see howe canne you subtract 52984732 out of 825000345б.

Scholer. First I sette downe the greatest summe, and after that I write under it the lesser number, beginning at the right side: and then my figures will stand thus.

8	2 5 0 0 0 3 4 5 6
	5 2 9 8 4 7 3 2

Then take I 2 from 6, and the rest is 4, which I write under them: then do I withdraw 3 from 5, and there remaine 2, which I write under them. Then take I 7 out of 4, but that I cannot, what shall I now do?

Note. Master. Marke well what I shall tell you nowe, howe you shall do in this case and in all other the like. If any figure of the neather summe be greater than the figure of the summe that is over him, so that it cannot be taken out of the figure over him; then must you put 10 to the over figure, and then consider how much it is, and out of that whole summe withdraw the neather figure, and write the rest under them. Can you remember this?

Scholer. Yes, that I trust I shall. Now then in mine example where I should haue taken 7 out of 4, and could not, I put 10 to that

that 4, which maketh 14, from it I take away 7: and there resteth 7 also, which I write vnder them.

Maister. So haue you done well, but now must you marke an other thing also: that whensoeuer you doe so put 10 to any figure of the ouer number, you must adde one still to the figure or place that followeth next in the neather line, as in this example there followeth 4, to which you must put 1, and make him 5, and then goe on as I haue taught you. Note.

Scholer. Then shall I say: 4 and 1 (which I must put to him for the 10 that I added to 4 before) make 5, which I should take out of 3, but that cannot bee, therefore must I put to it also 10, and then it will bee 13, from which I take 5, and there resteth 8 to bee written vnder them: and because of that 10 added to the 3, I must adde 1 to 8 that followeth in the neather line, and that maketh 9, which I should take out of 0 and cannot, therefore I put thereto 10, and that maketh 10: from 10 I take 9, and there remaineth 1, which I write vnder them.

Then doe I adde one likewise to the next figure beneath, which is 9, and that maketh 10, that 10 should I take out of the figure aboue, but I cannot, for it is 0, therefore I put 10 to it, and so take I 10 out of 10, and there resteth 0 to be written vnder them.

Then come I to the next figure, which is 2, and to him doe I adde 1, which maketh 3, that 3 I can not take out of nought, therefore of that nought I make 10, and thence do I take 3, so remaineth there 7 to bee written vnder them. Likewise doe I put 1 to 5 that followeth, and then is it 6, that would I take out of 5, and cannot, therefore I adde 10, to that 5 and make it 15, from which I rebate 6, and there remaineth 9, which I write vnder them. Now haue I spent all the neather figures, and what shall I doe more?

Maister. You should haue added 1 to the next figure following (if there had beene any, because you added 10 to the last figure before of the ouer line: but seeing there is no figure following, you must adde that 1 to the place following, and then deduct that 1 from the number aboue.

Scholer. Then shall I say because I borrowed 10 to the ouer 5, I must put 1 in the next place beneath, that is vnder 2: then must I subtract that 1 from 2, and there resteth 1, to bee written vnder that, in the ninth place. Now I haue no more to subtract, for there is not any figure remaining beneath, neither yet any vnite to be added, because I borrowed not 10 to the figure last before, and yet is there 8 remaining in the ouer line, which (I thinke by reason) shoulde bee sette at the end of the Figures in the lowest rowe which is vnder the

the line, for because there was nothing taken from it.

Maister. That is well considered, and reason teacheth so in deede.

Scholer. But sir, I beseech you, shall I alwaies when any number so remaineth alone (as this 8 did) write him vnder the line straight against his owne place?

Maister. Yea, what else? Whether they bee one or many: and this well remembred, you haue sufficiently learned Subtraction. Howbeit, because of certaine thinges that might deceiue you, if you did not take good heede to your working, I will propose to you an other example of many numbers to bee subtracted, as thus.

I receiued of a friend of mine to keepe 2869 crownes, of which at one time I deliuered him againe 500, at an other time 368, and at an other time 440, and at an other time 80, and an other time 64: now would I know how manie doe rest behinde. Therefore first I set downe my grosse summe and a line vnder it: and vnderneath it I set all the parcels, thus: and vnder them a double line.

2869	*Crownes receiued.*
500	
368	
440	*Deliuered.*
80	
64	

Then first I begin at the first place, and gather together the summe

of all those lines (saue the outermost) in their first figures, and so I doe with al the figures of the second place, and so forth, as I did in Addition, saue that I leaue out the highest rowe of numbers (as the line warneth me) and that summe so gathered betweene the double line, is the summe deliuered in all, which summe I doe afterwardes subtract out of the highest row of numbers, and the remainer doe I set vnder the neathermost line: as for example.

Note.

I set the summes as before: then doe I gather the first figures of all the places deliuered together, where I finde but 4 and 8, that make 12 (for three Ciphers encreaseth no summe in Addition, as you learned before) of the 12 therefore doe I write the digit 2 betweene the double line, and keepe the article 1 in my minde, till I come to the second place, where I finde 6, 8, 4, 6, that make 24, to them I put the article in my minde, and it is 25, of which I write 5 vnder the second place, and keepe the digit 2 in my minde for the third place, where I finde 4, 3, 5, that make 12, to the which I adde the 2 in my minde, and it maketh 14, thereof I write the 4 vnder the third place;

2869	*Crownes receiued.*
500	
368	
440	*Deliuered.*
80	
64	
1452	*Deliuered in all.*
1417	*Rest behinde.*

place: and because there remaineth no more figures to be added, I write the digit 1 in the fourth place, as you see in the example, and so it appeareth I haue deliuered in all a thousand foure hundreth fiftie two crownes.

Then come I vnto the subtracting of this summe betweene the lines, for by Addition it is equall to the fiue parcels ouer it. Therefore I proceed to subtract it from the ouermost summe, saying: 2 from 9, remaine 7, to bee written vnder them beneath the lowest line. Then in the seconde place I take 5 from 6, and there resteth 1, to be written vnder them. Then in the thirde place, 4 from 8 resteth 4. Last of all in the fourth place, 1 from 2 remaineth 1. And thus I see that after those 5 sums are subtracted from 2869, the Remainer is 1417.

Master. This is a verie sure and easie way for a learner: therefore I counsell you to practise it well.

Scholer. This I perceiue: but is there no shorter way and more speedie?

Master. Yes, when you are a while exercised in it: for you may, as fast as you can gather the numbers together, withdrawe them out of the highest summe. But if in quantitie those numbers added together, exceede the highest summe or vpper number then shall you (as before hath bin taught you) imagine to borrowe 10, 20, or 30, or mo, as neede shall re- An abridgment of the former manner of subtraction.

quire. and put them to the upper number, to helpe to further the abatement, reserving or restoring the Articles that you borrowed to the next place againe, and so still go forward till you haue ended your worke, as for example: In the last summe proposed, I gather first in the first place 4 and 8, that maketh twelue: which 12 I should deduct or take out of 9 in the upper number aboue the line. But I cannot, therefore I adde unto 9 an Article of 10, and that maketh the upper nnmber 19 : from whence I take 12, then there resteth 7. Then for the Article 10, I adde 1 to the nexte place of money deliuered: saying, 1 that I bring, and 6 make 7, and 8 make 15, and 4 make 19, and 6 make 25, which 25 I should take out of 6 in the upper number, but I cannot. Therefore I adde 2 tennes or 20 unto 6 in the upper number, and that maketh 26, then 25 out of 26 resteth 1, then the two tennes which I borrowed, or haue in mind, I adde to the next row, or summe deliuered saying: 2 that I bring, and 4 make 6, and 3 make 9, and 5 make 14, then 14 out of 8 I cannot take, but 14 out of 18 resteth 4. Now because there are no more places to be added, the one that I borrowed or haue in my mind, I rebate from 2 in the upper line, and there remaineth 1 which I set downe in the remainer line, and so my summe appeareth, as before, to be 1452 crownes. Lo thus haue you now a shorter way.

Scho-

Scholer. I like both wayes well, and I perceiue both well, yet as in the one the working ſeemeth ſomewhat long, ſo in the other it leaneth verie much (mee ſeemeth) to remembrance, and therefore may cauſe errour quickly, except a man haue a quicke and an exerciſed remembrance. But yet for the ſharpning of my witte, by your patience, if you will giue me leaue, I will trie what I can doe in ſuch a like ſumme, to worke it the ſhorteſt waye: whereuppon I would ſubtract out of 40308964, theſe three parcels

20003428
10002342
10101461

	40308964
	20003428
	10002342
	10101461

Therfore I ſet them firſt in due order: and then I gather the parcelles of the firſt place, which are, 8, 2, 1, that is 11, which I ſhould take or deduct out of 4, which is ouer him, but I cannot, therfore I adde an Article or one ten vnto 4, which maketh 14, then 11 out of 14, there reſteth 3 to be written vnder the firſt place betweene the two lines.

Then come I to the ſecond place, ſaying: 1 that I borrowed or haue in mind, and 6 make 7, and 4 make 11, and 2 make 13, which I cannot take from 16, therefore I adde 10 to 6, which maketh 16, & then 13 from 16, reſteth 3, which I write vnder the ſecond place betweene

the two lines.

Then come I to the third place, saying: one that I borrowed or haue in minde, and 4 make 5, and 3 is 8, and 4 make 12, which I should take out of 9 that is ouer them, but I cannot; therefore I adde 10 to 9, which make 19, then 12 out of 19, resteth 7.

Then come I to the fourth place, saying: one in minde and 1 is 2, and 2 make 4, and 3 make 7, which because it is a digit, I take from 8, and there remaineth 1.

After that I come to the fifth place, where are onely three Cyphers, which make nothing, then should I take that, that is to say nothing, from the figure ouer them, which is also a Cypher, therefore I must say thus: if I take nought from nought, there remaineth nought: so must I write a cipher vnder them. Then in the sixth place I finde but 1, which I take out of 3 ouer him, and the Remainer is 2, that must be written betweene the two lines in the sixt place. So I goe to the seuenth, where I finde onely Cyphers, and in the grosse summe ouer them a cipher also, therefore must I write their Remainer (which is nothing) with a Cypher also. Then in the eight and last place, I gather 1, 1, 2, that make 4, which if I take out of that 4 that is ouer them, there will nothing remaine. And that must be noted with a cypher betweene the two lines, as I haue often said, and so haue I ended my worke: and

the

the figures stand thus.

But Sir, I remember you taught mee that Ciphers should not come in the last place, for because they serue onely to encrease the value of other figures which follow the, & serue not those figures that goe before them: and now in my example I haue set two ciphers in the two last places.

40308964	
20003428	
10002342	
10101461	
00201733	

Maister. I commend you for your remembrance. And trueth it is, you should not haue set them here, but onely because that I would make you plainely to perceiue the Art of Subtraction. Therefore seeing that you doe now perceiue it, whensoeuer you would write down a Cipher, looke whether any other figures bee yet behinde. And if not, then let go the Cipher also, for it needeth not to write him in any latter places, where no other figure doth follow, except it bee (as I did now suffer you) to teach the vse of Subtraction the plainer.

Therefore your figures must stand thus when the worke is ended.

40308964	Charge.
20003428	Discharges
10002342	
10101461	
201733	

Scholer. Sir I do think with that that you taught mee before and by these two summes that you

taught me last also, that now I could subtract any summe.

Maister. So may you, if you haue marked what I haue taught you. But because this thing (as all other) must be learned surely by often practise, I will propound here two examples to you, wherein if you often exercise your selfe, you shall be ripe and perfect to subtract any other summe lightly; for in them is contained all the obseruances of whole number. And because you shall perceiue somewhat both how to doe it, and also whether it bee well done when you haue prooued to doe it, therefore haue I written vnder them both the Remainers.

30606	*Lent.*	30606	*Debt.*
10354 10249 163	*Repaide.*	10354 10249 163	*Paide.*
20766	*Paid in all.*	9840	*Rest.*
9840	*Rest to pay.*		

Scholer. Sir, I thanke you. But I thinke I might the better doe it, if you did shew mee the working of it.

Maister. Yea, but you must prooue your selfe to doe some things without my aide, or else you shall not be able to doe any more than you were taught: And that were rather to

learne

learne by rote (as they call it) than by reason. And againe there is nothing in this example or any other of whole number, but I haue taught you the rules of them already, sauing I haue not yet shewed you how you shall prooue this worke.

Scholer. Sir I pray you then shew it mee also.

Maister. For the performance whereof if you marke well what I sayd in Addition, you may easily perceiue what is to bee done for the proofe of Subtraction, and that is only performed by the aide of Addition, thus. Draw vnder the lowest number (which is your Remainer) a line, and then adde the summe paid in all (which before was made by the particulars) and the remainer together. And if these twaine being added together in one summe, do make the contents of the vpper number aboue the line, then is the Subtraction well wrought, or else not.

For the triall whereof I will propone one of our examples which are done, which was of 2869 crownes receiued to keepe of a friend.

Crownes	2869	*Receiued.*
	500	
Of which at sundry	388	
times I deliuered	440	
these particular sums	80	
	64	

which particulars in one Addition make, 1452. *deliuered in all.*
so resteth to my friend. 1417 *Rest to pay.*
2869. *Proofe.*

The proofe of Subtraction.

Now put or adde this Remainer, and the summe deliuered together, saying: 7 and 2 make 9, and 1 and 5 make 6, and 4 and 4 is 8, and 1 and 1 is 2, which in one summe amounteth to 2869, and is equall with the summe 2869 aboue the line, whereby I know the subtraction is well wrought, and this is the order and proofe hereof, and of all such like workes of subtraction.

Scholer. Then I trust by practise to attaine the vse of it. And is this all that I shall learne of Subtraction?

Maister. Yea, sauing that (as you haue seene in Addition) there are numbers of diuerse denominations, in which the working is not much vnlike, yet without some instructions be giuen of it, it might seeme to a learner more difficult, than in deede it is. Therefore I will briefly shew you the vse of it onely by one example or two.

A certaine man owed to me 14 l, 12 s, 8 d, of which he paide me at one time 4 l, 6 s, 8 d, at another time 3 l, and at an other 2 l, 3 s, 4 d, and last of all, 6 s, 8 d.

Now would I know what remaineth vnpaide yet, therefore I set my summes thus, euery

John Lucas his booke may the 5th 1686

uerie one in their due place: As pounds under pounds, shillings under shillings, pence under pence.

li	s	d
14	12	8
4	6	8
3	0	0
2	3	4
	6	8

Scholer. Syr, I pray you why do you write 2 li, for the common speach useth rather to say 40 s.

Maister. We must here use the Denomination that is greatest in any summe, so that we may not write according as we use to speake, saying: 16 d, 18 d, or likewise, 7 grotes, 8 grotes, 24 s, 40 s, 48 s, and such other, but we must write euerie Denomination that is in any summe by it selfe, namely shillings and pounds. So must we write for the last summes now named, 1 s, 4 d 1 s, 6 d: 2 s, 4 d: 2 s 8 d: 1 li 4 s: 2 li 8 s: and so forth of other like.

Note how the pen differeth from common order of counters.

Scholer. So that we may not write in Arithmetike pence, when the summe amounteth to shillings, nor shillings when the summe maketh pounds. Now if it please you, end your example.

Maister. When my summes are so set as I shewed, then according to the rules of Addition I gather all the particular summes which he payed me into one totall summe: directly to be set under them betweene the two lines, not medling with the, 14 li, 12 s, 8 d, as the line warneth me, therefore must I first begin with

the smallest denomination, saying, 8, 4, 8, is 20 d, which maketh one shilling and 8 pence: the 8 d I set downe under the place of pence, and the one shilling I keepe in minde to carrie to the next denominations of shillings. Then come I to the shillings, and saye: one that I bring or have in minde, and 6 is 7, and 3 is 10, and 6 make 16, which because it containeth not one pound, I set directly under the place of shillings. Then come I to the pounds whose parcels are 2, 3, 4, that is in all 9, that 9 do I set downe directly under the poundes: And so the totall or whole Addition of all the particulars paide, amounteth to 9 li, 16 s, 8 d.

li	s	d	
14	12	8	
4	6	8	
3			
2	3	4	
	6	8	
9	16	8	
4	16	0	*Rest.*

Now for the worke of Subtraction I must rebate that totall summe of Addition out of the higher number, that is to say, from the 14 li, 12 s, 8 d.

Therefore to performe the work, I say 8 pence out of 8 d remaineth or resteth nothing, therefore in the place of the rest or remaine, right under that denomination I set downe a 0. Then comming to the shillings, where I finde 16 which should be taken out of 12, but I cannot, therefore I imagine to borrow one of the next

de-

denomination, that is of the 9 l, and put that one pound so borrowed unto 12 s, that maketh 32 s.

Now 16 s out of 32 s resteth 16 s, which 16 s I set downe directly under the place of the rest.

Lastly comming to the pounds, saying, one pound in minde that I borrowed, and 9 make 10, then 10 out of 14, there resteth 4.

So doeth my whole rest or remaine appeare to be 4 l, 16 s, 0 d.

This I account the easiest way for a young beginner to practise, though it bee something long.

Scholer. Is there then any shorter way for this worke also?

Maister. Yes, as in this last example I will also shew you, for you may adde together the particular summes as they are set in order: beginning with the pence saying, 8, 4, 8, make 20 d, which 20 d you should take out of the 8 d above the line, but you cānot: therefore shall you borrow one of the next denomination, that is to say, one of the shillings, and put it to the 8 d that maketh 20 d, now 20 d out of 20 d, rest 0, which Cipher I set downe directly under them.

l	s	d
14	12	8
4	6	8
3	0	0
2	3	4
0	6	8
4	16	0

Then one shilling that I borrowed as had

in minde, and 6 make 7, and 3 make 10, and 6 make 16, then 16 out of 12 I cannot take, therfore of the next denomination I do borrow one pound, and put it to 12 s. which maketh 32 s, then 16 s out of 32 s, resteth 16 s.

Lastly I come to the pounds, saying: one pound in mind or that I borrowed, and 2 make 3, and 3 is 6, and 4 is 10, then 10 out of 14, there resteth 4.

So doth my remaine or rest appeare as before to be 4 l, 16 s, 0 d.

Scholer. This doe I perceiue very well, and if there be none other thing to bee learned in Subtraction, then may I come to Multiplication, for that you reckened to bee the next order.

Maister. Wee haue done in deede with the Art of Subtraction, as touching the working. But yet before wee goe to Multiplication, I will instruct you how to examine your worke whether it bee well done or not. For the performance whereof, if you marke what I sayde right now in the last manner of prowfe, you may easily perceiue what is to be done for this prowfe, which is onely made (as before was taught you) by the aide of Addition, thus.

The proof of Subtraction, in diuerse denominations.

Draw vnder the lowest number (which is your Remainer) a line, and then adde the summe paide in all, and the Remainer together. And if these twaine added together in one summe make the contentes of the vpper num-

number aboue the line, which in this example is 14 l, 12 s, 8 d. Then is the Subtraction well wrought or else not.

As for example, in our first summe which stood thus.

l	s	d	
14	12	8	
4	6	8	
3	0	0	
2	3	4	
	6	8	
9	16	8.	*Paid in all.*
4	16	0	*Rest.*
14	12	8	

Where in the title of pence I find 8 and 0: the 8 I set downe directly vnder him in the place of pence.

Then in the place of shillings I finde 16 and 16, which make 32 shillings, wherein is contained one pound and 12 shillings, the 12 s I set downe directly vnder them in their due place of shillings, and one pound I keepe.

Then comming to the pounds, saying: one that I keepe, and 4 is fiue, and 9 is 14, which 14 in due order I set downe directly vnder them, as this figure sheweth. Which is also 14 pound 12 s, 8 d, agreeing with the vpper number aboue. So I finde the work is good, and the Subtraction well wrought.

Now for the proofe of the latter Subtraction which we reckoned for the shorter worke. Drawe vnder the Remainer also a line, then adde that remainer and all the other numbers that ye did subtract before together, and write that that amounteth vnder the lowest line.

And if the summe that commeth thereof be equall to the highest number aboue, then is the Subtraction well wrought, or else not.

As for example also in the last summes which stood thus.

l	s	d
14	12	8
4	6	8
3	0	0
2	3	4
0	6	8
4	16	0
14	12	8

First in the title of pence I adde 8, 4, 8, that maketh 20 d, which containeth one shilling and eight pence.

The 8 I set downe vnder the lowest line in the row or title of pence, and the one shilling I kéepe to carie to the next denomination or place of shillings.

Then returning to the shillings, saying: one in minde, or that I kéepe, and 16 make 17, and 6 make 23, and thrée make 26, and 6 make 32 shillings, which amounteth to one pound, 12 shillings, the 12 shillings I set downe vnder the title of shillings, and one pound I kéepe or haue in minde to carie to the next denomination or place of poundes. Then come I to the poundes, saying, one that I bring, and foure make 5, and two make 7, and thrée is 10, and 4 make 14, then do I write 14 vnder the pounds, and so haue I ended the Addition, and I sée that the lowest line is like vnto the vppermost line in number, wherefore I know that I haue well done.

And thus now haue I taught you the Arte of Subtraction, and the meanes to proue whether

ther it be well wrought or not.

Many other workes and proofes might be shewed of Subtraction, but one of these, whether of them you please, are most apt and best allowed of any other worke or proofe, whether it be of l, s̃, d, or any other grosse summe whatsoeuer.

Scholer. Syr, I thanke you most heartily, for nowe I vnderstand well both the feate of Subtraction and his proofe.

Master. Therefore heere will I make an end of Subtraction, and will instruct you in Multiplication.

Multiplication.

Multiplicatiō is such an operation, that two summes produce the third: which third summe so many times shall containe the firste, as there are vnites in the second. And it serueth in stead of many Additions. As for example. When I would know how many are thirtie times forty eight: if I should adde 48 thirtie times, it would be a long worke. Therefore was this worke of Multiplication deuised, which shall do ỹ at once

that Addition ſhould do at many times.

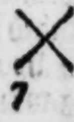
Scholer. I perceiue the commoditie of it partly, but I ſhall not ſee the full profit of it, till I know the whole vſe of it. Therefore ſir I beſeech you teach me the working of it.

Maſter. So I iudge it beſt, but becauſe that great ſummes can not be multiplied, but by the multiplication of digits, therefore I thinke it beſt to ſhew you firſt the way of multiplying them: As when I ſay: 8 times 8, or 8 times 9 and ſo forth. And as for the ſmall digits vnder 5, it were but follie to teach any rule, ſeeing they are ſo eaſie, that euerie child can do it. But for the multiplication of the greater digits, thus ſhall you do.

Firſt ſet your digits one ouer the other right, then from the vppermoſt downward, and from the neathermoſte vpward, drawe ſtraight lines, ſo that they make a croſſe, commonly called Saint Andrewes croſſe, as you ſee heere. Then looke how many each of them lacketh of 10, and write that againſt each of them at the ende of the lines, and that is called the difference: as if I would knowe howe many are 7 times 8, I muſt write thoſe Digits thus.

Digit difference.

8



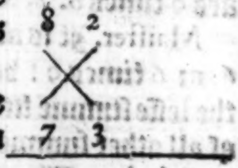
7

Then doe I looke howe much 8 doeth differ from 10 and I finde it to be 2, that 2

do

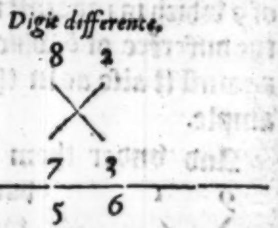
doe I write at the right hand of 8, at the end of the line thus.

After that I take the difference of 7, likewiſe from 10, that is 3, and I write that at the right ſide of 7, as you ſee in this example.



Then doe I draw a line vnder them, as in Addition thus.

Laſt of all, I multiply the two differences, ſaying, 2 times 3 make 6, that muſt I euer ſet vnder the differences, beneath the line: then muſt I take one of the differences (which I will, for all is like) from the other Digit (not from his owne) as the lines of the croſſe warne me, and that that is left muſt I write vnder the digits. As in this example. If I take 2 from 7, or 3 from 8, there remaineth 5: that 5 muſt I write vnder the digits:



and then there appeareth the multiplication of 7 times 8, to be 56. And ſo likewiſe of any other digits, if they be aboue 5, for if they be vnder 5, then will their differences be greater then thẽſelues, ſo that they cãnot be taken out of them. And againe, ſuch little ſummes euery child can multiply, as to ſay; 2 times 3, or 4 times 5,

and ſuch like.

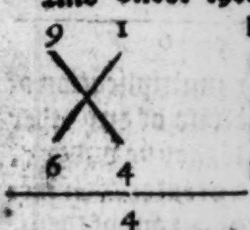
Scholer. Trueth it is. And ſeeing mee ſeemeth that I vnderſtand the multiplying of the greater digits, I will prooue by an example how I can doe it. I would know how many are 9 times 6.

Maiſter. It is all one in value to ſay 9 times 6, or 6 times 9: but yet the order is beſt to put the leſſe ſumme firſt, ſaying: 6 times 9, and ſo of all other ſummes.

Scholer. Then would I know, how many are 6 times 9: therefore I ſet the Digit thus, and make the croſſe thus.

Then doe I ſet their differences from 10 at the right ſide: the difference of 9 which is 1 againſt it, and the differẽce of 6 which is 4 againſt it alſo, as in this example.

And vnder them I draw a line. Then doe I multiply the Digits together, ſaying: one time 4 maketh 4, that 4 doe I write vnder the differences thus.



Then take I one of the differences from the other Digit, as one from 6, or elſe 4 from 9, and eache wayes there reſteth 5, which I doe write vnder the digits

digits thus. And so appea- reth the multiplication of 6 times 9, to be 54. Thus I see the feat of this maner of multiplication of digits.

9	1
6	4
5	4

Maister. Now might you goe straight to the multiplication of greater numbers, saue that both for your eas[illegible] surety in working, I wil draw you here a Table, whereby shall appeare the multiplication of all digits, and this is it that followeth.

1	1	2	3	4	5	6	7	8	9
	2	4	6	8	10	12	14	16	18
		3	9	12	15	18	21	24	27
			4	16	20	24	28	32	36
				5	25	30	35	40	45
					6	36	42	48	54
						7	49	56	63
							8	64	72
								9	81

In which Table when you would know the product in any multiplication of Digits, seeke your first or last Digit in the greater figures, & from it goe right forth toward the right hand, till you come vnder the number of your second Digit, which is in the highest row: and then the number that is in the meeting of the rowes of

litle squares (which come directly frō both your propounded Digits) is the multiplication that amounteth of them. As if I would know by this Table the multiplication of 7 times 9; seeke first 7 in the greater figures, and then go right forth toward the right hand, till you come under 9 of the highest row, in which place where you so come under the other Digit (as here for example you come under 9) is alwaies contained the ofcome or product, which you seeke: and that place we terme to be in the common angle, in respect of the two numbers so taken on the out sides, as here in that common angle, where the rowes of little squares (directly proceeding frō 7 and 9) do meete, you haue 63, which 63 is the summe of this multiplication of 9 by 7.

Scholer. This is very good and ready. And so may I finde the multiplication of any Digits. But now how shall I doe in greater summes?

Maister. When you would multiply any summe by another, you shall marke that it is the meetest order to set the greatest number highest, which is the place of the number that must be multiplied: & likewise the lesser number under it, for that is the place of the multiplier, or Multiplicator, that is to say, the number by which multiplication is made: and is in English alwaies put before this word, Times: in such speaking when I say, 20 times 70. And the number that followeth this word Times,

is

is that which must be multiplied.

Therefore when I would multiplie one number by another, I must write the greatest highest, and the lesser vnder it, as in Addition. And vnder them must I draw a line. As for example: If I would multiplie 264 by 29, I must set them thus.

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 264
  29
-----
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¶ Of which numbers thus set downe to be multiplied, may be formed a question as thus.

There are 29 men, and each man hath 264 lambes. The question is how many lambs they haue in all. *

To the performance whereof I must multiplie euerie figure of the higher rowe by euerie figure of the neather row: and that that amounteth, I must set vnder the line, as thus, First I do multiplie 4 by 9, saying: 9 times 4 (or 4 times 9, which is all one) and that maketh 36, as the Table before of digits doth declare, of that 36 I must write the 6 that is the digit, vnder the 9, and the Article 3 I keepe in mind to carie to the next place.

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 264
  29
-----
   6
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Then come I to the seconde figure of the higher row, which is 6, and say: 9 times 6 make 54, and with the 3 in my minde make 57, the 7 I set downe vnder the 2, and the 5 I keepe in minde.

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 264
  29
-----
  76
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After that I come to the next figure which is 2, and do multiplie it by nine, and that maketh 18, and with 5 that I haue in minde maketh 23, wherefore because it is the laste worke of that multiplier, I set it downe in order as you see.

264
29
2376

And so haue I ended the first figure of the multiplier. Wherefore I giue it now a fine dash with my pen.

Then begin I with the next figure, and multiplie it into all the higher figures as thus.

First two times 4 make 8, that 8 do I write vnder the second place: for euermore the digit or first figure of Multiplication that amounteth of the first figure of the higher number, must be set vnder the Multiplier of it, and the other in their order toward the left hand.

264
29
2376
8

Scholer. I vnderstand you thus: that the digit of the summe amounting of the multiplication of the first figure of the higher rowe, by the first figure of the lower rowe or Multiplier, must be set vnder the first place: and that that amounteth of the same first figure by the second multiplier, must be set vnder the second place, and so of the other, if there be more multipliers.

Master. So meane I indeede: and if there amount but a digit, then must it be set vnder

the

the multiplier.

And nowe to goe forth: I multiplie by the ſame 2, the ſecond figure of the higher rowe, which is 6, ſaying: 2 times 6 make 12: whereof I write the digit 2 vnder the third place, & the article 1 I keepe in mind.

264
29
2376
28

Then do I multiply the laſt figure of the higher ſumme by that ſame 2, ſaying: 2 times 2 is 4, and with the one that I haue in minde maketh 5, which 5 I write vnder the fourth place. And ſo haue I ended the whole multiplication: wherefore I alſo giue the 2 a daſh with my pen, thus: and ſo I doe euer as ſoone as I haue diſpatched anie Digit by which I do multiplie: And the ſummes ſtande thus.

264
~~2~~9
2376
528

Then muſt I draw a line vnder all thoſe ſums that amount of the multiplication, and muſt adde al them into one ſumme, as in this example you may plainly ſee.

264
~~2~~9
2376
528
7656

Where in the firſt place I finde but 6, and therefore write I it vnder the line. Then in the ſecond place 8 and 7 make 15, whereof I write 5, and keepe one in my minde, and ſo forth, as you learned in Addition. And ſo appeareth the whole ſumme to be 7656, which

amounteth of the Multiplication of 264 by 29, and that is the iuſt number of Lambes the 29 men had.

Scholer. If there be no more to be obſerued in it, then can I do it, I ſuppoſe, as by this example I ſhall proue.

¶ There is a peece of ground which containeth 1365 yardes in length: And 236 yardes in breadth, I would know how many yardes ſquare there is in all this peece of grounde, which numbers I ſet downe with the greater aboue, and the leſſer vnder as you ſee.

1365
236

Then do I multiplie fiue by ſixe, ſaying: 6 times 5 make 30: of which I write the Cipher in the firſt place and the Article three I keepe in minde to carie vnto the nexte place.

1365
236
0

Then do I by the ſame 6 multiplie the ſecond figure of the higher ſumme, which is 6, ſaying: 6 times 6 make 36, and 3 in my minde make 39, of which I write the 9 vnder the ſeconde place, and the Article 3 I keepe in minde.

1365
236
90

Then do I multiplie the third figure which is 3, by the ſame 6, and that maketh 18: and with 3 in my minde make 21. Then 1 I ſet downe, & 2 I keepe in my mind.

1365
236
190

Then

Then come I to the laſt figure of the higher ſumme, and multiply it by 5, ſaying: 6 times 1 make 6, and 2 in my minde make 8, that 8 doe I write vnder the fourth place. And ſo haue I ended the firſt multiplier, and daſh him ſlightly with my pen.

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 1365
  236
-----
 8190
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Then begin I with the ſecond multiplier, and ſay firſt, 3 times 5, that maketh 15, of which I ſet the 5 vnder the ſecond place, becauſe that the multiplier is there and the article 1 keepe in minde.

```
 1365
  136
-----
 8190
   5
```

Then come I to the ſecond figure, that is 6, and multiply it by 3, which maketh 18, and with one in minde maketh 19, the 9 I ſet downe vnder the third place, and 1 I keepe in minde.

```
 1365
  236
-----
 8190
  95
```

Then come I to the third figure, which is 3, and multiplie it by 3, ſaying: 3 times 3 make 9, and with one in my mind make 10, the Cipher I ſet vnder the fourth place, and the article 1 I keepe in minde.

```
 1365
  236
-----
 8190
 095
```

And then comming to the laſt figure 1, I multiply it by 3, and it maketh 3, and with the one in mind it maketh 4: which 4 I ſet in the laſt place, and then haue I

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 1365
  236
-----
 8190
4095
```

ended two of the multipliers, and the ſummes ſtand as you may ſee in the latter end of the page going before. And then I giue 3 his daſh.

```
1365
 236
----
8193
4095
  0
```

Then come I to the third multiplier, and multiply it into euery figure of the higher ſum. and firſt I ſay: 2 times 5 make 10, of which I ſet the cipher vnder the multiplier in the thirde place, and the Article 1 I keepe in minde.

And ſo multiplying the ſecond figure 6, by that ſame 2, there amounteth 12, and with 1 in my mind make 13, whereof I write the digit 3 vnder the fourth place, and the article 1 I keepe in minde.

```
1365
 236
----
8190
 30
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Then do I multiply the ſaid 2 by the third figure of ye higher ſumme, which is 3, and that maketh 6, & with the one in minde make 7, which 7 I ſet downe vnder the fifth place, as appeareth by the example.

```
1365
 236
----
8190
4095
730
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Then come I to the laſt place, and multiply that 1 by 2, and there amounteth 2, which I ſet in the firſt place, and then doth the ſumme ſtand thus.

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1365
 236
----
8190
4095
2730
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And ſo haue I ended the whole

whole multiplication.

But now (as you taught me) to know what this whole summ is, I must adde all those parcels together, and then vnder the line will appeare as you may see, the grosse or totall summe, that is, 322140. Whereby I knoe there is so many yardes square in that peece of ground. *

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   1365
    236
  -----
   8190
  4095
 2730
 ------
 322140
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Master. This is well done, and like a workeman.

Scholer. Then me thinketh I could call it well downe, when I know whether I had well done or no.

Master. It is to be proued by 9, as Addition was, but the surest proofe is by Diuision, and therefore I will reserue that proofe by Diuision till you haue learned the Arte of Diuision. And anon I will shew you how it is commonly proued.

¶ But first for your further instruction in this exercise of Multiplication, I will without example more trie your cunning, and so make an ende. And the question is this. I would know how many daies it is, since the Natiuitie of our Lord and Sauiour Jesus Christ vnto this present yeare, 1590. Which to performe, you must multiplie by the dayes in one whole yeare, which are 365.

Scholer. Now for that you haue giuen me

so much light in to the question, you shall see I will handsomely finish the worke, for according to your former instructiōs I set them downe with a line vnder them thus.

1590
365

Then say I, 5 times 0 is 0, the 0 I set downe vnder the first place, as here appeareth. Then say I, 5 times 9 make 45, the digit 5, I set downe in the second place vnder 9, and the article 4, I keepe in minde to be added to the next Multiplication. Then saying, 5 times 5 make 25, and 4 in minde makes 29, the 9 I set downe in the third place, and 2 I keepe in minde. Then comming to the last figure I say, once 5 is 5, or 5 times 1 is 5, and 2 in minde make 7, that seuen do I set downe vnder the fourth place. And thus haue I ended my first multiplier, and therefore I giue it a dash with my penne.

1590
365
7950

Then come I to the second Multiplier which is 6, and doe likewise multiplie it into the vpper number, saying, 6 times 0 is 0: the 0 I set down in the second place right vnder his multiplier. Then say I 6 times 9 make 54, the 4 I set downe vnder the thirde place, and fiue I keepe in minde. Then say I, 6 times 5, make 30, and 5 I keepe in minde, make 35, the digit 5 I set downe in the fourth place, and 3 I keepe

1590
365
7950
9540

keepe in minde. Then say I 6 times 1 is 6: or once 6 is 6; and 3 in minde make 9, which I set downe next: and so haue I ended two multipliers; wherefore I dash the 6 with my penne.

Then beginne I to multiply the third multiplier into the ouer number, saying, 3 times 0, is 0: the 0 I sette downe in the third place right vnder his multiplier: Then say I 3 times 9 make 27, the Digit 7 I sette downe in order next; and 2 I keepe in minde, then I say 3 times 5, is 15, and 2 in minde is 17: the 7 I set downe and one I keepe. Lastly I say once 3 is 3, and 1 I keepe is 4, which I set downe orderly next: And so haue I ended the Multiplication, and my figures stand thus.

		1	5	9	0
		3	6	5	
		7	9	5	0
	9	5	4	0	
4	7	7	0		

Maister. I commend you for your diligence, the worke is very perfectly done, which parcels if you now adde together into one summe, it will bee 580350, which is the grosse or totall summe of that Multiplication, and declareth the number of dayes since our Lord and Sauiour his Incarnation, vnto the end of 1590 yeares, besides 397 daies and 12 houres, for leape yeares.

Scholer. This is marvellous me thinketh, that such great matters may so easily bee atchieued by this Art, which heretofore I euer thought had beene impossible, as infinite sortes

of people are of that minde.

Maister. Truth it is that knowledge hath no greater enemy then ignorance, for this is one of the least of ten thousand things that may be done by this Art, as hereafter you shall be able to iustifie.

Scholer. This manner of multiplication I perceiue, if there be no more in it.

Maister. Yes there are other formes and helpes for ease and shorter labour of the worke of Multiplication, but I will remit them till you haue a little tasted Diuision, where also the like helpe into Diuision may be vsed, and so therefore vnder one example for both, will I shew you both ease in Multiplication and also in Diuision.

But sith the other formes and workings do nothing differ from these workes in effect, but onely in setting of the numbers, I will ouerpasse them till a more meeter place and time. And now will I instruct you in Diuision, so that you thinke your selfe sufficiently to perceiue what I haue taught you.

Scholer. Yes sir I thanke you, but I do not perceiue how to examine my work, to try whether I haue well done or no, therefore as you promised me ere while, I pray you first shew me how I shall prooue it.

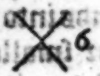
Maister. That is commonly vsed by the proofe of 9, as you learned before in Addition, sauing that it differeth from that forme in diuerse

uerse respects: As for example.

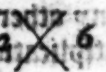
First you must make a crosse after this maner. 

Then must you examine your summe that should be multiplied, & looke what remaineth after casting away of 9, that set you at the one side of the crosse: then examine the multiplier, and whatsoeuer remaineth in it, after casting away 9 so often as you can, write that at the other side of the crosse: then must you multiply those two numbers together, and looke what amounteth thereof, if it be under 9, write it at the higher part of the crosse: but if it be aboue 9, then take thence 9 as often as you can, and write the rest at the head of the crosse. As first for example, we will proue the example you put forth of the peece of ground that contained 1365 yards in length, and 236 yards in breadth.

Therefore first I take away all the nines from the summe to bee multiplied, saying: 5 and 6 make 11, cast away 9, rest 2, then 3 and 2 make 5, and 1 is 6, that 6 I write at the one side of a crosse thus.

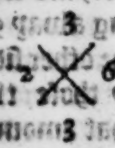


Then do I examine the multiplier, which is 236, wherein when the 9 is cast out, there remaineth 2, that therefore I set at the other side of the crosse.



Then doe I multiply 6 by 2, and it ma-

keth 12, from which 12 I withdraw 9, then re-steth 3, that 3 doe I set at the head of the crosse. Then doe I examine the grosse summe amounting of the Multiplication, which is 322140, where I finde 9 once, and 3 remaining, that 3 I set at the foote of the crosse, and then I set it to agree with the other 3 at the top of the crosse, and so knowe I that I haue done well: for if they two do differ, then were my worke vaine, and the multiplication false.



This is the common proofe, but the most certaine proofe is by Diuision, of which I will anon instruct you.

Scholer. Sir, what is the chiefest vse of multiplication?

Master. The vse of it is greater than you can yet vnderstand: howbeit, these plaine commodities it hath, that if you would resolue any greater and whole value into many small and lesse portions: as if you would change pounds into shillings, pence or any other greater or smaller parcels, by Multiplication you shall doe it speedily and easily. Also if you should neede to adde one summe to it selfe, or to any other oftentimes, you shall doe it by Multiplication much more speedily, readily, easily, and surely, than by often and sundry Additions. Take you these commodities grossely shewed for an answere at this time,

and hereafter I will more abundantly make you to perceiue the vse of it.

Diuision.

Scholer.

WEll Sir, then in Diuision I pray you to instruct mee: But me thinketh by the name of it, that it should be alone with Multiplication: for I call that Diuision, when any thing is parted into diuerse and many partes.

Diuision what it is.

Master. You take it as it is taken commonly: howbeit, if you marke well, you shall perceiue that it is quite contrarie to Multiplication, and doth not parte one thing or fewe things into manie, but contrariewayes it bringeth many parcels into fewe, but yet so, that these few taken together, are equall in value to the other many: for by Diuision pence are turned into shillings, and shillings into pounds: as for example of 120 shillings, it maketh 6 poundes, so are 120 turned into 6, which is a smaller number: but then, if you consider the Denominators, you shall see that

they are such, that one of the latter is equall to 20 of the first, & so in value the summes are one, though in number they do farre differ, and the latter summe is the lesser, and so is it alwaies in Diuision: howbeit, yet in the working, the sum is parted by another, and thereof doth it take the name.

Scholer. I thinke I shall better vnderstand the reason of the name, when I knowe the vse of the worke, therefore now would I gladly learne that.

Master. Diuision is a distributing of a greater summe by the vnites of a lesser. Or Diuision is an Arithmeticall producing of a third number, in respect of two propounded numbers: which third number shall so often containe an Vnite, as the greater of the two propounded numbers doth containe the lesser. So that, euen as Multiplication did seeme to serue in stead of many Additions, so Diuision may seeme to be in place of many Subtractions: Because the third number briefely expresseth, how many times the lesser of your two propounded numbers may be subtracted from the greater: as in practise will more plainely appeare. Therefore as you may perceiue, vnto Diuision are required three numbers: the first, which should be diuided, and that must (generally) be the greater: and the second, by which the other must be diuided, and that is (generally) the lesser, and is called

Diuision what it is.

the

the Diuisor. And the third which answereth to the question, How many times: and therefore is called the Quotient.

The first must be first written, and the second so to set vnder it that the last figure of the lower number bee right vnder the last of the higher, contrariwise to the workes of the other kindes of Arithmeticke: for in them the two first figures were set euer meete one vnder the other, but in diuision the last figures must be set meete, except it chance so, that the last figure of the Diuisor be greater than the last of the higher number, for then you shall set the last of the Diuisor, vnder the last saue one of the higher number, as for example. A generall rule for placing the figures. An exception.

If you should diuide 365 (which are the summe of the dayes of a yere) by 28, which are the dates of a common moneth, then should you set them thus.

365
28

But if you would diuide these 365 dayes, by 52, which is the number of weekes in one yeare, then should you set them thus.

365
52

Likewise if I would diuide the same 365 by 4, which is the sum of the quarters of a yeare, then must I set them thus.

365
4

Scholer. Syr, this do I vnderstand; but how now should I do to diuide the one by the other.

Master. You must begin with the last figure next the left hand, and see how many times the last figure of the Diuisor may be taken out of the last figure of the ouer number, and that shall you note within a croked line towards your right hand. As for example.

A generall rule for placing the figures.

I would diuide 365 by 28, then set I those two summes thus.

365 (
28

And I looke how many times I may finde 2 (which is the last figure of the Diuisor) in 3, (which is the last of the number to be diuided) and considering that I can take 2 out of 3 but once, I make a croked line at the right hand of the numbers, and within it I set 1, and that is called the Quotient number, as I told you. Then because that when 2 is taken out of 3, there remaineth 1, I must write that 1 ouer 3, and deface or cancell the 3 and the 2, then will the figures stand thus.

An exception.

Quotient numbers.

1
365 (1
28

Then must I goe to the next figure of the Diuisor, and take it likewise so many times out of the figures that be ouer it, and looke what doth remaine, that I must write ouer them, and cancell them, as in this example.

Therefore now doe I take once 8 out of 16, and there remaineth 8, which 8 I must set ouer the 6, and cancell or crosse out the 16, and the 8 of the Diuisor: And then will the fi-

gures

gures stand, thus.

And so haue I once wrought.

Scholer. So I perceiue that you take the neather figure not onely out of the other that is right ouer him, but out of that with the other also that remaineth before, and are written toward the left hand.

28
365
28

Maister. So must you doe: for you must so take the Diuisor out of the ouer number, that there remaine not ouer it so great a summe as it selfe is, for then were your worke in vaine.

But yet againe here must you marke, that when you seeke how many times the last figure of the Diuisor may be found in the number ouer him, that you looke also whether you may as often finde all the figures following in those that are aboue them, (considering all the remainders, if there be any) if not, take your Quotient lesse by one, and then prooue againe: and so still, till you finde a meete Quotient: And by that meete Quotient must you alwaies multiply your Diuisor, and the product set vnder your Diuisor, so that his first figure stand vnder the first figure of your Diuisor, and the second vnder the second, and so forth: and then subtract that product from the number to be diuided, that standeth directly ouer it, as you haue seene me doe.

Note.

When you haue thus wrought once, then

must you begin againe, and write your diuisor a new, nearer toward the right hand by one place, as in this example, you shall set 2 under 8, and 8 under 5, thus.

28
365 (1
288

Then (as before) seeke how many times you may take your deuisor out of the number ouer him now.

2

Scholer. That may I doe here 4 times.

Maister. Truth it is that you may finde 2 foure times in 8: but then marke whether you can finde the figure following so many times in the other that is ouer him. Can you finde 8 foure times in 5?

Scholer. No, neither yet once.

Mark how to consider this kind of remainer.

Maister. Therefore take 2 out of 8, once lesse.

Scholer. That is 3 times.

Maister. Well, then 3 times 2 make 6: if I take 6 out of 8, there remaineth 2: which 2 with the 5 following, make 25, in which summe I find 8 thre times also, and therefore I take 3 as a true quotient, and write it within the crooked line of the Quotient, before the one thus.

28
365 (13
288
2

Then say I: 3 times 2 make 6, then 6 out of 8 resteth 2, therefore I cancell the 2 and the 8, and write ouer it the 2 that doth remaine, thus.

2
28
365 (13
288
2

Then

Then doe I take 8 as many times out of 2, saying: 3 times 8 make 24, and if I take 24 out of 25, there remaineth 1: so thē I cancell 25 and 8, and ouer the 5 I set 1, thus.

2
281
365 (13
288
2

Or you might (after you found 3 to be a fit Quotient) straight way haue multiplied the whole Diuisor 28, by that 3, at once: which giueth 84, which being set vnder 28, and duely subtracted from 85, of the number diuided, giueth 1, the remainer of the whole diuision: as before you had. Worke which way you list: here you see also the forme.

81
365 (13
288
24
8

And now haue I done with diuiding, for I can finde my diuisor 28 no more in the ouer summe.

Scholer. No except you would part the 1 that remaineth into 28 partes.

Maister. That is well sayd, and so must we doe in such cases, when there remaineth any thing: but I will let that passe now, and will make you perfect in diuision of whole numbers, and will hereafter teach you particularly of broken Numbers, called Fractions.

Now if you doe perceiue the order of Diuision, then doe you diuide this summe 136280 by 452.

Scholer. First I sette downe the number

that should be diuided, then doe I set the diui-sour vnder it, so that the last figure of it bee right vnder the last figure of the ouer number. Then will it bee thus.

1	3	6	2	8	0
4	5	2			

Maister. Can you take the last of your diuisour (which is 4) out of 1, which is the last of the ouer number?

Scholer. I had forgotten, because the last of the Diuisor cannot be taken out of the last of the ouer number, in so much as it is the greater, therefore must I set the diuisor one place more forward, toward the right hand thus.

1	3	6	2	8	0
	4	5	2		

And then must I looke how often I may finde the last figure of the Diuisour (that is 4) in 13, which thing I may doe 3 times, therefore doe I say, 3 times 4 is 12, which I take out of 13, and there remaineth 1. Then doe I make at the right hand of my summes a crooked line, and write before it my Quotient 3: and I cancell 13 and 4 and ouer the 3 I set the 1 that remaineth, and then the figures stand thus.

	1					
1	3	6	2	8	0	(3
	4	5	2			

Then doe I multiply the same Quotient into euery figure of the diuisor, and withdraw the summe that amounteth out of the numbers

hers ouer them, as first I say: 3 times 5 make 15, which I take from 16, and there resteth 1, I cancell therefore 16 and 5, and write o-uer the 6 that 1 that re-maineth thus.

```
  1 1
1 3 6 2 8 0  (3
  4 5 2
```

Thus do I say like-wise, 3 times 2 makes 6, which I take out of 12, and there resteth 6, therefore I cancell the 12 and the 2, and ouer the 2 I write the 6 that remaineth thus.

```
  1 1 6
1 3 6 2 8 0  (3
  4 5 2
```

Then should I set forward the diuisor, into ye next place toward the right hand thus.

```
  1 1 6
1 3 6 2 8 0  (3
  4 5 2 2
    4 5
```

Master. But you may see, that ouer the 4 is no figure, therefore must I set the Diuisor yet forwarder by an other place.

And marke, whensoeuer it chaunceth soe, that you should set forward the Diuisor, & that it cannot stande there, because there is no number ouer the last place, or if there be anie, it is lesser then the last figure of the Diuisor, then must you remoue the Diuisor yet once a-gaine: and because that his first place of re-mouing serued not to subtract him so much as once, therefore you shall write in the quotient a cipher 0. And if you should by chaunce neede to doe so oftentimes, for euerie time write a

cipher in the Quotient. The reason of this, will I shew hereafter.

Scholer. Then must I set my summes thus.

```
  116
136280
 45252  (30
   4
```

And because I remoued the diuisor, so that I ouer skipped one place, I must write a Cipher in the quotient: and then must I seeke a new quotient, as in this example I must say. How many times 4 is there in sixe (and sith it can be but once, therefore do I write 1 in the quotient: and then say I: 1 time 4, taken out of 6, remaineth 2, I cancell the 6 and the 4, and write 2 ouer them thus.

```
   2
  116
136280  (301
 45252
   4
```

```
   2
  116
136280  (301
 45252
   4
```

Then say I againe, once 5 out of 28 remaineth 23, I let the 2 stand as it did, and ouer that 8 I set 3, cancelling the 8 and the 5 vnder it, thus.

```
   2
  1163
136280  (301
 45252
   4
```

Master. You might as well haue sayed, once 5 out of 8, and so remaineth 3, but nowe go forth.

Scho-

Scholer. Then once 2 out of 0, cannot be, what shall I now do?

Master. Borrow of the next number that is behind, (for there is 230) and do as you learned in Subtraction in a like case.

Scholer. Then must I borrowe 1 of the 3 comming behinde next, and make that 0 to be 10: & then take I 2 out of 10, and there resteth 8. And because I borrowed one of the 3, I must cancell the 3, and write 2 ouer it: then doth ẏ figure stand thus.

```
    22
  11638
  136280   (301
  45252
    4
```

Master. Now haue you done, and yet remaineth 228, and your quotient sheweth you, that if you diuide 136280 by 452, you shall find your diuisor in your greater number 301, that is, CCC times and once, and 228 remaining.

And in the other example, where I diuided 365 by 28, the quotient was 13, and one remained; whereby I know that in a yeare (which containeth 365 dayes) there are 13 moneths reckoning 28 dayes (or 4 weekes) iust to a moneth, and 1 day more.

Scholer. Why then do we call a yeare but 12 moneths?

Maister. Of that at a more conuenient time will I fully instruct you: but nowe it is not conuenient to entangle your minde with a-

ther things, then do directly pertaine to your matter. Therefore if you remember what you haue heard, you haue learned a short manner of Diuision, which I would haue you often to practise, so that you may be perfect in it, and hereafter I will shew you certaine other proper points touching it.

Scholer. Then I pray you yet tell me, how I shall examine and trie my worke, whether I haue done well or no, that though no man be by for to tell me, yet I may perceiue it my selfe.

Master. Some men (yea and commonly) doe trie it by the rule of 9, as in all the other kindes, saue that their order is. First they cast away 9, as often as they can, out of the Diuisor, & that that remaineth, they set at one side of a crosse: as in our first example, the diuisor was 28, from which you may take 9 thrée times & 1 remaineth, which they set by a crosse thus.

Then do they likewise examine the Quotient (which in our example is 13) and from thence they do caste away 9 as often as they can, and the Remainer they set at the other side of the crosse, and then multiplie they together those two Remainers: and to it that amounteth, they adde the Remainer of the Diuision, if there were any, from that whole summe they withdraw 9 as often as they can,

and the reſt they ſet at the head of the croſſe: as in our example the quotient is 13, from which take 9, and there remaineth onely 4, & therefore muſt you ſet 4 at the other ſide of the croſſe, thus.

Then multiply 4 by 1, and it yeeldeth but 4, thereto adde the remainer of the diuiſion (which was 1) and it will bee 5, which ſumme doeth not amount to 9, and therfore muſt be ſet wholly at the head of the croſſe, as you ſée here.

And this number on the head of the croſſe, is the firſt proofe, to which if you finde another like in the number that was diuided, then haue you done well.

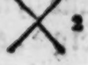
Therefore now ſhall you likewiſe examine the whole ſumme that was diuided, and take away 9 as often as you can, and that that remaineth, ſet at the foote of the croſſe: and if it be equal to that in the head of the croſſe, then haue you well done, or elſe not.

As in our exãple the whole ſumme was 365, which maketh 14, from that take 9, and there reſteth 5, which ſet at the foote of the croſſe, thus.

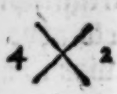
And you ſhall ſee that they agrée: therefore haue you well done.

Scholer. Now will I likewiſe examine

our ſecond example, where the Diuiſor was 452, which maketh 11: from thence I take 9, and the 2 that remaineth I ſet at the right ſide of the croſſe, thus.



Then examine I the quotient, which was 301, where I finde but onely 4, that doe I ſet at the other ſide of the croſſe thus.



Then do I multiply 4 by 2, and it maketh 8, to that doe I adde the remainer of the diuiſion (which was 228, and it maketh 12) & they two make 20, wherein I finde twiſe 9, and 2 remaining, that 2 muſt I ſet at the head of the croſſe thus.

Then doe I examine the whole number to be diuided, which was 136280, where I finde twiſe 9, and 2 remayning, which I ſet at the foote of the croſſe, thus.

And becauſe that it doth agrée with the figure at the head of the croſſe, I know that the diuiſion is well wrought.

Maſter. This is the common proofe, howbeit, the more certaine working is by the contrary kinde, as to prooue Diuiſion by multiplication, thus.

Againe of the proofe of Diuiſiō.

Multiply the quotient by the diuiſor, and it

if the summe that amounteth bee equall to the summe that should be diuided, then haue you well diuided, else not.

Howbeit, this must you marke, that if there remained any thing after the Diuision, that must you adde to the summe that amounteth of the Multiplication: as in our first example the quotient was 13, and the Diuisour, was 28: Now multiply the one by the other, and the summe will bee 364: so that if you adde the one that remained after the Diuision, then will it be 365, which was the summe that should be diuided, and therefore I know that I haue well done.

Daddy

Daddy

Scholer. Now will I proue the same in the second example whose Diuisour was 452, and the quotient 301: these doe I multiply together, and there amounteth 136052, to which if I adde the 228 that remayned, then will it be 136280, which was the whole summe to be diuided, and therefore I perceiue that I haue well done.

Dadd

Maister. This is the surest way, to examine Diuision by Multiplication: and contrariwise the surest proofe of Multiplication, is by Diuision.

And therefore according to my promise, now will I shew how you may proue Multiplication by Diuision.

A proofe of Multiplication by Diuision

When you haue ended Multiplication, and would know whether you haue well done

or not, set the grosse summe that amounteth of the multiplication ouermost, and diuide it by the multiplier: and if the quotient be the same number that should be multiplied, then haue you well wrought, else not: as in that example where wee multiplied 264 by 29, the grosse summe was 7656.

Now if you will know whether that multiplication be true, you shall diuide that 7656 by the multiplier 29, and you shall perceiue that the quotient will bee 264, and that is a token that you haue well wrought.

Scholer. By your patience I will prooue that: and first I set downe the grosse summe and the multiplier, not after the rule of multiplication, but after the rule of diuision, for now that number is become the diuisor, that was before the multiplier, I shal set them therefore thus.

7656
29

Then shall I seeke how many times 2 in 7 that may be 3 times, and 1 remaineth: but then may not 9 bee found so often in 16, therefore must I take a lesser quotient, that is to say, 2: then say I twise 2 maketh 4, which I take out of 7, and there remaineth 3, then doe I cancell 7 and 2, and ouer 7 I write 3, and in the quotient I set 2, so the figures stand thus.

3
7656 (2
29

Thē say I forth, 2 times 9 make

9 make 18, which I abate out of 36, and there resteth 18, then cancell I 3, and ouer him set 1, and likewise I cancel 6 and 9, and ouer them I set 8, so that thus stand the figures.

1
38
7656 (2
29

Then do I set forward the Diuisor by one place, and seeke a newe Quotient, that is to say, how many times 2 are in 18, which I finde to be 9 times, but then can I not finde 9 so many times in 5, therefore I take a lesser Quotient, as to say, 8: but yet is that too great: for if I take 8 times 2 out of 18, there remaineth but 2, and I cannot finde 8 times 9 in 25, therefore yet I take a lesser Quotient, that is 7, which is also too great, for if I take 7 times 2 out of 18, there resteth 4, but now I cannot take 7 times 9 out of 45, therefore yet I seeke a lesser quotient, as to say, 6: then say I, 6 times 2 make 12, that I take out of 28, and there remaineth 6, so I cancel the 18 and the 2 & write 6 ouer 8, thus.

16
38
7656 (26
299
2

Then say I forth: 6 times 9 maketh 54, that take I out of 65, and there remaineth 11, and the figures stand thus.

1
16
381
7656 (26
299
2

Then must I set forth the diuisor againe, and seeke a new quotient, which will be 4: for

though I may finde 2 in 11 fiue times, & 1 remaine, yet I can not finde 5 so often in 16, therefore I set the figures thus.

```
   1
  26
 381
7656 (264
2999
  22
```

And the 4 in the Quotient I multiplie into the figures of the diuisor, saying: 4 times 2 maketh 8, which I take out of 11: and there resteth 3, therefore I cancell the 11 and the 2, and set 3 ouer the first place of 11, thus.

```
   1
 263
 381
7656 (264
2999
 22
```

And then doe I say forth, 4 times 9 maketh 36, which I take from 36, and there remaineth nothing, so that the quotient of this Diuision, where 7656 is diuided by 29, is 264, which doth declare that if 264 be multiplied by 29, the summe will be 7656 And thus I perceiue now how both multiplication is proued by diuision, and Diuision also by Multiplication.

Master. Nowe haue I ended the fiue common kindes of Arithmetike: for as touching Mediation, Duplation, Triplation, and such other, they are no seuerall kindes of Arithmetike, but are contained vnder the other: for Mediation is contained vnder Diuision, and is nothing else but diuiding by 2: and so are Duplation & Triplation contained vnder Multiplication: for Duplation is nothing else but multiplying by 2, & Triplation is multiplying

by

by 3, of which I will onely propose an example, for the rules you haue heard alreadie.

If you would meditate or diuide into 2, this sum, 4531010, you shall set 2 for the Diuisor, worke as you learned before as thus.

4531010 (
2

An exãple of Mediation.

Then I finde 2 in 4 two times, therefore my Quotient must be 2: so I cancell 4 and 2, and remoue the Diuisor forwarde, thus: as the worke requireth, & as before in diuision hath bene declared.

4531010 (2
22

Which Mediation or Diuision by 2, being finished, you shall haue by your Quotient 2265505, which is the halfe of 4531010, as you may trie by duplation: for double that quotient, or multiplie it by 2, and the same number will amount.

Duplation

I will no longer tarie about these, seeing they are but members of the other kinds. But here now according to my promise I will teach you certaine easie formes both of Multiplication & of Diuision, and first of Multiplication.

If you would therefore multiplie any sum by 10, you shall neede to do no more but adde a cipher before his first place, as for example: 36 multiplied by 10, make 360.

Easie formes of Multiplication.

Likewise if you would multiplie any sum by 100, put two ciphers at his beginning.

So if you would multiply any sum by a thousand, adde three Ciphers to the beginning of it.

Scholer. This do I well perceiue, and also the reason of it.

Master. I will omit all reasons till our next méeting, when I shall tell you the reasons of all other partes of Arithmetike also: and as to our matter now, looke (as I haue tolde you) that you both remember it, and also often practise it.

And now haue you learned how to multiply easily by 10, 100, 1000: and of like manner may you do with any other of that sort.

But now, if you will multiplie by 20, 30, 40, and so forth: or by 200, 300, and such like, where there is one cypher in the first place, or many orderly in the first places, you shall take away those ciphers, and multiplie the summe onely by the other figure or figures. (if they be many) and then at the beginning of the summe that amounteth, you shall set so many ciphers as you tooke away.

Example of 2873, which I would multiplie by 300. First I omit the two ciphers from the multiplier, and I multiplie the summe by only 3 that is left, and it amounteth to 8619: before which I put the two Ciphers that I before omitted or tooke away, and then it is 861900. And that is the summe that amounteth, when 2873 is multiplied by 300.

Scholer. And if there were two or more figures beside the Ciphers, I must onely take away the Ciphers, and multiplie by the other

figures

figures, as I learned before: as if I woulde multiply 93648 by 25000, I should take away the thrée ciphers, and multiply the same by 25, and then at the beginning of that totall summe, should I adde the thrée ciphers againe.

Maister. Euen so: but and if it chance the number that should be multiplied, or both the summes as well the number that should bee multiplied as the multiplier, to haue ciphers in the first places, euermore omitte the Ciphers, and worke by the rest. But remember to restore as many Ciphers to the amounting summe, as you bated before, as in this example: 30200 shall bee multiplied by 206: I shall onely take away the two Ciphers from the greater number, and then multiply 302 by 206, and afterward adde the two Ciphers againe. But if I would multiply the same 30200 by 2060, I shall not onely take away the two Ciphers from the number that should be multiplied, but also I may take away the one Cipher from the multiplier, and then must I adde three Ciphers to the summe that amounteth: but take héede that you take away no Cipher that commeth after any signifying figure, as in this last example, you may not take away that in the fourth place of the higher number, neither that in the third place of the multiplier: howbeit, yet this you may doe: If one Cipher or more come in the middest

of your summes, you may multiply by the other figures, and ouerskippe them, but so, that you giue euery figure his due place, as thus: I will multiply 3026 by 2004, therefore I set them thus.

3026
2004
24
1208

And thus do I multiply them: First 4 times 6 make 24: I set the 4 vnder the first place, and keepe the 2 still in my minde: Then say I againe: 4 times 2 maketh 8, and the 2 that is in my minde maketh 10, I sette downe the cipher 0, and keepe the article 1 in my minde. Then 3 times 4 is 12, and the 1 in my minde maketh 13, I set downe the figure 3, and keeping the 1 still in my minde hauing no more places of the vpper number to multiply it withall: I put it downe next 2 in the fift place.

But now when I come to the next place being a cipher 0: I let it goe, because it multiplieth nothing: and likewise the second cipher.

But then when I do come to the 2, and multiply it into the 6 of the ouer number, you must take heede (according as I taught you in Multiplication) that the first number amounting of the multiplication, be set right vnder the multiplier, & the other orderly toward the left hand, according as you may see in this example,

3026
2004
12104
6052
6064104

which

which being finished with the Addition thereof gathered together, will stand as this example sheweth.

Which is in deede wrought so much sooner and shorter by ouerskipping of the two ciphers: which otherwise if the same example were wrought at length, it would haue had two workings more as by the same example here also set downe doth appeare.

3026
2004
12104
0000
0000
6052
6064104

Scholer. Sir, I thanke you: for I see great ease in this way of Multiplication, and if you can shew me such like in Diuision, you shall greatly further me.

Maister. Yes, I will teach you some easie wayes in Diuision also, and first this: If you would diuide any summe by 10, you shall only with your pen make a square line, betweene the first figure of your summe and the second, and then haue you done: for the whole number that followeth the line, standeth for the quotient, and the figure that is before the line, is the remainer: as for example, 3648 diuided by 10, will stand thus.

364|8

Where 364 is the quotient, and betokeneth that so many times are 10 in 3648: and the 8 after the line, is the remainer, which cannot be diuided into 10, but by breaking it

into fractions, wherewith I will not meddle yet.

And so likewise if you would diuide any summe by 100 with your pen, you shall cut away the two first figures, and if yee would diuide by 1000, you must cutte away the 3 first figures, and so of any other diuisor, whose last figure is 1, and the other be ciphers, looke how many Ciphers the diuisor hath, and so many figures at the beginning shal you cut away with the squire line, and they stand alwayes for the remainer, because they are lesse than the diuisor, and cannot be diuided by it, and the other figures that bee behinde the line, stand for the Quotient.

But now if your Diuisor haue any other figure in his last place than 1, and in all his other places haue ciphers, looke how many Ciphers there be, cut away so many of the first figures of the number that should be diuided, and diuide the rest that followeth the line, by that figure that is in the last place, as if it were the whole diuisor.

Example of 64284, which I would diuide by 300, here must I cut away the two first figures, (for so many Ciphers my diuisor hath) and must diuide the rest by 3, which is the figure in the last place of the diuisor. First therefore I part away the two first figures, and the sum standeth thus.

642|84

Then doe I diuide 642 by 3, and the quotient

Diuision Multipl

tient is 214, for in 6 I finde twise 3, and in 4 once, and 1 remaining, which 1 with the 2 next before, doth make 12: wherein I finde 3 foure times: & this is a ready way to turne shillings into pounds: for sith one pound doth containe 20 shillings, I must diuide the whole number of shillings by 20 therefore easily to doe it, I see that my diuisor hath one cipher, and therefore I cut away one figure from the beginning of the whole summe of shillings, and then I doe mediate or diuide by 2 the other figures or summe that followeth.

Scholer. I will put an example.

If you would diuide 64287 shillings by 20, that is to say: If you would turne so many shillings into pounds, I must cutte away the first figure, that is 7, and diuide the rest, that is 6428 by 2, so shall the quotient bee 3214 whereby I knowe that 64287 shillinges make 3214 pounds, and seuen shillings remaining.

Maister. Now proue by Multiplication whether you haue well done or no.

Scholer. The quotient is 3214, which I do multiply by the diuisor 2, and it doth amount to 6428.

Maister. Hereby may you perceiue not only that you haue well done, but also how by diuision you may turne shillings easily into pounds: And contrariwise, by Multiplication you may turne pounds into shillings.

123456789101112131415

But here shal you see amongst diuerse men, diuers formes of such diuisiō, but if you marke what I haue told you you shall perceiue easilie all their wayes: for some men do not cut away so many of the first figures of the summe that they would diuide, as there are Ciphers in the first places of their Diuisor, but they set all their Ciphers orderly vnder the first places of the number that they woulde diuide, and then with the other figure (or figures if they be manie) they diuide the rest of their summe.

Another maner of the abridgment.

Example. If they would diuide 725931, by 3400, they set their summes thus.

7	2	5	9	3	1
3	4		0	0	

And then doe they diuide orderly till they come to the Ciphers: for there they stay and end their worke, as in this example: They seeke how often 3 may be found in 7, which is 2 times, and one remaining, therefore they set 2 in the quotient, and cancel 3, and 7, and ouer 7 they set the 1 that remained thus.

1						
~~7~~	2	5	9	3	1	(2
~~3~~	4		0	0		

Then do they go forth saying: two times 4 maketh 8, which they take out of 12, and there remaineth 4, thus.

~~1~~	4					
~~7~~	~~2~~	5	9	3	1	(2
~~3~~	~~4~~		0	0		

Then renew they the Diuisor forward, and seeke howe often 3 may be found in 4, which is

is but once, & 1 remaineth, then set I 1 in the quotient, & cancell 3 and 4, and ouer them they set that 1 thus.

```
 1
 1 4
 7 2 5 9 3 1   (21
 3 4 4 0 0
 3
```

Then take they once 4 out of 15, and there resteth 11: or else more easilie: Take once 4 out of 5, and there resteth 1, so they cancell the 4 and 5 and set 1 ouer them thus.

```
 1
 1 4 1
 7 2 5 9 3 1   (21
 3 4 4 0 0
 3
```

Then set they forth the diuisor againe, and seeke howe many times 3 are in 11, which they find 3 times, and 2 remaining: so they set 3 in the quotient, & cancell 11 and 3, and ouer them set 2 thus.

```
 1 2
 1 4 1
 7 2 5 9 3 1   (213
 3 4 4 4 0 0
 3 3
```

Then do they multiplie 4 by 3, which maketh 12, that withdraw they out of 29 and there resteth 17, of which the 7 must be set ouer the 9, and the 1 ouer the 2, thus.

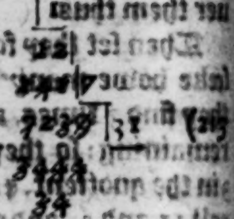
```
 1
 1 2
 1 4 1 7
 7 2 5 9 3 1   (213
 3 4 4 4 0 0
 3 3
```

And now are the two ciphers next ensuing, so that the diuisor can no more be set forward, and therefore is the diuision ended, and the

Remainer is 1731.

Nowe the Quotient, which is 213, doth declare, that if you diuide 725931, by 3400, you shall finde it therein 213 times: and there remaineth 1731, so shall you finde it, if you worke as I taught you, by cutting away the two first figures, because of the two ciphers.

Note. But this must you marke (as you may perceiue by this last example) that if there bee left any other Remainer in the summe that was behind the square line, that the Remainer must be set to the latter ende of the first Remainer, which was cut away with the square line: as if you would diuide 725931 by 34000, after the forme that I taught you, then would your summes appeare thus.



So that 17, which remaineth after the line, must be set to the 31 (that was cut away with the line) in higher places, as you see here: where that 17 with the 31, do make 1731.

¶ And here will I make an end of Diuision, sauing that I do request you to exercise your selfe well herein by manie summes, till you haue attained some expertnesse therein. For the reasons and conclusions thereof are so manie and so aualeable for all sortes of men whatsoeuer, that if I should speake of the infinite

finite vses thereof, I should rather lacke words then matter. And therefore recommending it to your iudgement hereafter, vppon your further trauell into this Art. I will here end this treatise, representing vnto you one example or simple question of Diuision and Multiplication in stead of many, which is this.

There are 4 brasse peeces: The first of them at a shot spendeth 9 pound of poulder, the second spendeth 5 pound, the third 4 pound, and the fourth 2 pound. They are all appointed against the battery of a hold, and there is allowed by the maister Gunner 700 pound of poulder to be spent by these 4 peeces in this assault. The question is two fold: The first how many shot each peece shall iustly make about with this seuen hundred pound of poulder. And lastly how many pound of poulder ought iustly to bee allowed to each peece for his true proportion.

Scholer. Why Sir you make mee smile, to beare me in hand, that these two demaundes may be simply resolued by Multiplication and Diuision.

Maister. Truely that they may, and that may you by and by worke your selfe with a litle labour: First adde together their quantities of poulder, that is 9 pound, 5 pound, 4 pound, and 2 pound, all which make 20: Diuide the 700 pound of poulder by that 20, and your quotient giueth 35, as here appeareth.

which sheweth for most certainety that they shall make iust 35 shottes about.

1
700
200 (35
2

Scholer. Sir, all this haue I done, and I see it is so, but whether it be true or not, I cannot tell.

Maister. To try the truth of the same, multiply the first peece, that spends 9 pound by 35, and you shall see his allowance, which is 315 pound of poulder. Multiply also the second peece that spends 5 pound by 35, and you shall finde 175 pound his allowance: then 4 by 35, and you shall find 140 pound his allowance. Lastly multiply 2 by 35, and you shall finde 70 pound his allowance. All which 4 particular summes you shall adde together by Addition, as here appeareth; and it maketh iust 700 pound, and so is the question truly absolued.

315
175
140
70
700

Scholer. Truly Sir these excellent conclusions do wonderfully more and more make me in loue with the Art.

Maister. It is an Art that the further you trauell, the more you thirst to goe on forward. Such a fountaine that the more you draw, the more it springs. And to speake absolutely in one word (excepting the study of Diuinity, which is the saluation of our soules) there is no study in the world comparable to this for delight

light in wonderfull and godly exercise: For the skill hereof is wel knowen, immediatly to haue flowed from the wisdome of God into the heart of man, whom he hath created the chiefe image and instument of his praise and glory.

Scholer. The desire of knowledge doth greatly encorage me to be studious herein: and therfore I pray you ceasse not to instruct mee further into the vse thereof. *

Maister. With a good will. And now therefore for the further vse of these two latter, that is Multiplication and Diuision, I will briefely shew you the feate of Reduction.

Reduction.

Reduction is, by which all summes of grosse denomination may bee turned into summes of more subtile denomination. And contrariwise, all summes of subtile Denomination, may bee brought to summes of grosser denomination.

Scholer. What call you grosse denomination, and subtile denomination?

Grosse denomination.

Maister. That I call a grosse denomination

which doth containe vnder it many other subtiller or smaller: As a pound in respect to shillings, is a grosse denomination: for it is greater than shillings, and containeth many of them. And shillings in comparison to pounds, are a subtile denomination, for because they are lesser than pounds, and many of them are contained in one of the other: as so, likewise of other things, whatsoeuer thing is compared to other, if it be a greater and containeth many of them, it is a grosser denomination: but if it be lesser, so that many of them are in the other, then are they called subtile denominations: whereby you may perceiue, that one denomination may be called a grosse denomination, and also a subtile (that is to say, a great and a small) in diuerse comparisons. For shillings compared to poundes are a subtile or small denomination: but compared to pence, they are a grosse or great denomination.

Subtile denominations.

Scholer. Now I vnderstand the name, I pray you teach me the vse.

Maister. The vse is easily learned, if you remember what you haue learned before. For if you will reduce any summe of a grosse denomination into a summe of a smaller or subtiller denomination, you must consider how many of that subtiller denomination doe make one of the grosser denomination, and by that number or numerator doe you multiply the other summes: as if you would reduce 20 poundes into

To reduce grosse denominations to subtile.

into shillinges, you must consider that in a pound are included 20 shillings, therfore multiplie the one 20 by the other 20, and there will amount 400, whereby you may knowe, that in 20 pound are contained 400 shillings. Likewise if you would reduce 30 shillings into pence, considering that in 1 shilling are 12 pence, you must multiplie 30 by 12, and it will be 360: whereby you finde, that in 30 shillings are contained 360 pence. And thus may you reduce any grosse denomination into a more subtiler, by Multiplication, if you knowe how many of the lesser doe make the greater: of which thing I will anon giue you a briefe Table for the most accustomed kinds of money, weights, measures and time, and such like, whereby you may know how often each subtile Denomination is contained in the Grosser, when you shall neede it for the foresayde kinde of Reduction. And also the same shall serue you, if you would reduce any summe of a subtiler Denomination, into a summe of a grosser Denomination: For in such Reduction you must consider (as in the other forme) how many of the smaller do make the greater, and by that number must you deuide the other summe, and the Quotient will declare howe many of the greater Denomination, are comprehended in that summe: as for example: If you would knowe how many shillings are contained in 3840 pence, consider that 12 pence doe make

To reduce subtile denominations to grosse.

1 shilling: you must diuide that 3240 by 12, and your quotient will be 270, whereby you know that so many shillings are in 3240 pence. But and you would knowe further, how many pounds are in those 270 shillings, seeing that euerie pound containeth 20 shillings, diuide that 270 by 20, and it will be 13, and 10 remaining, whereby you may knowe that in 3240 pence, or 270 shillings, are 13 pounds and 10 shillings. For euermore the Remainer must be named by the name or denomination of the summe that was diuided, which in this place were shillings. And thus may you doe with any other kindes of Denominations.

¶ Wherefore to the intent you may haue a certaine light or knowledge in most common coines, weights & measures (which is the most chiefe and principallest thing in trafficke to be known) I haue in ech Reductions as they come in order, set down certaine instructions incident thereunto. And first I haue hereunto added this Table, wherin is comprehended not onely our currant and common coines, but also the most part of the vsuall coines of Christendome, with their iust weightes and value currant in this realme of Englād, inteding at the latter end of my Addition to this booke, to write of the ordinary money vsed in diuerse places, & their common values currant for traffike, with the manner of their exchanges from place to place, &c.

A

A Table of the names and valuation of the most vsuall Gold Coines throughout Christendome with their seuerall weight of Pence and Graines; & what they are worth of currant money English.

The names and titles of the Gold.	The weight Pẽce,	Graines.	The Value Shil.	Pence.
Royall	4	23	15	0
Halfe Royall	2	11	7	6
Old Noble	4	9	13	4
Halfe Noble	2	4	6	8
Angell	3	8	10	0
Halfe Angell	1	16	5	0
Salute	2	5	6	4
2 parts of Salute	1	11	4	2
George Noble	3	0	9	0
Half George Nob.	1	12	4	6
First crowne K.H.	2	9	6	4
Base crowne K.H.	2	0	5	0
Great Soueraigne	10	0	30	0
Souera. K. H. best	3	14	10	8
Soueraigne K. H.	4	0	10	0
Edward Souer.	3	14	10	0
Elizab. Soueraigne	3	14	10	0

The names and titles of the Gold.	The weight		The Value	
	Pece,	Graines.	Shil.	Pence.
Elizab. Crowne	1	19	5	0
Vnicorne of Scot	2	10	6	0
Scottish crowne	2	5	6	0
French Noble	4	16	13	4
All the sortes of French crownes	2	5	6	0
Old French crown	2	5	6	0
Flaunders Rider	2	6	6	6
Gelders Rider	2	2	3	6
Phillips Royall	3	10	10	0
Phillips Crowne	2	5	5	0
Collen Gilden	2	2	4	8
New And. Gild,	2	3	5	0
Flaunders Noble	4	10	12	0
Halfe Flaun. noble	2	6	5	0
Flem. Angell best	3	6	9	0
Flād royal or Key.	3	10	10	0
Carolus Gilden	1	21	3	6
Flaunders Royall	2	6	5	0
Saxon Gilden	2	2	4	8
Flaunders crowne	2	5	6	0
Phillips Gilden	2	3	4	2
Halfe Phil. Gilden	1	1	2	1

The names and titles of the gold.	The weight.		The value.	
	Pence.	Graines.	Shil.	Pence.
Golden Lion	2	16	7	8
3. parts of gold Li.	0	21		5
2.3.parts of gol.Li.	1	19	4	11
Dauids Gilden	2	2	4	0
Horne Gilden	1	12	4	11
Old Andre. Gild.	2	3		10
Cruſado long cros	2	0	6	0
Cruſado short cros	2	6	6	2
Myl rayes	4	20	13	4
Halfe Mill rayes	2	10	6	8
Portigue. 1. ounce	2	16	58	0
Golden Caſtilio	2	23	8	10
Ducket of Aragon	2	6	6	6
Hungarie Ducket	2	7	6	4
Double Piſtolat	4	8	11	8
Single Piſtolat	2	4	5	10
Ducket of Floren.	2	5	6	4
Double Ducket	4	11	13	0
Single Ducket	2	6	6	6
Dou.duc. of Rome	4	13	12	8

Currant

Of Siluer coines currant in this Realme.

The Edward Crowne of 5 s.
The Edward halfe Crowne of 2 s, 6 d.
The Edward Shilling, halfe Shilling, and the three pence.
Philip and Maries Shilling, & halfe Shilling.
The Mary Groat, and Mary Two pence.
Queene Elizabeths Shilling, 6 d, 4 d, 3 d, 2 d, 1 d, 3 ob 3 q, and ob.

Note.

It is to be vnderstood (gentle Reader) that whereas the waight is called by the name of a penny, it is not meant a penny of siluer money, but a penny of Goldsmiths waights, which is 24 Barly cornes dry. And 20 of these pence make an Ounce: and 12 of these ounces make a Pound Troy: So that if a man haue not the waight wherewith to weigh any peece that may come to his hand, he may doe it with the Barly graines or cornes being dry, and taken out of the middle of the eare.

Here would I now expresse the values of sundry other coines of diuerse Countries, but for three causes I now refraine. The first and chiefest is, because they are not currant by the Statues of this Realme. An other cause is, by reason they are so vncertaine, that they be neuer long at one rate. And againe they are so different in so many places, that it were mat-

matter inough for a great booke, to speake sufficiently of them all. Howbeit, yet because you shall not be altogether ignorant of them, I will shew you the values of some that are most in vse, and first of Fraunce.

The most common money are Deniers, Soulx and Franks. Twelue Deniers make 1 s, 20 Soulx make one Franke, so that as you see, these three kindes are like in the rate to pence, shillings, and pounds with vs, but that this is the difference, that their Denier is but the ninth part of our penny, and so their Soulx (commonly called Sowses) go nine to our shilling, and nine of their Frankes to an English pound of money: So that three of their Franks make a Noble. And by those three may you practise how to reduce Frech money into English money, according as I haue set foorth here following. French coines.

2160 deniers make 240 d, or 20 s.

3240 deniers make 360 d, or 30 s.

8352 deniers make 929 d, or 2 l, 17 s 4 d.

2160 soulx make 240 shillings. And so of other in like rate. As for the rest of their coines I omit them till hereafter that you haue some vnderstanding in broken numbers.

But now, as for the Coines of Flaunders they bee so changeable, that you must know them from time to time, else you cannot reduce them into our money certainely. But yet that you may haue an example of their Flanders coines.

money to exercise you withall, you shall take those that bee most common: as Stiuers both single and double, Groats Flemmish, Carolus, and Gildens. A Flemmish Groat is a little aboue thrée Farthings English. A single Stiuer is 1 d. ob. q. halfe farthing. The double stiuer is 3 d farthing. The double stiuer Carolus is 4 d. ob. halfe farthing. Then there is also the Carolus Gilden which is worth 20 stiuers. And the Flemmish Noble is worth 3 Carolus Gildens, and 12 stiuers.

French coines.

So that if you would conuert Flemish money, or any other kinde of money whatsoeuer it be, iustly into sterling, you must reduce it q. & into the smallest part of English money that is in that coyne, as for example. If I would reduce 368 double stiuers into English money, considering that a double stiuer containeth 3 d farthing, you shall first looke how many farthings be in the double stiuer, and you shall finde them 13, therefore multiply the summe of the stiuers by 13, and then haue you their value in farthings, which is 4784. Now if you diuide that by 4, then there will appeare the number of pence: but better it were to diuide it by 48 (for so many farthings are in one shilling) and then will the quotient declare the summe of the shillings.

Flanders coines.

Likewise, if you would reduce any sum of single stiuers into English money, you must multiply the summe first by 13, and then haue you

you reduced them into a certaine summe, that is to wit, halfe farthings: which summe if you diuide by 8, then will amount the summe of pence: or if you diuide it by 96, the summe of shillings will appeare.

But marke this in all Diuision, when yee do reduce to bring one Denomination into another, if there be any Remainer after the Diuision, that must be named by the Denomination of the grosse summe that was diuided: as for example. I would bring 254 farthings into pence, therefore I doe diuide that 254 by 4. for so many farthings make 1 penie, and the quotient is 63, which is the summe of the pence, and then remaineth yet 2, which are farthings still, as one may proue by diuiding. And this must be marked in all Diuision, namely when it is done for Reduction. Note well.

¶ Touching Danske money, they haue their Soulz, whereof 20 is a Liuer: which is two shillings sterling. They haue also their Grash, whereof 30 make a Gildrene, which is 4 6 sterling. They haue also Dollors, and their common or old Dollor is 35 Grash. New Dollors they haue, which be diuerse, some valued at 24 Grash, some at 26, and some at 30. And thus much I thought good to adde to the Author touching Danske money.

Concerning Spanish money, whereof the most common, are Cornados, Maruedes, Royals and Duckets: 6 Cornados make a

Mam

Marueide, 36 Marueides maketh 1 Ryal, and 11 Ryals maketh one Ducket, so the Ducket containeth 374 Marueides, which is about 5 shillings, 10 pence sterling. Therefore if you would conuert 124 li 5 s sterling into Duckets, consider that pence is the least value or Denomination named in this question, therefore reduce 124 pound 5 shillings into pence, and it maketh 29820 pence: which if you diuide by the pence that a Ducket is worth, which is 70, you shall haue for your quotient 426 Duckets, your desire.*

Note well. Weightes. Thus much haue I saide of money. now will I shewe you in like sort the distinction of weightes, after the Statutes of Englande, where the least portion of weight is commonly a Graine, meaning a Graine of Corne or wheate, drie and gathered out of the middle of the eare. Of these Graines in times passed, 32 wayed iust 1 peny of Troy, and then was but 20 pence in an Ounce. But now are there 46 pence in an Ounce, so that there are not fully 14 Graines in one peny. But now of Ounces after Troy rate (which is the standard of England) 12 do make 1 pound.

A graine. A peny of Troy. An ounce.

Haberdepoise weightes. But commonly there is vsed another weight called Haberdepoise, in which 16 Ounces make a pound. Therefore when you would reduce Ounces into poundes, you must consider whether your weightes be Troy weight or Haberdepoise: and if it be Troy weight, you must

must diuide your ounces by 12, to bring them to pounds, but if it be Haberdepoise, you must diuide them by 16. Now againe, there be greater weights which are called an hundreth, halfe a hundreth, and a quarterne, and also a halfe quarterne, &c. A hundred weight.

Scholer. Why? so there may be reckoned 20 pound, 40 pound, 200 pound, and such innumerable.

Master. All these are numbers of weight, but they haue not common weightes made to their rate, as the other haue. And againe, these that I did name are not iust in number as they seeme by their name, for an hundreth is not iust 100, but is a hundreth and twelue pound. And so the halfe hundreth is 56: the quarter 28, and the halfe quarter 14. And these be the common weightes vsed in most things that are solde by weight.

Howbeit there are in some things other names, as in wooll, 28 pound is not called a quarterne but a Todde: and 14 pound is not named halfe a quarterne, but a Stone, and the 7 pound halfe a Stone. Other names because they differ in many places, and agree in few, I let them passe. weights. Todde. Stone.

But a Sacke of wooll by the statutes, is limited to be 26 Stone. Sacke.

¶ Nowe in Cheese, though it be solde by the hundreth, and by the Stone in some places, yet the verie weightes of it are Cloues Cheese weights.

and Weyes. So that a Cloue containeth 8 pound: and a Wey 32 Cloues, which is 256 pound, that is 12 score and 16 pound: And so much weyeth the weigh of Suffolke chéese: And the like weight is or should be the barrell of Suffolke Butter.

The Wey of Essex Chéese containeth 16 score, and 16 pound: and so much is also the Barrell of Essex Butter.

Measures for liquor.

Now of weights are made other measures, both for graine & liquor. For a pound in waight maketh a Pint in measure, so that 8 pound (or 8 pintes) doe make a gallon: halfe a Gallon is named a Pottle: and halfe a Pottle is called a Quart, which containeth two Pintes. Now aboue a Gallon the next measure is a Firkin: then a Tertian, a Kilderkin, or halfe Barrell, and a Barrell. And by those measures are sold commonly Ale, Béere, Wine, and Oile, Butter and Sope: Salmon, Herrings, and Eeles.

A pint. Gallon. Pottle. Quart. Firkin. Tertian. Kilderkin. Barrell.

Ale measures.

But as these be vnlike things, so the measures of their vessels doe differ: for the measures of Ale are as followeth.

Of Ale	the Firkin	conteyneth	8	Gallons.
	ẏ Kilderkin		16	
	the Barrell		32	

Of béere	the Firkin	conteineth	9	Gallons.
	ẏ Kilderkin		18	
	the Barrell		36	

Sope

Sope meaſures, both Firken, Kilderkin, and Barrell, ſhould be equall to Ale meaſures. Moreouer, the ſtatutes doe limite the waight of euery of thoſe thrée veſſels being empty. Sope meaſures.

A Barrell	to	26	
Halfe Barrell	weigh	13	pounds.
A Firken	empty	$6\frac{1}{2}$	

Herrings are to be ſold by the ſame meaſures that Ale and Sope be ſold by. Herring.

Herrings alſo are ſold by the tale, 120, to the hundred, ten thouſand to the laſt.

Salmon and Eeles haue a greater meaſure. Salmon and Eeles.

	the Butte		84	
Salmon	the Barrell	hol-	42	Gallons.
& Eeles	halfe Bar.	deth	21	
	the Firkin		$10\frac{1}{2}$	

Howbeit, ſome ſtatutes did limite Eels veſſels equall with Herring veſſels.

Now as for wine veſſels they are ſeldome ſmaller than Hogsheades which are of 63 gallons: euery Hoggeshead is two Barrels: yet there are many other wine veſſels, but of them all, ſée this Table, and marke the meaſures one to another. Wine meaſures.

Of wine and oile,	the Rondlet	hol- deth	$18\frac{1}{2}$	Gal- lons.
	the Barrell		$31\frac{1}{2}$	
	ÿ Hogshead		63	
	the Tertiã		84	
	the Pipe		126	
	the Tonne		252	

Tertians. But you shall marke that there bee other kindes of Tertians: for there bee Tertians (that is to say) Thirdles of Pipes, of Hoggesheads, and of Barrels, as well of other things as of wine.

Butte. Also of Malueseys, and Secke, &c. the halfe Tonne is not called a Pipe, but rather a But.

And thus much haue I thought meete to tell you at this time.

Scholer. And is that alwaies true?

Maister. I haue tolde you how it should bée, but how it is, I may not say: how they doe differ dayly from their iust measure, that Gagiers can tell you better than I. But I will let this passe now, and speake briefly of the other measures.

Drie measures. And as of weightes there did spring the liquid measures, (whereof I spake last) so of the same springeth dry measures: as Peckes, Bushels, Quarters, and such like, whereby are measured corne and like graines: and salt, lime, coales, and other like. And this is the order and quantity of them.

A pecke. A Pecke is the measure of two Gallons.

A

A Bushell containeth foure peckes. A Bushe[ll.]
A quarter holdeth eight bushels. Quarte[r.]
A wey conteyneth sixe quarters. Wey.

These are the common names and measures, but in diuerse places there bee diuerse sortes.

The bushell in many places is 2 bushels: but then is the bushell there called a Strike. Strikes.
And in some places halfe a quarter is called a Cornoke. But these diuersities are too many to tell you briefly them all. And againe, sith they are against the law and statutes, I count them vnmeete to be vsed.

But now remaineth yet another kinde of measures, whereby men meate length and breadth, and thicknesse, and those are an inch, a foote, and such other: whose names and quantities this table sheweth. Measure to meate length, breadth, and thicknesse.

3 graines of barly in length make an inch. An inch.
12 Inches make a foote. Foote.
3 foote make a yard. Yard.
3 foote and 9 inches make an Elle. Elle.
5 yards and a halfe, make a perch. Perch.
1 perch in breadth and 40 in length, do make a rodde of land, which some call a rodde, some a yard land, and some a Farthendele.
2 farthendels make halfe an acre of ground,
4 Farthendels make an Acre. Acre.

¶ More 40 rods in length doe make a furlong: 8 furlongs make a mile, which contayneth 320 perches.

So that an English mile grounded upon the statute, is in length 1760 yards, 5280 foote, and 63360 inches.

Somewhat greater than the Italian mile of 1000 paces, and 5 foote to a pace.

Here might I tell you many things else touching measure, and also howe to reduce strange measures to our measures, but because it cannot be well done without the knowledge of Fractions, which as yet you haue not learned, I will let them passe till another time, that I haue taught you the knowledge of broken numbers.

The parts of time. Scholer. But yet Sir of the parts of time I pray you tell me somewhat.

A day. An houre. Weeke. Moneth. Yeare. Maister. You know that a naturall day hath foure and twenty houres, and euery houre hath sixty minutes. It needeth not to tell you, that seuen daies make a weeke, and foure weekes make a common moneth, and thirteene moneths make a yeare, lacking one day, and certaine houres, and minutes: But of that I shall instruct you hereafter.

Here will I make an end of Reduction for this time, which though it be counted no kinde seuerall of Arithmetike, yet you see it is no lesse needefull to be knowne, or easier to bee done, than any of the other.

Scholer. Mary Sir, it seemeth vnto mee much harder than any other sort, for it requireth the knowledge of so many thinges: but

but now ſir when you ſee time, I am readie to learne forth: for as much of Reduction as you haue taught me, I remember, but and if I doe at any time forget, I ſhall haue recourſe to the tables you ſet forth for me.

Maiſter. So doe you, for it will not be remembred without exerciſe. But in as much as you vnderſtand ſo much as we haue intreated of, I will now alſo inſtruct you in Progreſſion.

Progreſſion.

ALthough vntill this day the moſt part of writers haue defined Progreſſiō as a compendious kinde of Addition, yet truly it is not ſo: for Progreſſion (as the very nature of the word doth informe any man) is a going forward and proceeding in numbers, & that regularly and orderly, whoſe place is aptly choſen to be very neare, or rather next after the expoſition of the foure principall parts of Arithmetike, for in it after a moſt eaſie maner, are all the foure former parts exerciſed & practiſed: and not onely Addition, as cuſtomably is done. Which cuſtome hath bene the cauſe why it hath ſo ſpecially bene named a kinde of Addition, and defined to be a quicke

and briefe Addition of diverse summes proceeding by some certaine and reasonable order.

You shall also understand, that there are infinite kindes of Progressions, but for you (as yet) two are sufficient to be exercised in: of which the one I call Arithmeticall, and the other Geometricall.

Arithmeticall Progression.

Arithmeticall Progression is a rehearsing or placing downe of many numbers, number after number, in such sort that betweene every two next numbers rehearsed or placed downe, the difference, diversitie, or excesse be equall and alike.

Scholer. Syr, I thanke you for that you have both opened unto me what Progression is truly, and also why it is here placed. But I pray you with an example make plaine your definition.

Master. Examples cannot want, seeing all reasonable creatures naturally use the order of one kinde of Arithmetical progression (which therefore is also named Naturall) whensoever they distinctly doe count or number anie multitude by one: saying, 1, 2, 3, 4, 5, 6, whereby the proceeding from number to number, and everie one surmounting and exceeding his fellow next before by a like quantitie (which here is 1) declareth the same to be Arithmeticall Progression. And for the more plainenesse, I set it downe in this manner.

The

The common excesse. 1 1 1 1 1

The Progression 1 2 3 4 5 6

Scholer. This is most euident. And I thinke that I am able to tell you nowe of any Progression Arithmeticall propounded, what is that common excesse or difference whereby it procéedeth if this order be kept in it.

Maister. What say you of 3, 6, 9, 12, 15?

Scholer. They excéede eachother by 3. And that may I set downe in such euident order, as you did your example of naturall Progression, in this wise.

The common excesse 3 3 3 3

The Progression 3 6 9 12 15

Master. And do you not also now perceiue, that the whole table of Multiplication may be made by the order of Progression Arithmeticall: either if you will begin at the first number of any of them on the left hand, and so procéede right ouerthwart: or at any of the first numbers of the vpper row, and go directly downeward.

Scholer. I pray you let me consider the thing a litle, and I will answer you.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

By this triall I perceiue it now verie well: for the common excesse or difference betwéene any two next, is continually as much as the first number of euery row, either from the left hand ouerthwart taken, or from any of the vppermost ouerthwart rowes downeward.

To know the totall summe of an Arithmeticall progression.

Master. Nowe then if of any such Progression you would spéedily know the total summe, much quicklier than by common rules of Addition: first tell how many numbers there are (which numbers here we call places or parcels) and if they be odde, write their summe downe by it selfe, as in this example, 2, 4, 6, 8, 10, 12, 14. where the numbers are 7 as you may sée: therefore set downe 7 in a place alone: then adde together the first number and the last, as in this example; adde 2 to 14 and that maketh

maketh 16, take halfe of it, and multiply by the 7, which you noted for the number of the ſame places, and the ſumme that amounteth, is the ſumme of all thoſe figures added together, as in this example: 8 multiplied by 7 makes 56: and that is the ſumme of all the figures.

Scholer. That will I worke by an other example. I would know how much this ſumme is, 5, 8, 11, 14. 17, 20, 23, 26, 29. I tell the places, and they are 9, that I note. Then I put the firſt number 5, and the laſt 29, together, and they make 34. I take the halfe of it, that is 17, and multiply by 9, and it maketh 153. That you ſay is the ſumme of all the numbers.

Maiſter. So ſhall you finde it if you try it.

Scholer. How ſhall I try it?

Maiſter. By your common Addition: for if you adde all the parcels together, you ſhall ſee the ſame ſumme amount, if you did worke well. And that manner of Addition trieth all kindes of ſumming any Progreſſion.

Scholer. Then can I ſumme a Progreſſion if the numbers of the parcels be odde. But what if they be euen? as in this example, 1, 2, 3, 4, 5, 6, 7, 8.

Maiſter. When the number of the parcels is euen, then note that alſo as you did before, and likewiſe adde the firſt ſumme to the laſt, and by the halfe of the number of the places doe you multiply it: as in your example, the

parcels

parcels are 8, that I note: then adding the first summe to the last, there amounteth 9, that doe I multiply by the halfe of parcels, that is by 4, and it maketh 36, which is the summe of the parcels.

☞ But if you will take one rule for these both, doe thus. Multiply the halfe of the one by the other whole, and the summe will amount all one. For sometime it chaunceth that the number of the parcels bee odde, so that their halfe cannot be taken: and sometime it chaunceth the Addition of the first number and the last, to bring forth an odde number, so that the halfe of it cannot be taken: but they will neuer bee both odde.

A general rule.

Scholer. Then I perceiue this, if there be no more longing to it.

Maister. This is inough for Progression Arithmeticall: howbeit there is an other maner of Progression called Geometricall, when the numbers increase by a like proportion, that is, if the second number containeth the first, 2, 3, or 4, times, and so foorth: then the third contayneth the second so many times also: and so the fourth, the third, and the fifth, the fourth, wherefore I set these three examples.

Progression Geometricall.

3,	6,	12,	24,	48.
1,	3,	9,	27,	81.
2,	10,	50,	250.	

Here in the first example you see, that euery number containeth the other (that goeth next

next before him) 2 times: and in the ſecond example 3 times, in the third example 5 times. Now if you would know how to finde eaſily the ſumme of any ſuch numbers, doe thus: Conſider by what number they be multiplied, whether by 2, 3, 4, 5, or any other, and by the ſame number doe you multiply the laſt ſumme in the Progreſſion.

Scholer. I pray you worke it by this example, 2, 8, 32, 128, 512, 2048, which I haue framed by procéeding from 2, and continually multiplying by 4.

Maiſter. Then muſt I multiply the laſt ſumme (which is 2048) by 4 alſo, and it will be, 8192. Now muſt I bate from this ſumme the firſt number of this progreſſion, which here is 2, then reſteth 8190, which ſumme I muſt diuide by 1 leſſe then was the number that I multiplied by. Seeing then I multiplied by 4, I muſt diuide by 3, ſo diuiding 8190 by 3, the Quotient will bee 2730, which is the ſumme of all the Progreſſion. And now to prooue whether you can doe the ſame, I giue you theſe numbers to adde by this rule, 3, 15, 75, 375, 1875, 9375, 46875.

Scholer. I cannot well tell by what number this Progreſſion doth encreaſe.

Maiſter. In any ſuch doubt, doe thus: Diuide the ſecond number by the firſt, and the Quotient will ſhew you the number that ex-

gendreth the Progression.

Scholer. Then is that number in this example, 5, for so many times is thrée in 15.

Maister. So is it. Now worke as I taught you.

Scholer. The last number is 46875, which I multiply by 5, and it yéeldeth 234375, from which I abate the first number of the Progression, that is 3, and there resteth 234372, which I diuide by 4, for that is one lesse than 5, and the quotient is 58593, which is the whole sum of the Progression.

Maister. If you remember well this, you haue learned the Art of Progression, both Arithmeticall, and also Geometricall, which you may proue either by Subtracting of each number alone from the summe, and so will there nothing remaine: or else by adding together of all the parcels, for so will the same sum amount.

¶ And now for the vse and better vnderstanding of both these rules, I will propone vnto you certaine pleasaunt and necessary questions Arithmeticall and Geometricall. And to the performance of their workings, such necessary rules and documents, as are requisite for the vnderstanding of them, or any such like.

A certaine Mercer sold 20 yardes of veluet to bee paide in 12 wéekes, by Arithmeticall proportion: that is to wit, to receiue the first wéeke

weeke 6 shillings, the second weeke 12 shillings, the third weeke 18 shillings. And so forth increasing the number of weekes by 6 shillings, till the twelfth and last weeke were expired: the question is, how many pounds he had for 20 yardes of veluet.

To the performance of this question and such other the like, I set forth the 12 paiments in such sort, as for example here appeareth in the margent.

6
12
18
24
30
36
42
48
54
60
66
72

Then touching the adding together of these summes without the aide of Addition, according to the rules I taught you in Progression Arithmeticall, I note the number of the places, which are 12, then adding the last number of the Progression which is 72, and the first number together make 78, and multiplying 78 by halfe the number of the places, which is 6, amounteth to 468 shillings, and in pounds maketh 23 pounds, 8 shillings. And so much hath the Mercer for his 20 yards of veluet, which is nigh about 23 s, 5 d. a yard.

Scholer. I understand this worke veris wel: but is there any proofe for the iustifying hereof, as you haue of other workes?

Master. The worke of it selfe (being so perfectly wrought) that in your proceeding and going forward from number to number, each number exceeding his fellowe by an equall or like quantitie, is all that is demaunded for iustifying of the same: yet notwithstanding be-

cause your request is reasonable, I will propone an example for the proofe hereof.

The proofe of the last question.

A certaine man is bound to pay for twentie yardes of Veluet, the summe is 23 pound, 8 s: and it is to be payed weekely in twelue weekes or termes by Arithmeticall Progression. The question is therefore to know with what number the same Progression is to be begun, and continued in such equall portion Arithmeticall, that in 12 weekes the same may iustly be accomplished.

For the solution whereof, and of all such other like, reduce 23 l 8 s. all into shillings, which maketh 468 shillings.

A generall rule.

Then adde 1 vnto 12 the number of the termes, it maketh 13, which 13 you shall multiplie by halfe the number of the termes which is 6, it maketh 78: then diuide 468 by 78, and you shall finde 6 in the quotient, which is the true number that shall begin and continue the said Progression. That is to say, the first weeke 6 shillings, the second, 12, s and the thirde weeke 6 s more, which is 18 s, & so euerie weeke as they rise, 6 s more than the weeke before, as is manifest in the question aforesaid.

A Farme is to be solde to be payed by the weekes in a yeare, the first weeke to pay 4 s, the second weeke 8 s: the third weeke 12 shillings and so forth, increasing each number by 3 till the number of 52, which are the number of weekes in a yeare be expired. The question is what

what the price of the farme commeth to.

Scholer. I doubt not but by that you haue alreadie taught me, to end this question verie well. wherfore I set forth the Progression with his excesse 52 times.

Master. Nay, stay a while. And here for your further ease, (to abridge you of great labour that appeareth to fal out in this question, and so may do in many other the like, if a question were proponed of 100 or 200 places or mos, and that this question nor any other the like can be ended, vnlesse you know absolutely what the last number of the Progression at the 52 place is or ought to be) I will giue you a generall rule how to knowe the last number of any Progression Arithmeticall, as well as if you had ordinarily procæded by continual Addition, till you had come to the last worke, which is this.

Multiplie the excesse by a number lesser by one then the number of the places, and thereto put the first number of the Progression, and you shall haue your desire. A generall Rule.

Scholer. This rule is well worth the noting: for if I vnderstand you aright, I consider that my excesse is 4, which I multiplie by 51, which is one lesse then the number of the places, and it maketh 204: whereunto I adde the first number of the Progression which is 4, and then it is 208: which you say is or should be the last number of the Progression.

Maister. This is a most approoued truth if there were neuer so many places.

Scholer. This rule is so easie, that I were much to blame, if I do not remember it. For by the benefite hereof I haue such an ease and light into this excellent Arte, that my first entrance doth seeme to passe a great many mens further studie and longer continuance.

Master. Many moe considerations coulde I propound you in these Arithmeticall progressions: but these are sufficient for a tast, to giue you occasion to thinke that Rules of knowledge and Arts are infinite capable of enlargement.

Scholer. Happie were I, if I did but well vnderstand that which is already inuented and written. But these things in my simple fantasie offer themselues to be greatly beneficiall into the aide of Progression. Therefore now I will go forward with your question.

Now considering that the 52 and last place is 208, I adde thereunto the first number of the Progression which is 4, it maketh 212, which I multiplie by halfe the number of the places, which is 26, and it amounteth to 5512 shillings. And so much is the totall summe or Adition of this Progression: which maketh 275 l, 12 s as appeareth here by my tables.

Master. I like well your labour, and commend you for your diligence. I will here propone one example more, and therewithall for this

this time will end progreſſion Arithmeticall.

A certaine man bought 20 Elles of Holland to be payd in 17 wéekes or termes by Progreſſion Arithmeticall. And the firſt wéeke to pay 1 s̄, 8 d̄, the ſecond wéeke 3 s̄, 4 d̄, the third wéeke 5 s̄, the fourth wéeke 6 s̄, 8 d̄, and ſo forth each wéeke ſucceeding 20 pence more then the wéeke before. The queſtion is what the ſumme of his 20 Elles commeth to.

Scholer. Becauſe here is mention made both of ſhillings and pence, I feare there is ſome harder matter contained herein, than in the other before, therefore I pray you worke it your ſelfe, and I will diligently marke your labour.

Maiſter. There is no more to be done in this, than in the other before, but becauſe your requeſt is ſo reaſonable, be attentiue vnto me.

Firſt by our generall rule I ſeeke to finde out the laſt number in the ſeuentéenth place, what this Progreſſion ought to be. Therefore here in my tables multiplying the exceſſe 20 by 16, which is 1 leſſe then the number of the termes or places, and it commeth to be 320, and thereunto adding the firſt number of the Progreſſion which is twenty pence, all is 340 pence, or 28 s̄, 4 d̄: for ſo much ought the laſt number of the payments to be.

Then finally to know what the whole 17 places amount vnto, I adde the firſt number

of the Progression and the last together, both which make 360: Now because 17 is an odde number, whose halfe cannot be taken, I take the halfe of 360 which is 180: and multiplying 180 by 17, it commeth to 3060 pence, which maketh as you see by division 12 £, 15 s. And so much is the buier to pay for his 20 elles of Holland. Which 3060 pence if you divide by 20, the number of Elles that was bought, you shall finde 12 s, 9 d, and so much payd he for an Elle one with an other.

The Proofe.

A certaine man doth owe 12 £, 15 s, to bee payde in 17 weekes or termes by Arithmeticall Progression. The question is to knowe with what number hee shall beginne and continue the Progression in such equall proportion, as the same may be truely payde and satisfied in 17 weekes.

The answere.

First I reduce 12 £, 15 s, all into pence: which as you see here in my Tables make 3060 pence, that I let stand by a while.

Then I adde 1 to 17, the number of the places or termes, which maketh 18, which I should multiply by halfe the number of the weekes or termes, which is $8\frac{1}{2}$ which $8\frac{1}{2}$ multiplied

tiplied by 18, cannot well be done vnleſſe you were acquainted with Fractions or broken numbers, therefore you ſhall let that paſſe, and multiply 17 by the halfe of 18: which is 9, (for that is all one with the multiplication of $8\frac{1}{2}$: and the multiplication of 9 into 17 maketh as you ſee 153, with which number you ſhall diuide the 3060 pence before ſayd: and the Quotient bringeth forth 20 pence, which is the firſt number or payment to begin the progreſſion withall: and ſo each weeke ſucceeding to riſe 20 d more than the weeke before, and thereby in 17 weekes ſhall the 12 pounds, 15 ſhillings, be paid, as before was ſufficiētly declared. Thus much for Progreſſion Arithmeticall.

Scholer. Certainely Sir I know not how to render you condigne thankes for theſe benefites ſhewed me, which me thinketh are ſo eaſy, delightfull and pleaſant, that I count my ſelfe happy to be in your company.

Maiſter. I am glad you delight ſo well herein, which is an Art of wonderfull dexterity to all ſortes of men of what degree or profeſſion ſo euer they bee. And now will I propone a queſtion or two of Progreſſion Geometricall.

A Mercer hath twelue yards of ſatten which he valueth at 16 ſhillings the yard, and ſelleth the ſame 12 yards to an other man to bee payde as followeth. That is to wit for the firſt yard to haue one ſhilling, for the ſecond yard two ſ:

for the third yard 4 s, for the fourth yard 8 s, and so forth doubling each number following till the twelfth and last yard. The question is who hath made the better bargaine of the buier or seller.

First you may sette downe 12 the number of the yards, as you see here in this example. And against each number the number of shillings due to be payde, as the order of Duplation or the multiplication by 2 teacheth.

s	
1	1
2	2
4	3
8	4
16	5
32	6
64	7
128	8
256	9
512	10
1024	11
2048	12

Then resorting to the adding up or summing of this progressiō, where I consider that the increase of this summe proceeded by the multiplication of 2. And therefore after I have drawne a line under the 12, worke and multiply the last summe by 2 also, and it yeeldeth 4096: from whence I abate the first number of the Progression which is one, and then resteth 4095: which I should divide by 1 lesse than I did multiply by, but seeing it is one, I neede not to divide it: for 1 (as I have sayd before) doth neither multiply nor divide, therefore I take that summe 4095 for the whole summe of the shillings, which by Reduction amoun-

amounteth to 204 li, 15 s, and ſo much hath the Mercer for his 12 yardes of ſatten: which is 17 li, 1 s, 3 d a yarde. But I thinke you will buy none ſo deare.

Scholer. No Syr by the grace of God this yeare. *

Maſter. Then what ſay you to this queſtion? If I ſold vnto you a horſe hauing 4 ſhoes, and in euerie ſhoe 6 nailes, with this condition, that you ſhall pay for the firſt naile 1 ob: for the ſecond naile 2 ob: for the thirde naile 4 ob: and ſo forth doubling vntill the ende of all the nailes: Now I aſke you how much would the price of the horſe come vnto?

Scholer. Firſt to know the number of the nailes, I muſt multiplie 6 by 4, and it maketh 24. Then will I doe thus: I will write the number of the nayles euerie one in order from 1 to 24, and againſt each number of the nailes the ſumme of halfe pence duely, as the order of Duplation or Multiplication by 2 teacheth, and as in this next figure following appeareth.

Then doe I reſort to the rule of Summing by this Progreſſion, where I conſider that the increaſe of this ſumme procéedeth by the multiplication of two, as the laſt example did. And therefore multiplying the laſt ſumme by 2 alſo, and it yéeldeth 16777216, from which I do abate the firſt number which is 1, and then reſteth 16777215, which I ſhould

1	1
2	2
4	3
8	4
16	5
32	6
64	7
128	8
256	6
512	10
1024	11
2048	12
4096	13
8192	14
16384	15
32768	16
65536	17
131072	18
262144	19
524288	20
1048576	21
2097152	22
4194304	23
8388608	24

diuide by 1 leſſe than I did multiplie: but ſeeing that it is 1, I neede not to diuide it, for 1 (as you haue before ſayed) doth neither multiplie nor diuide, therefore I take that number 16777215 for the whole ſumme of the halfe pence, which by Reduction I finde to be 699050 s, and 7 d ob: that is, 34652 pound, 10 s 7 pence, halfepeny.

Maſter. That is well done, but I thinke you will buy no horſe of the price.

Scholer. No Sir if I be wiſe.

Maſter. Well then, aunſwer me to this queſtion.

A Lord deliuered to a bricklaier a certen number of loades of Bricke, whereof he willed him to make 12 walles, of ſuch ſorte, that the firſt wall ſhould receiue two thirdles of the whole number: and the ſecond two thirdles of that that was left. And ſo euerie other two thirdeles of that that remained: and ſo did the bricke-

Brickelayer: and when the 12 walles were made, there remaineth 1 loade of Bricke.

Now I aske you, how many loade went to euerie wall, and how many loade was in the whole?

Scholer. Why Syr it is vnpossible for me to tell.

Master. Nay it is verie easie if you marke it well. Marke well that I sayed, that euerie wall should receiue 2 thirdles of the summe that was left. Now take away two thirdles from any summe, and you must néedes grant that that which remaineth is 1 thirdle of the summe last before: example of 9, from which if you take 2 thirdles, there will remaine thrée, which is one thirdle of 9. Likewise from thrée bate two thirdles: and there will remaine one.

Scholer. This is true, and nowe I perceiue, that the least wall had but two loade of Bricke.

Master. And by the same reason may you know how many loade euerie wall had, according as this figure following doth shewe, and likewise what the whole summe of bricks was: for if you make 12 summes, multiplying by 3, still from the last remainer, as you may sée here on the left side of the table, there will appeare all the remainers of the whole wall: and if you multiplie the last of those 12 summes by 2 also, then will that be the summe of the loades which were deliuered vnto the bricke-

Bricke-layer.

Againe, if you do double euerie Remainer, as you may see at the right side of this table, those numbers will shew the summe of loades that went to each wall: whereby you may perceiue, that each wall was three times so great as the next lesser.

Scholer. Loe now it appeareth easie enough. Now surely I see that Arithmetike is a right excellent Art.

Maister. You will say so when you know more of the vse of it: for this is nothing in comparison of the other poyntes that may be wrought by it.

Scholer. Then I beseech you sir, cease not to instruct me further in this wonderfull cunning.

The remainer after euerie wall.		Loades due to each wall.
1	12	2
3	11	6
9	10	18
27	9	54
81	8	162
243	7	486
729	6	1458
2187	5	4374
6561	4	13122
19683	3	39366
59049	2	118098
177147	1	354294

Summe of the 531441 Loades deliuered.

The

The Golden Rule.

Maiſter.

BY order of the ſcience (as men haue taught it) there ſhould follow next the extraction of rootes of number, which becauſe it is ſomewhat hard for you yet, I will let it paſſe for a while and will teach you the feate of the rule of Proportions, which for his excellency is called the Golden Rule. Whoſe vſe is, by three numbers knowne, to find out another vnknowne, which you deſire to know, as thus. If you pay for your boorde for three moneths 16 s, how much ſhall you pay for 8 moneths.

To know this and all ſuch like queſtions, you ſhall conſider which two of your 3 numbers be of one denomination, and ſet thoſe two the one ouer the other, ſo that the vndermoſt be it that the queſtion is aſked of: as in my queſtion 3 and 8 be both of one denomination, for they both be moneths, and becauſe 8 is the number that the queſtion is aſked off, I ſette them one ouer the other, and 8 vndermoſt, thus, with ſuch a crooked draught of lines. Then doe I ſette the

3

8

other number which is 16, against 3 at the right side of the line, thus.

3 — 16
8 —

And now to know my question, this must I doe: I must multiply the lowermost on the left side, by that on the right side, and the summe that amounteth, I must divide by the highest on the left side. Or in plainer wordes thus; I shall multiply the number, of which the question is asked (which is called the third number) by the number of another denomination, (which is called the second,) and the summe that amounteth must I divide by the summe of like denomination, which is called the first. Then for the knowledge of this question, I multiply 8 into 16, and there amounteth 128, which I divide by 3, and it yeeldeth 42 shillings, and 2 s. remaineth, which I turne into pence, and they be 24 d. of which the third part is 8 pence, so the third part of 128 s. is 42 s. 8 d: which summe I write at the right hand of the figure against 8 thus.

The third number. The secōd number. The first number.

3 — 16 s.
8 — 42 s. 8 d.

Hereby I know, that if three moneths boording doe come to 16 s. that 8 monethes boording will come to 42 shillings, 8 pence: and likewise of any other like question.

But here must you marke, that the first number and the third be of one denomination, and also the second and the fourth, the which

which you seeke: or else be of such denominations, that you in working may bring them into one. As if a man should aske mee this question.

¶ Twelue weekes iourneying cost mee 14 French Crownes at 6 s̃ the peece, how many pounds is that in one yeare. Here you see no two numbers of one denomination: but yet in working you may turne them into like denominations, as thus: turne the one yeare into 52 weekes, and the fourth summe will bee French Crownes, by the order of the working: Then to know this question, multiply the third summe 52 by the second 14, and the summe will be 728: that diuide by your first number 12, and the quotient will bee 60 Crownes: and 8 Crownes remaining: which if you turne into shillings, they will be 48 shillings, which if you doe diuide by your first number 12, the quotient will be 4: which signifieth 4 s̃, but these 60 French Crownes (which make 18 pounds, with the 4 shillings: for the summe that aunswereth to the question: and it is the iust expences of a yeare:) And the summe will be thus.

15		14
52		60 — 4 s̃

And take this euermore for a generall rule touching this whole Art, that the doubtfull or vnknowne number that you would be resolued of, shall alwayes bee sette in the third place. Note also the first number and the third

muſt euer be of one nature and denomination, or elſe muſt in working be brought to like denomination, and then of neceſſity muſt the other number be in the ſecond place. *

Remember alſo, that the place of the firſt number is the higheſt on the left ſide: and the place of the ſecond right againſt it on the right ſide: the place of the third number is vnder the firſt, as by thoſe examples you haue ſeene.

Scholer. This I truſt I can doe.

Maiſter. But and if the queſtion bee asked thus: In 8 weekes I ſpend 40 s, how long will 105 ſhillings ſerue mee? Here you ſee that 8 weekes anſwers himſelfe, and ſaith 40 ſhillings. But how long time 105 ſhillinges will ſerue, you know not. Therefore you ſhall ſet 105 in the third place, according as I told you euen now. And the firſt place muſt alwaies be of the ſame nature or Denomination, that the third is of, which here is 40. Then muſt 8 needs be that other. Now multiply 105 by 8, and it will be 840, which if you diuide by 40, it will yeeld 21, which is the fourth number, and ſheweth how many weekes 105 ſhillings will ſerue, if you ſpend 40 s in 8 weekes.

The figure of this queſtion is this: as if you ſhould ſay: If 40 s ſerue for 8 weekes, 105 ſerue

ſhillings.	weekes.
40	8
105	21

serue for 21 weekes.

Other diuersities there be of working by this rule, but I hadde rather that you would learne this one well, then at the beginning to trouble your minde with many formes of working, sith this way can do as much as all the other, and hereafter you shall learne the other more conueniently.

¶ And for your further aide and instruction to make you better acquainted with this Golden Rule, I haue here propounded 6 questions and their answers, which I thinke most conuenient and meete to preferre the desirous to perfect vnderstanding. The first foure are all braunches of one question sprung out of the best tree, (for a young learner to taste of) that groweth in this Ground of Artes, for that no manner of question in the Rule of 3 whatsoeuer it be, can be proponed, but it must be comprehended vnder the reason or stile of one of these foure.

The questions be these.

If 15 Elles of Cloth cost 7 ł, 10 s: what comes 27 Elles to at that price? Answer. 13 ł, 10 shillings.

If 27 Elles cost 13 ł, 10 s: what are 15 Elles worth? Answer. 7 ł, 10 s.

If 27 Elles cost 13 ł, 10 s: how many elles shall I haue for 7 ł, 10 s? Answer. 15 Elles.

If I sell 15 elles for 7 li, 10 s: how many els are to be deliuered for 13 li, 10 s: Ans. 27 elles.

If 8 pound of any thing cost 16 s 6 d: what money is to be receiued for 49 pound? Answer. 5 li, 1 s, 0 d, ¼.

If 4 li of any thing cost 17 d: what money will 8765 pounde of that commoditie cost? Answer. 155 li, 4 s 3 d, q.

Of all which questions I omit the worke of purpose, that you should whet your wit therby at conuenient leisure, to clime each braunch, and gather the fruite of them, and doe minde now, before we make an end of this rule, to giue you some instruction of the Backer rule of 3, whose order is quite contrarie to this that you haue learned.

For in this Rule hitherto, euermore looke how much the third number is greater than the first, so much the fourth number is greater than the second. And contrariwaies: looke how much the first summe is greater than the third, (if it do chance so) so much is the second summe greater than the fourth. But in this Rule, there is a contrarie order, as this: that the greater the third summe is aboue the first, the lesser the fourth summe is beneath the second: and this rule therefore you may call the Backer rule, as in example.

The backer Rule.

Question of buying cloth.

If I haue bought 30 yardes of cloth, of two yardes breadth, and would buy canuas of three yardes broade to line it withall, howe many

yardes should I néede?

Scholer. Why there is none so broad.

Master. I doe not care for that, I doe put this example onely for your easie understanding. For if I should put the example in other measures, it would be harder to understande. But nowe to the matter: if you would knowe this question, set your numbers as you did before: but you shall multiplie now the first number by the second, and that ariseth thereof, you shall diuide by the third: which thing if you do héere, I meane if you multiplie 30 by 2, it will be 60: which summe if you diuide by 3, there will appeare 20: whereby I knowe, that if 30 yardes of cloth of two yardes broade, should be lined with canuas of thrée yardes broad, 20 yards of canuas would suffice, as this figure sheweth.

Breadth.	*Length.*
2	30
3	20

And now because you found fault with my example, how say you, perceiue you this?

Scholer. Yes Sir I suppose.

Master. Then answer me to this question: how many Elles of canuas of Elle breadth, will serue to line twenty yardes of Say, of 3 quarters broad.

Scholer. In good faith sir I cannot tell, for I know not how to bring the summes to like Denominations.

Master. Then will I tell you: sith there is mention here of quarters, and againe euerie one of the measures both Elles and yards may be parted into quarters, do you part thẽ so both in the breadth, and length, and then put forth the question by quarters.

Scholer. Then shall I say thus. How many quarters of canuas of fiue quarters broad, will line 80 of thrée quarters broad.

Maister. Now answer to the question.

Scholer. First I will set them downe in their forme thus, for 5 is ioyned with the question, and is therfore the third number: then is 3 the number of the same denomination, I meane because they be both referred to breadth. Nowe I multiplie 80 by 3, and it is 240, which I diuide by 5, and it yéeldeth 48. Then say I, that 48 quarters of 5 quarters broad, will suffice to line 80 quarters of 3 quarters broad.

Breadth	*Length*
3	80
5	

Master. Turne the quarters againe into Elles and yardes.

Scholer. Then I say, that 9 Elles and thrée quarters of a yard of elle broad, will serue to line 20 yardes of thrée quarters broad, as this figure sheweth.

Breadth	*Length*
3	80
5	48

¶ Master. Nowe what say you to this ques-

question: I lent my friend 400 li for seuen moneths, how much money ought hee to lend me againe for 12 moneths, to recompence my courtesie shewed him. Can you aunswer to this?

Scholer. Yes Sir I suppose; for I will set downe my Numbers thus: where I multiply 7 into 400, and it

Moneths.	*Pounds.*
7	400
12	

maketh 2800, which I diuide by 12, and it yeeldeth 233 li, and there is 4 li remaining of my diuision, what shall I doe therewith?

Maister. Turne the same 4 li into s. and then diuide it by 12 as you did before.

Scholer. Well Sir it shall be done, so haue I 6 s for my quotient, and yet remaineth 8 s vppon my diuision.

Maister. You must also reduce that 8 s into pence, which maketh 96, and diuide that also by your first diuisor.

Scholer. So haue I done, and I find 8 pence for my quotient and nothing is left.

Maister. This must you alwayes do, when any thing remaineth vpon your diuision: whether it be money, waight, measure, or any kinde of thing whatsoeuer. This rule is so profitable for all estates of men, that for this rule onely (if there were no more but it) all men were bound highly to esteeme Arithmeticke.

By this rule may a Captaine in warre worke many things, as Maister Digges in his Stratiaticos doth declare: onely now in this my simple Addition, for a tast and encouragement, I will enlarge the Authour with a question or two more, wishing you, and every my Countreymen or Gentlemen whatsoever, that by nature be any thing given to Military affairs, to bee familiar and acquainted with this excellent Art, the which hee shall finde not onely at the Sea, but also in the Campe and field service, aboundantly to aide him, either in fortification, paying of souldiers wages, charges of ordinaunce, poulder, shot, munition and instruments whatsoever, as for example.

Question of prouision touching an armie.

If it should chaunce a Captaine which hath 40000 souldiers to bee inclosed with his enemy, that hee could have no fresh purveyance of victuals, and that the victuals which hee hath, would serue that army but onely three moneths, how many men should hee dismisse, to make the victuall to suffice the residue eight moneths?

Scholer. As you taught me, I set the numbers, thus, saying: If 3 monethes suffice 40000, to how many will 8 monethes suffice?

Moneths.	*Men.*
3	40000
8	

To know this, I multiply the first number

ber 3 into the ſecond 40000, and it yeeldeth 120000, which ſumme I diuide by 8, and there wil be in the quotient 15000, which if I do ſubtract from 40000, the remainer will declare that he muſt diſmiſſe 25000 as this figure ſheweth.

Moneths.	*Men.*
3	40000
8	15000

¶ Maiſter. Now aunſwere me to this queſtion: If 136 Maſons in a moneth bee able to build a fort to preſerue the ſouldiers from the enemy, and ſuch expedition requireth that I would haue the ſame finiſhed in eight dayes, how many workemen ſay you is there to bee appointed?

Schol. As you taught mee I ſet the numbers thus, ſaying: if 28 daies require 136 Maſons, what number of men by the like proportion will 8 daies require.

28	136
8	

To know this, I multiply the firſt number 28 into 136: and it yéeldeth me 3808: which I diuide by 8, and my quotient is 476, which is the iuſt number of Maſons that ſhall ſupply this worke. And now me thinke theſe queſtions are very eaſie.

Maiſter. Truely if you take delectation herein, you ſhall finde this art not onely eaſie, but wonderfull pleaſaunt and profitable. Now therefore one queſtion more I will pro-

pose, and so leaue of this rule in whole numbers, vntill we come to the vse of it in broken numbers, for had you the vnderstanding of broken numbers perfectly, not onely in this rule, but in all other, the question that in sight or appearance seemeth to be 100 times more harder to absolue, may thereby bee wrought as soone, or sooner than this.

Scholer. Your wordes doth greatly encourage me to be studious to attaine whole numbers, but might I once attaine to be a practitioner in broken numbers, I should thinke my selfe happy.

Maister. What say you then to this question? If 48 Joiners in two daies make 200 light horsemen staues (esteeming the worke but 12 houres a day) and such neede requireth that 384 Joyners are sette to the finishing of those 200 staues, in what time say you, will they make them vp?

Scholer. I see here that I must turne my 2 daies into houres. And so doing I set my numbers thus.

48 —— 24
384 ——

Saying, if 48 men are 24 houres, 384 men will make an end quickly. For it is grounded vppon an olde prouerbe, many hands make quicke speede.

I multiply 48 into 24, and it amounteth to 1152, which I diuide by 384: and my quotient

quotient is 3 houres, which is my desire.

I take this for a note worthie the marking, either in the rule of three, forward, or backeward. when the two numbers are multiplied together, the product is of the same nature and Denomination that the seconde number is of.* Note.

Master. Well sith you perceiue nowe the vse of this rule, I will shew you other which ensue of the same, and first the double rule which is so called, because there is in it double working, by which thing onely it differeth from this. The double rule.

Scholer. Then by an example I shall vnderstand it well enough.

Master. So shall you, and let this be the example, If the cariage of 1 C weight (that is to wit 1 1 2 pound) 30 miles, doe cost 12 pence, how much will the cariage of 5 C weight cost, being caried 100 miles? Of cariage

Scholer. I pray you shew me the working of it.

Master. You must make 2 workings of it: the first thus. If 1 C weight cost 12 d, how much wil 5 C weight cost? Set your figure thus

C. Weight.	Pence.
1	12
5	

And multiplie 5 by 12, and thereof amounteth 60, which if you diuide by 1, the Quotient will be still 60, that is the price of 5 C

weight for 30 miles.

Then begin the second worke, saying: If 30 miles cost 60 d. how much will 100 miles cost? Set your figure thus.

Miles	*Pence*
30	60
100	

Then multiplie 100 by 60, whereof amounteth 6000, which being diuided by 30, will yeeld 200 d. Then you may say that so many pence shall cost the cariage of 500 pound weight 100 miles, after the rate of 12 pence for the 100, caried 30 miles.

Scholer. Now I perceiue it also.

¶ Master. These and such other like questions of the double rule of 3, are to be answered much sooner, at one onely working by the Rule of 3 composed by fiue numbers, which anon I will shewe you, and then when you haue the vse thereof, you may vse which way you thinke good.

Scholer. Syr I thanke you much for your courtesie, and I long now till this rule be ended, that I may see how I shall behaue my selfe with that newe rule of 5 numbers, for that I haue euer since you taught me hitherto, in the Golden Rule both forwarde and backeward wrought but with three numbers onely. *

Master. But yet a while wee will goe on forwarde with this double rule of 3: therefore answer me to this question: 30 bushelles of

Question of sowing.

wheate

wheate sowed, yeelded in one yeare 360: how many will 80 bushels yeeld in seuen yeare? I meane sowing euerie yeare of those seuen, still 80 bushels.

Scholer. First I say, that if 30 bushelles will yeelde 360 in one yeare, then 80 bushelles will yeeld 960 in one yeare. Then for the second worke I say: If one yeare yeeld 960, then seuen yeare will yeelde 6720: as these two figures do shew.

Seede.	*Encrease.*	*Yeares*	*Encrease.*
30	360	1	960
80	960	7	6720

But now Syr, if I set forth 30 bushels of corne to another man for 7 yeare, agreeing so that he shall sowe euerie yeare the whole encrease of the corne, and I at the ende of those seuen yeares to haue the halfe of the whole encrease: I would know howe many bushelles will there amount to my part, supposing the encrease to be after the rate of the last question, for 30 bushels in one yeare, to yeeld 360. Question of corne.

Master. In such a question you must haue so many seueral workings, as there be yeares, as for example: In the first yeare 30 bushels yeelde 360: then to knowe the yeelding of the second yeare, I must say: If 30 yeeld 360, how many yeeldeth 360? Worke by this your rule, & you shall finde 4320. Then say for the third

yeare: If 30 yeeld 360, how many will 4320 yeeld? you shall haue 51840, and so euerie yere multiplying the whole encrease by 360, and diuiding it by 30, the increase of the next yeare will amount, as these 7 figures following doe orderly declare: where I haue set 7 letters for the 7 yeares, of the which the first is set without Arte, because that is the encrease which you do presuppose: and the last number of each other doth shew the encrease of the yeare that it standeth for, which the letters do declare, so that the encrease of the 7 years is 1074954240 bushels: how many quarters that is, and also how many weies, you may by Reduction soone finde.

a	b
30 —— 360	30 —— 360 360 —— 4320
c	d
30 —— 360 4320 —— 51840	30 —— 360 51840 —— 622080

e

30 —— 360
622080 —— 7464960

f

30 —— 360
7464960 —— 89579520

g

30 —— 360
89579520 —— 1074954240

Now with one question more I will proove you. If 6 mowers do mow 45 Acres in 5 daies, how many mowers will mowe 300 Acres in 6 dayes?

Scholer. If 45 Acres require 6 mowers, then 300 Acres requireth 40. Now againe: if 5 dayes require 40 mowers, then 6 dayes needeth but 33 mowers.

Maister. Why doe you not make mention of the two that remaineth in the last Diuision? for the last part of the question is wrought by the Backer rule, where the first number 5, is multiplied into the second, that is 40, whereof amounteth 200, which if you diuide by the third number 6, the quotient will bee 33, as you sayde, but then will there remaine two, which cannot well be diuided into sixe partes: howbeit, you may understand by the sirt part of two the third part of one mans worke, which you must put to the 33; or else you must say, that 33 workemen will end all the 300 acres in sixe dayes, saue two mens worke for one day, or two dayes worke for one man. But such broken numbers called Fractions, you shall hereafter more better perceiue, when I shall wholly instruct you of them.

¶ Maister. Yet one question more of field matters I will propone, and so I will make an end of this double Rule of 3.

Scholer. With all my heart sir I thanke you, and I will dispatch it as soone as I can

because I would faine see the order of the next rule of 5 numbers.

Maiſter. If a Captaine ouer a band of men, did ſet 300 Pioners a worke, which in 8 houres did caſt a trench of 200 Rods: I demaund how many Laborers will be able with a like trench in three houres to entrench a Campe of 3400 Rods.

Scholer. I thinke I am now in the Backehouſe ditch, for I know not well which way to goe about it: And beſides that, truely I thinke I ſhall neuer come to preferment that way my growth is ſo ſmall.

Maiſter. You know not how God may raiſe you hereafter by knowledge and ſeruice into the fauor of your Prince, for the auaile of your Countrey.

Example for Nauigation: Sir Francis Drake, a man greatly honoured for his knowledge, is not the talleſt man, and yet hath made as great a venture for the honour of his Prince and Countrey, as euer Engliſh man did.

Scholer. Sir, I thanke you for your good encouragement. My minde though I bee little is as deſirous of knowledge as any other: I haue pondred now a litle of it, and thus I ſet forth the worke.

Rod.	*Men.*
200	300
Rod.	
3400	

Say,

Saying, if 200 rod require 300 men, what shall 3400 rodde require? I multiply 3400 by 300: and it yeeldeth 1020000: which I diuide by 200, and my quotient is 5100 men.

Then I must say for my second worke, if in 8 houres 5100 men be able to discharge it, how many shall performe the same in 3 houres? Now if I would worke by the Golden Rule of proportion forward, I should finde a lesse number of men, because three houres is lesse then eight houres: but because reason teacheth mee that the lesser the time is, wherein the trench must be made, the more labourers I ought to haue, thereuppon I vse now the Backer Rule, as in example. And I haue in my quotient 5000. So many Pioners must I haue to entrench the Campe in three houres.

Maister. You haue answered the question very artificially: And truely I commend you for your diligence and apt vnderstanding: and now according to my promise, I will (in whole numbers) giue you a little taste of the Rule of 3 compounded of 5 numbers.

The Rule of 3 compound of 5 Numbers.

THis Rule of 3 compoſed, is diſtinct for moſt needefull queſtions, into two ſeuerall partes or workings: and there belongeth vnto it alwaies 5 numbers, whereof in this Rule being the firſt part, the ſecond number and the fift are alwayes of one nature and like denomination, which rule is to be wrought thus: you muſt multiply the firſt number by the ſecond, and that ſhall bee your diuiſor: then againe, multiply the other thrée numbers, the one by the other, and their product ſhall be your diuidend.

And now according to my promiſe, wee will firſt worke the queſtion of weight and cariage which I deliuered you in the double rule of 3: to be abſolued by this rule, which was this.

If the cariage of 1 C weight 30 miles coſt 12 d, what will the cariage of 500 weight ſtand me in being caried 100 miles?

C. weight.	*Miles.*	*Pence.*	*C. weight.*	*Miles.*
1	30	12	5	100

Now marke well how theſe fiue numbers

ſtand

stand: Then multiply the first number by the second, as 30 by 1, which maketh but 30, and that number keepe for your Diuisor. Then multiplie the other three numbers the one into the other: that is to witte 12 by 5, which maketh 60: Lastly 60 by 100, which as you see here in our Tables ariseth to 6000, which 6000 you shall diuide by the product of the two first numbers, which here is 30. And as you see there is found 200 pence, which is the duetie that yee ought to pay for the cariage of 5 C weight 100 miles, after the rate of 12 d the hundreth, and agreeth with the conclusion of the double rule of 3.

Scholer. Sir I thanke you, it is euen so.

Master. Yet note this for a generalitie in this rule, looke what nature or Denomination your middle number is of (which heere are pence) and of the like Denomination or nature is alwaies your quotient. Note.

Scholer. Well now, and it please you by your patience, I will see how I can ende the question then next following of 30 bushels of wheate sowed, which in one yeare yeelded 360, how many then wil 80 bushels yeeld in 7 yere, sowing euerie yere of those 7 stil 80 bushels and according to your reasōs I set my numbers thus.

Bushels.	Yere.	Bushels.	Bushels.	Yeres.
30	1	360	80	7

80
———
28800
7
———
201600

Where I multiplie 30 by 1, and it maketh 30 my diuisor: then multiplying the other 3 numbers the one into the other, as here appeareth in my tables: they make 201600: which I diuide by 30: and my quotient is 6720 bushels my desire, for so much also it came at two workings by the rule of 3.

Master. Yet one question more I will propound vnto you and so leaue this rule, till it please God hereafter, that I may make you worke it in broken numbers.

What comes the interest of 258 pound for fiue moneths to, after the rate of 8 li taken in 100 li for 12 moneths.

Scholer. Syr this is yet within the compasse of some reasonable vsance. Therfore to minister equitie in this case, I will see how I can worke the same, which I set downe thus: praying you if I haue not done well to shew me mine errour.

li	*moneths.*	li	li	*moneths.*
100	—12—	8	-258—	5

Master. Proceed, you haue done very well.

Scholer. Then I doubt not by the grace of God but to ende it: I multiplie 100 by 12, it yeeldeth 1200: and the three other numbers multiplied together, produceth 10320: which I deuide by 1200: and my Quotient is 8 pounds. Then according as you haue taught me heretofore, I turne the 720 li that is left, into

into shillings: and diuiding it by my first number, my quotient is 12 shillings. So I answere that the lone of 258 pound for 5 Moneths, after the rate of 8 pound in the 100 poūd for a yeare, comes to 8 pound 12 shillings.

Maister. You say true, I commend your diligence, now behold the manner of the second part of this rule.

Maister. In the second part of this rule of 3 composed: the third number is like unto the first. And the rule is to be wrought as thus: you shall now contrary to the last rule multiply the third number and the fourth together: and that product shall be your diuisor. Then multiply the fift by the second, and the product thereof by the first: and that is the number that shall bee diuided. For example, I propound this question, for a proofe of my last question of interest. A Merchant hath receiued 8 pound, 12 shillings, for interest of certaine money for 5 moneths terme, which hee receiued after the rate of 8 pound in the 100 pound, for a yeare. The question is now how much money was deliuered to raise this interest: Behold therefore the manner,

The second part of the rule of 3 compoūd.

The proof of the last question.

how the que-	li. *monethes.*	li. *monethes.*	li.	s.
stion is set	100 —— 12.	8 —— 5	—— 8 —	12
forth.				

Scholer. Sir I perceiue it very well: and according to the doctrine which you prescribed for the working thereof: if it please you now

it is set downe, I thinke I can follow the worke.

Maister. Nay stay a while, and afore you worke, marke well how I deliuer a reason for the perfite vnderstanding of this Rule, which is thus: If 8 li in 12 moneths do yéeld me 100 li, to take 8 li, 12 s, for 5 moneths, must needes yéeld a great deale more.

So vpon the knowledge that I haue in this Art, the first part of this Rule is answerable to the rule of thrée forward: And this latter part accordeth to the rule of 3 backeward.

Scholer. Sir, I yeelde you most hearty thankes for these your last instructions, they haue giuen me great light into these two rules, whereby I may the better by deliberation conceiue how to vse them hereafter, when occasion shall require.

Note. Maister. You say well, goe to now if you will, and try your cunning in the question: But this note take with you by the way, in as much as here is mention made of shillings: turne all your money as you worke into shillings, for your more ease in worke.

Scholer. If it please you to behold me a litle, I will quickly end it: for I haue but my first: my second: and my last number to be multiplied together for my diuidend: And my third into my fourth for my diuisor.

100 li,

li Moneths. li Moneths. li s

100 —— 12 —— 8 —— 5 —— 8 — 12

20	20	20
2000	160	172
12	5	
4000	800	
2000		
24000		
172		
48000		
168000		
24000		
4128000		

4128000
8 00

Which 4128000 I diuide by 800, and my quotient is 5160 shillings, which in pounds yeeldeth 258, my desire.

Maister. I will here for this time in whole numbers end this rule, and I will instruct you in the rules of Felowship. You may at your conuenient leisure for your exercise worke the same by the rule of 3 at twice. And for your ayde and incoragement herein, I set downe here a profer how to apply it.

Moneths. A li s Pounds. B li

5 —— 8 12 8 —— 100
12 —— 412 4/5 s: 412 4/5 —— 258 li

The Rule of Felowship.

But now wil I shew you of the rule of Felowship or company, which hath sundry operations, according to the divers number of the cōpany. This rule is somtime without difference of time, and sometimes there is in it difference of time. First I will speake of that without difference of time, of which let this be an example,

Foure Merchants of one company made a banke of money diversly, for the first layd in 30 l', the second 50 l', the third 60 l', and the fourth 100 pound, which stocke they occupied so long, till it was increased to 3000 pound. Now I demaund of you, what should each man receive at the parting of this money?

Scholer. I perceive that this rule is like the other, but yet there is a difference, which I perceive not.

Maister. Then I will shew it to you. First by Addition you shall bring all the particular summes of the Merchants into one summe, which shal be the first summe in your working by the Golden Rule, and the whole summe of the gaines by that stocke shall bee the second summe. Now for the third summe, you shall

set

set the portion of each man one after an other, and then worke by the Golden Rule, and the fourth sum will shew you each mans gaines: as in example.

The parcels of those foure Merchants make in one summe 240 pound: set that in the first place, the gaines in the second, & the first mans portion of stocke in the 3 place thus.

240 3000
30

Now multiply the second by the third, and it will be 90000, which you shall divide by 240, and there will appeare 375 l, thus.

240 3000
30 375

And that is the gains for the first man.

Now for the second man, set the 50 l, that he brought in the third place, and worke as before: and his part will be 625 pound, as this figure sheweth.

240 3000
50 625

Likewise for the third man set his money which was 60 l, and his part of gaines will bee 750 pound, as here appeareth.

240 3000
60 750

And so for the fourth man, if you set his sum which is 100 pound, his gaines will be 1250 l, as the worke will declare.

240 3000
100 1250

Scholer. This I perceiue: but is there

any way to examine whether I haue well done or no?

Maiſter. For the triall hereof adde together all their 4 portions, and if their Addition make the whole ſumme of the gaines, then is the worke well done.

Scholer. That will I try by and by, the foure parcels are theſe, which added together, make 3000, which is the iuſt ſumme of money that they gained, whereby I know the worke is well done.

375
625
750
1250
3000

Maiſter. Well now another example will I put to you, not of gaines, but of loſſe: for one reaſon ſerueth for both.

If three Marchants in one ſhippe and of one felowſhip, had bought Marchandiſe, ſo that the firſt had layd out 200 poundes, the ſecond 300 pounds, and the third 500 pounds, and it chanced by tempeſt that they did caſt ouer boord into the ſea Marchaundiſe of the value of 100 li, how much ſhould each man beare in this loſſe?

Scholer. If I ſhall doe in this as you did in the other queſtion, then muſt I ioyne their three portions together, 200, 300, and 500, which maketh 1000. Then ſay I, if 1000 loſe 100, then ſhall 200 looſe 20, and 300 ſhall looſe 30, and 500, ſhall looſe 50, as by

theſe thrée figures it doth appeare plaine.

1000 — 100 1000 — 100
200 — 20 300 — 30

1000 — 100
500 — 50

Maſter. Well, ſith now you haue done theſe, I will propound a queſtion of more importance, which ſhall make you not onely the abler to vnderſtand this rule, but alſo it will greatly aide you in the next Rule of Felowſhip with time, if ſuch néede be that your money be of diuerſe denominations.

For this may not bee forgotten in all ſuch queſtions: if the number be of diuerſe kindes you muſt by Reduction bring it into one kinde, that is to ſay, to the leaſt value that is named in the queſtion. And likewiſe ſhall you doe, if the time be of diuerſe kindes, as ſome yeares, ſome moneths, wéeks and daies, you ſhall make all months, wéeks or daies, according as ẏ leaſt name of time in the queſtiõ is: As for example. Note.

Firſt in diuerſity of money. Thrée companions bought 2000 ſheepe, and payd for them 241 l, 13 s, 4 d, of which ſumme one paide 101 l, 10 s. The ſecond paide 82 l, 17 s, 10 d. And the third payde 57 pound, 5 ſhillings, 6 d: How many ſhéepe muſt each of them haue? Anſwere: The firſt ſhall haue 840. The ſecond 686. And the third 474. And that muſt you worke Queſtion of ſheepe.

thus.

First considering that your money is of diuerse denominations, you shall (by Reduction) bring it all into the smallest denomination which is in it, that is to say, pence, and so wil the totall summe be 58000 pence.

Now if you turne each mans money into pence also, the first mans summe will be 24360 pence: the second mans summe 19894 pence. And the third mans money will be 13746 pence.

Now to know how many sheepe euery man shall haue, let the whole summe of money that is 58000 pence be set in the first place, and in the second place set the number of sheepe, and then orderly in the third place sette each mans money, and then multiplying the third and the second summes together and diuiding that that amounteth by the first, there will appeare the number of sheepe that each man ought to haue: as these three figures doe shew.

a

58000 —— 2000
24360 —— 840

b

58000 —— 2000
19894 —— 686

c

58000 —— 2000
13746 —— 474

Scholer. Why doe you sette the money in the first place, seeing in the question you say,

2000

2000 shéepe cost 58000 d, and not thus 58000 d, cost 2000 shéepe.

Maister. You remember, I taught you at the beginning of this Golden Rule, that the first and third numbers must bee of one name, and of like things: and euermore the number that the question is asked of, must bee sette in the third place. Now is the question plainely this: If foure men bought 2000 sheepe for 58000 pence, how many sheepe shall each man haue?

But séeing in this question there ought more respect to be had to the summe of money, than to the summe of the persons, (for in the sums of money is there proportion toward the shéepe, and not in the number of persons) therefore must we turne the question thus.

If 58000 pence bought 2000 sheepe, how many did 24360 buy? Againe, how many did 19894 pence buy? and how many bought 13746 pence?

Scholer. I perceiue it reasonable, and so shall I doe in all like questions.

Maister. Euen so. But for easinesse of the worke marke this: whensoeuer the first and second numbers haue ciphers in the first places, you may both in the multiplication and in the diuision leaue out those ciphers, so that you leaue out like many out of both summes. as in this question the first number 58000 hath three ciphers, and so hath the second, that

is 2000: therefore cast away their ciphers, and so will the first number be 58, and the second 2: set them in their places, and worke according to the rule, and you shall perceiue that it will be all one, sauing that this is the shorter and easier way, as these three figures doe shew.

a

58 — 2
24360 — 840

b

58 — 2
19894 — 686

c

58 — 2
13746 — 474

And this you see is both easier, and also the more certaine way to know the answere to this question.

Scholer. Truth it is as you say: but sir, me seemeth I might aske a further question here, not onely how many sheepe each man should haue, but also what euery sheepe cost.

Maister. That question doth not onely belong to this Rule, but may also bee discussed by Diuision, especially if the questions number be one only: as thus. Diuide the totall sum 58000 pence by 2000 (or 58 by 2, omitting the ciphers) and the quotient will be 29 pence, that is 2 shillings, 5 pence, howbeit, by this rule you may doe it, and best when the number of the question doth exceed 1: as if I should aske this

this queſtion, 2000 ſheepe coſt 58000 d, how much did 20 coſt? Then ſhall I ſet my figure thus,

2000 ——— 58000
20 ———

And doing after the rule, there will amount 580 pence, that is 2 l. 8 s, 4 d: the price of one ſcore: But if you will vſe that eaſie way that I did teach you, you may chaunge the firſt and ſecond number thus.

2 ——— 58
20 ———

Iudgment

Thus doe you perceiue the vſe of the rule without time. And that you may as well perceiue the ſame with diuerſity of time, I propoſe this example. The rule of fellowſhip with time.

Foure Marchants made a common ſtocke, which at the yeares end was encreaſed to 35145 l. Now to know what ſhal be each mans portion of gaines, you muſt know each mans ſtocke and time of continuance. Queſtion of a banke.

The firſt man of theſe foure layd in 669 l, which he did take from the ſtocke againe, at the end of 10 monethes. The ſecond man layde in 810 l, for 8 moneths. The third layd in 900 l, for 7 moneths. And the fourth layd in 1040 l, for 12 moneths.

This queſtion ſhal you examine as you did ẏ other before, ſauing that where as in the third place of ẏ figure you did ſet ech mās ſum alone, here you ſhall ſet the ſame being multiplied by the number of their time, & likewiſe in the firſt place of the figure you ſhal ſet ẏ number which Note. A general rule.

mounteth of their whole summes so multiplied by their time, and added into one whole sum, as thus.

The first mans summe is 669 li, which I multiply by 10 (that was the number of his time) and it maketh 6690. The second mans summe 810 li multiplied by 8. (which was his time) maketh 6480. The third mans summe 900 li, multiplied by 7 (for that was his time) yeeldeth 6300. The fourth mans summe was 1040 li, and his time 12, multiply the one by the other, and it will be 12480.

These foure summes thus multiplied by their time, must be set orderly in the third place of the figure: and in the first place must bee set the whole summe of all foure, which is 31950, and the gaine must be in the second place, which is 35145. Now to end the question, I say first: If 31950 did get 35145, what did 6690 gette? Answere, 7359 li, as by this figure appeareth.

a
31950 — 35145
6690 — 7359

Likewise the second man had to his part 7128 li, the third must haue 6930 li. And the fourth man shall haue for his part 13728 pound, as these figures doe plainely declare.

Scholer.

	b			c	
31950		35145	31950		35145
6480		7128	6300		6930

	d	
31950		35145
12480		13728

Scholer. This I like very well: but what proofe is there of this worke?

Maſter. The ſame that I taught you for the other, to adde all the portions together, and if they agree to the whole ſumme, then is your worke well done.

Scholer. That wil I prooue in this example. The foure parcels are theſe, which if I adde together there will amount 35145, and that was the whole ſumme: whereby I perceiue the worke is well done.

7359
7128
6930
13728
35145

Maſter. If it fall out otherwiſe, be ſure it is not well done.

Scholer. Then doe I underſtand this worke alſo very well, but what haue I now to learne?

Maſter. There are many other excellent partes behinde, of which I will not as now, make mention, becauſe that without the knowledge of Fractions they cannot bee duely taught, and much leſſe underſtanded. There-

fore will I propoſe to you two or thrée queſtions more, (that thereby you may better perceiue the vſe of this rule and all other like) and ſo make an end for this time.

¶ Thrée partners by ſome ill aduenture ſuſtained the loſſe of 160 l, whereof the firſt layd into the common ſtocke 200 l, for 10 moneths. The ſecond layd in 350 l, and the third 100 l, but for how long the two latter, is vnknowne. But breaking off their partnerſhip, the firſt found him ſelfe a looſer 80 l, the ſecond 56 l, and the third 24 l. The queſtion is, for how long time was the money of the two latter in company.

For the ſolution hereof and of ſuch other like, you muſt alſo multiply the firſt mans 200 l that he put into the ſtocke by his time of continuance, which was 10 moneths, and it maketh 2000: wherefore now I affirme, if his money that loſt 80 l, multiplied by his time, make 2000: what ſhall his money make that loſt 56 l, and his that loſt 24 l, which two numbers I commit to the triall of the rule of 3 at 2 workings thus: If 80 giue 2000, what giueth 56? And againe, if 80 giue 2000, what giueth 24?

80 — 2000
56 — 1400

80 — 2000
24 — 600

To conclude, if you now diuide 1400 the ſecond mans portion by 350, which was his

his stocke that he layd into company; you shall finde in your quotient 4 monethes, and for so long time did the second man put his money into the common stocke.

Lastly if you diuide the third mans new laying in, which was 600, by 100, which was his stocke that he put into company: the quotient declareth his time of continuance, which was 6 moneths. And thus is the question resolued.

Scholer. Sir I haue attentiuely beheld your working, and the more we trauell herein, the more me thinke I am in loue with this excellent Art.

Maister. Then what say you to this question?

There is in a Cathedrall Church 20 Cannons, and 30 Vicars, those may spend by yeare 2600 l. but euery Cannon must haue to his part 5 times so much as euery Vicar hath: how much is euery mans portion, say you? Question of Canons

Scholer. I pray you make the answere your selfe also, so shall I perceiue best the meanes to answere to such other like.

Maister. In this question you must doe as in those before sayde, that haue diuersity of time, for here is diuersity of portions: Therefore shall you multiply the number of the persons by their difference of portion: (as you did in the other by time.) Then must you multiply the 20 (which is the number of Cannons)

by 5, (for that is the number of their portion) ſo will it be 100: Then 30 (that is the number of Vicars) by 1, (that is the number of their portion) and it will be 30: put thoſe two ſummes together, and they make 130: then ſay thus: If 130 ſpend 2600 pound, what may 100 ſpend: The rule ſheweth 2000 pound.

Againe for Vicars: If 130 ſpend 2600 pound, what may 30 ſpend? Anſwere 600 pound, as theſe figures ſhew.

130	2600	130	2600
100	2000	30	600

But if euery Canon ſhould haue ſo often times 4 l, as the Vicar ſhould haue 3 l, then ſhould I multiply 20 by 4, (that were 80) and 30 by three (that were 90) and then both were 170. Then ſhould the figures be ſet thus.

	l s d		l s d
170	2600	170	2600
80	1223,10,7,	90	1376,9,5

But this ſort is too hard for you, by reaſon of the Fractions, therefore I will let it reſt to that place.

And by this rule you ſee what the twenty Canons may ſpend, which ſumme if you diuide by 20, you ſhall ſee each Canons portion: and ſo of the Vicars, if you diuide their ſumme by 30, the quotient will declare euery Vicars portion.

The

The ſecond Dialogue.

The accounting by Counters.

Maſter.

Now that you haue learned the common kindes of Arithmeticke with the pen, you ſhal ſee the ſame Arte in Counters: which feate doth not onely ſerue for them that cannot write & read, but alſo for them that can doe both, but haue not at ſometime their pen or tables with them.

This ſort is in two formes commonly: The one by lines, and the other without lines. In that that hath lines, the lines do ſtand for the order of places: and in that that hath no lines, there muſt be ſet in their ſtead ſo many Counters as ſhall neede, for each line one, and they ſhall ſupplie the ſtead of the lines.

Scholer. By examples I ſhould better perceiue your meaning.

Maiſter. For example of the lines, loe here you ſee ſixe lines, which ſtand for ſixe places, ſo that the

100000

10000

* 1000

100

10

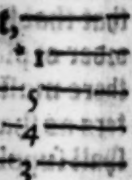
1

neathermost standeth for the first place, and the next aboue it for the second, and so vpward, till you come to the highest, which is the sixth line, and standeth for the sixth place. Nowe what is the value of euerie place or line you may perceiue the figures which I haue set on them, which is according as you learned before in Numeration of figures by the penne: for, the first place is the place of vnites or ones: and euerie Counter set in that line betokeneth but one: and the second line is the place of 10, for euerie Counter there, standeth for 10. The third line the place of hundreths, the fourth of thousands, and so forth.

Scholer. Sir, I do perceiue that the same order is here of lines, as was in the other figures by places, so that you shall not need longer to stand about Numeration, except there be any other difference.

Master. If you do vnderstand it, then how will you set 1543?

Scholer. Thus as I suppose.



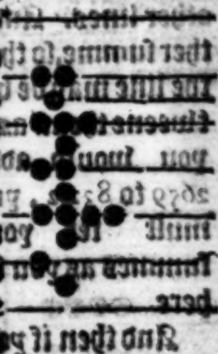
Master. You haue set the places truly, but your figures be not meet for this vse: for the meetest figure in this behalfe, is the figure of a Counter round, as you see here, where I haue expressed y same sum.

Scho-

Scholer. So that you haue not one figure for 2, nor 3, nor 4, and so foorth, but as many Digits as you haue, so many Counters you set in the lowest line: and for euery 10 you set one in the second line: and so of other. But I know not by what reason you set that one Counter for 500 betweene two lines.

Maister. You shall remember this, that whensoeuer you neede to set downe 5, 50, or 500, or 5000, or so foorth any number whose Numerator is 5, you shall set one Counter for it in the next space aboue the line that it hath his denomination of: as in this example of that 500, because the numerator is 5, it must bee set in a voide space: and because the denominator is hundred, I know that his place is the voyde space next aboue hundreds, that is to say, aboue the third line.

And further you shall marke, that in all working by this sort, if you shall set downe any summe betweene 4, and 10, for the first part of that number you shall set downe 5, and then so many Counters more, as there rest numbers aboue 5. And this is true both of Digits and Articles. And for example I will set downe this summe 287965, which summe if you marke well, you neede none other examples for



learne the Numeration of this forme.

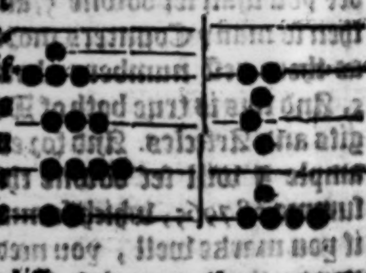
But this shall you marke, that as you did in the other kinds of Arithmeticke, set a pricke in the places of thousands, in this worke you shal set a Starre, as you see before.

Scholer. Then I perceiue Numeration: but I pray you, how shall I doe in this Art to adde two summes and more together?

Addition.

Maister.

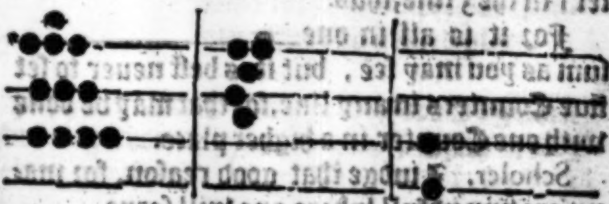
THe easiest way in this Art, is to adde but two summes at once together: howbeit you may adde more, as I will tell you anon. Therefore when you will adde two summes, you shall first set downe one of them, it forceth not which, and then by it draw a line crosse the other lines. And afterward set downe the other summe, so that the line may be betweene them: as if you would adde 2659 to 8342, you must set your summes as you see here.



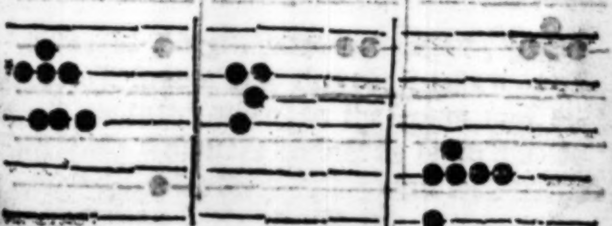
And then if you list, you may then adde the ones

one to the other in the same place: or else you may adde them both together in a new place: which way because it is most plainest, I will shew you first.

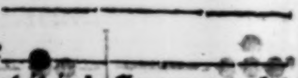
Therefore will I begin at the vnites, which in the first summe is but 2, and in the second summe 9, that maketh 11. Those doe I take vp, and for them I sette 11 in the new rowme, thus.



Then doe I take vp all the Articles vnder a hundred, which in the first summe are 40, and in the second summe 50, that maketh 90: or you may say better, that in the first summe there are foure articles of 10, and in the second summe 5, which maketh 9, but then take heede that you set them in their right lines as you see here.



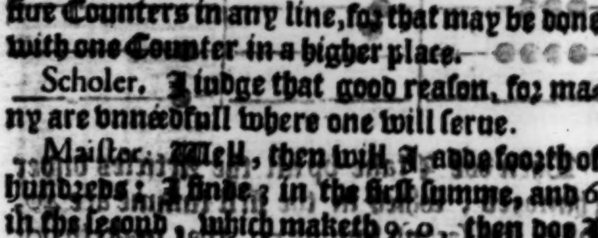
Where I haue taken away 40 from the first summe, and 50 from the second, and in their stead I haue set 90 in the third roome, which I haue set plainely that you might well perceiue it: howbeit, seeing that 90 with the 10 that was in the 3 roome already doth make 100, I might better for those 6 coūters set 1 in the 3 line, thus.



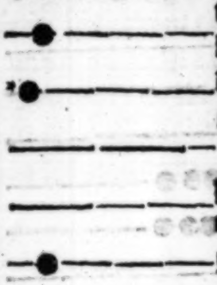
For it is all in one sum as you may see, but it is best neuer to set fiue Counters in any line, for that may be done with one Counter in a higher place.

Scholer. I iudge that good reason, for many are vnneedfull where one will serue.

Maister. Well, then will I adde forth of hundreds: I finde 3 in the first summe, and 6 in the second, which maketh 900, then doe I take vp, and sette in the third roome, where is 100 already, to which I put 900, and it will be 1000: therefore I set one counter in the fourth line for them all, as you see here.



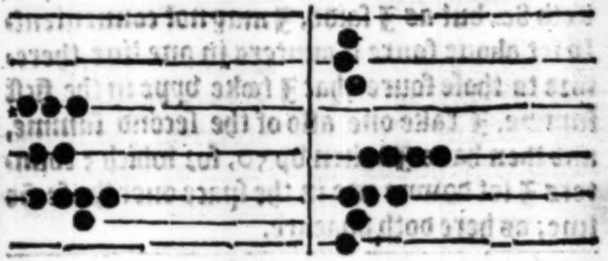
Then adde I the thouſandes togeather, which in the firſt ſumme are 8000, and in the ſeconde 2000, that maketh 10000: them do I take vp from thoſe two places, and



then it appeareth as you ſée to bee 11001, for ſo manie doth amount of the Addition of 8342 to 2659.

Scholer. Syr, this I doe perceiue: but how ſhall I ſet one ſum to another, not changing them to a third place.

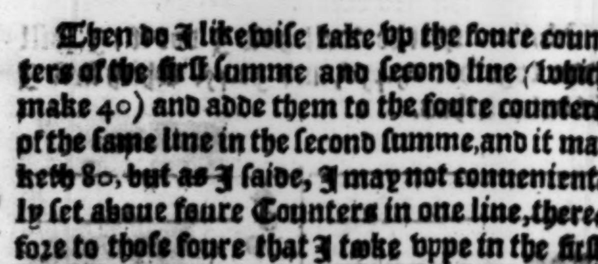
Maſter. Marke well how I doe it: I will adde together 65436, and 3245, which firſt I ſet downe thus.



Then do I begin with the ſmalleſt, which in the firſt ſumme is 5, that doe I take vp and would put to the other 5 in the ſecond ſum, ſauing that 2 Counters cannot be ſet in a voide place of 5, but for them both I muſt ſet 1 in the

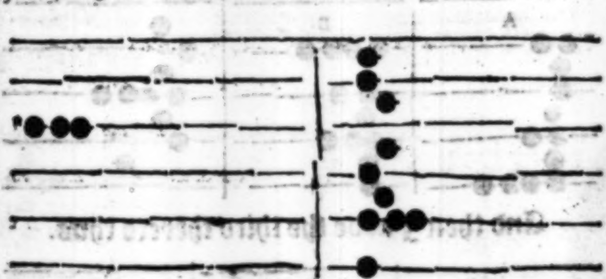
ſecond line, which is the place of 10: therefore I take vp the fiue of the firſt ſumme, and the 5 of the ſecond, and for them I ſet 1 in the ſecond line, as you ſee here.

Then do I likewiſe take vp the foure counters of the firſt ſumme and ſecond line (which make 40) and adde them to the foure counters of the ſame line in the ſecond ſumme, and it maketh 80, but as I ſaide, I may not conueniently ſet aboue foure Counters in one line, therefore to thoſe foure that I take vppe in the firſt ſumme, I take one alſo of the ſecond ſumme, and then haue I taken vp 50, for which 5 counters I ſet downe one in the ſpace ouer the ſecõd line, as here doth apaeare.

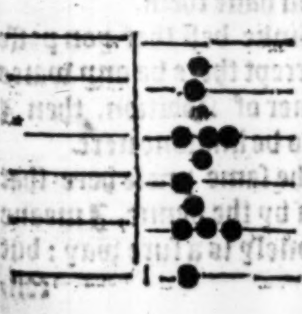


And then is there 18, as well with those 4 Counters, as if you had set downe the other 4 also.

Now doe I take the 200 in the first summe, and adde them to the 400 in the second summe, and it maketh 600, therefore I take up the two Counters in the first summe, and three of them in the second summe, and for them five, I set 1 in the space aboue, thus.



Then I take the 3000 in the first summe, unto which there are none in the second summe agreeing, therefore I do onely remoue those three counters from the first summe into the second, as here doth appeare.

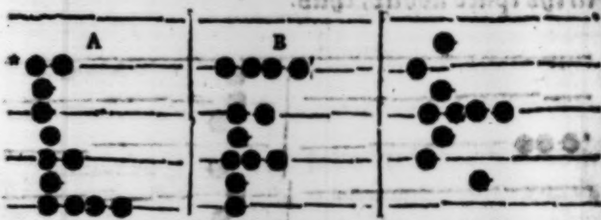


And so you see the whole summe that amounteth of ỹ Addition of 65436 with 3245 to be 68681.

And if you haue marked

marked these two examples well, you néede no further instruction in Addition of 2 onely summes: but if you haue more than two sums to adde, you may adde them thus.

First adde two of them, and then adde the third and fourth, or more if there be so many: as if I would adde 2679 with 4286 and 1391. First I adde the two first summes thus.



And then I adde the third thereto thus.

And so of more, if you haue them.

Scholer. Now I thinke best that you passe forth to Subtraction, except there be any waies to examine this manner of Addition, then I thinke that were good to be knowne next.

Maister. There is the same proofe here that is in the other Addition by the penne, I meane Subtraction, for that onely is a sure way: but

con-

considering that Subtraction must bee first knowne, I will first teach you the Art of Subtraction, and that by this example.

Subtraction.

I Would subtract 2892 out of 8746. These summes must I set downe as I did in Addition: but here it is best to set the lesser number first thus.

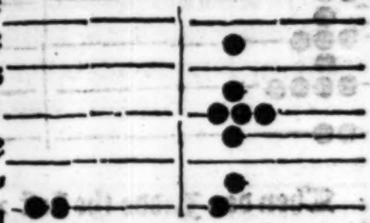
Then shall I begin to subtract the greatest numbers first (contrary to the vse of the penns) that is the thousands in this example: therefore I finde amongest the thousands 2, for which I withdraw so many from the second summe (where are 8) and so remaineth there 6, as this example sheweth.

Then

Then doe I likewise with the hundreds, of which i[n] the first summe I find 8, and in the second summe but 7, out of which I cannot take 8, therefore this must I do: I must looke how much my summe differeth from 10, which I find here to be 2, then must I bate for my summe of 800, one thousand, and set downe the excesse of hundreds, that is to say, 2, for so much as 1000 is more then I should take vp. Therefore from the first summe I take that 800, and from the second summe (where are 6000) I take vp one thousand, and leaue 5000, but then I set down the 200 vnto the 700 that are there already, and make them 900, thus.

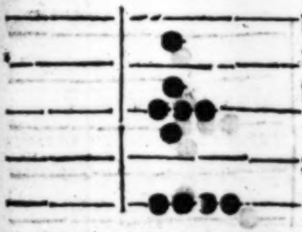
Then come I to the Articles of tennes, where in the first summe I finde 90, and in the second summe but onely 40. Now considering that 90 cannot be abated from 40, I looke how much that 90 doth differ from the next summe aboue it, that is 100, or else (which is all to one effect) I looke how much 9 doth differ from 10, and I finde it to bee 1, then in

the stead of that 90, I do take from the second summe 100: but considering that is 10 too much, I set downe 1 in the next line beneath for it, as you see here.



Hauing that here I haue set one counter in the space, in stead of 5, in the next line.

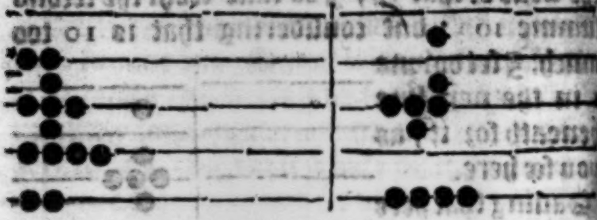
And thus haue I subtracted al, saue two which I must bate from 6 in the second summe, and there will remaine 4, thus.



So that if I subtract 2842 from 8746, the remainer will bee 5854.

And that this is truely wrought, you may prooue by Addition: for if you added to this remainer the same summe that you did subtract, then will the former summe 8746 amount againe.

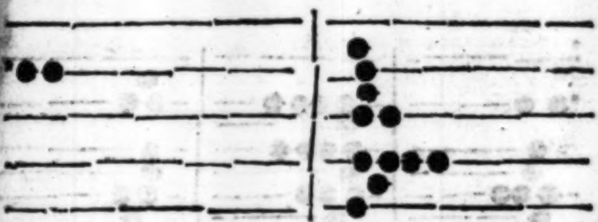
Scholer. That will I prooue: and first I set the summe that was subtracted, which was 2892, and then the remainer 5854, thus.



Then doe I adde the firſt 2 to 4, which maketh 6: ſo take I vp 5 of thoſe Counters, and in their ſtead I ſette 1 in the ſpace, and 1 in the loweſt line, as here appeareth.

Then doe I adde the 90 next aboue to the 50, and it maketh 140, therefore I take vppe thoſe 6 Counters, and for them I ſet 1, to the hundreds in the third line, and 4 in the ſecond line thus.

Then do I come to the hundreds, of which I find 8 in the firſt ſumme, and 9 in the ſecond, that maketh 1700: therefore I take vp thoſe 9 Counters, and in their ſtead I ſet 1 in the fourth line, and 1 in the ſpace next beneath, and 2 in the third line as you ſee here.



Then is there left in the firſt ſumme but onely 2000, which I ſhall take vp from thence, and ſet in the ſame line in the ſecond ſumme, to the one that is there alreadie: and then will the whole ſumme appear as you may well ſee, to be 8746 which was the firſt

groſſe ſum, and therefore I doe perceiue that I had well ſubtracted before.

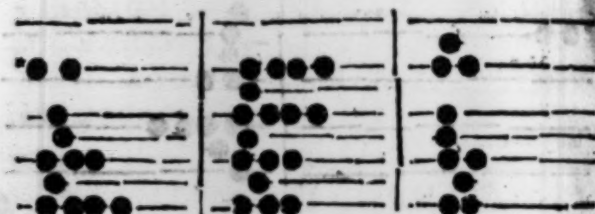
And thus may you ſee, how Subtraction may be tried by Addition.

Scholer. I perceiue the ſame order heere with Counters, that I learned before in

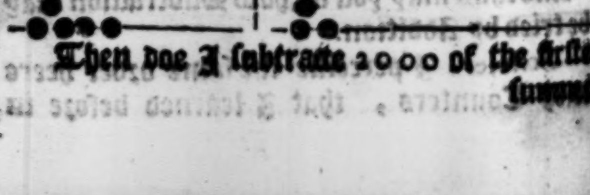
figures.

Maſter. Then let me ſée howe you can trie Addition by Subtraction.

Scholer. Firſt I will ſet forth this example of Addition, where I haue added 2189, to 4988. And the whole ſumme appeareth to be 7177.



Nowe to trie whether that ſumme be well added or no, I will ſubtract one of the firſt two ſummes from the third, and if I haue well done, the Remainer will be like that other ſumme, as for example, I will ſubtract the firſt ſumme from the third, which I ſet thus in their order.



Then doe I ſubtracte 2000 of the firſte

ſumme

summe from the second summe, and then remaineth there 5000, thus.

Then in the third line I subtract the 100 of the first sum from the second sum, where is only 100 also: and then in the third line resteth nothing, as you may see in the example following.

Then in the secōd line with his space over him I find 80, which I should subtract from the other summe: then seeing there are but onely 70, I must take it out of some higher summe, which is here onely 5000: therefore I take up 5000: and seeing that is too much by 4920: I set downe so many in the second roome, which with the 70 being there already, do make 4990, and then the summes do stand thus.

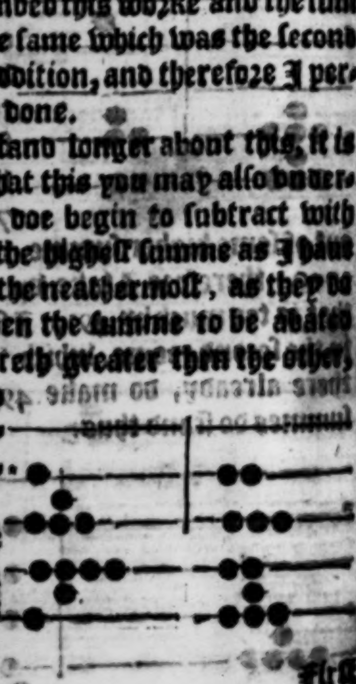
Yet remaineth there in the first summe, 9 to be abated from the second summe, wherein that place of unittes doth appeare onely 7: then must I bate a higher summe, that is to say 10, but seeing that 10 is more then 9 (which I should abate) by 1, therefore shall I take vp one counter from the second, & set downe the same in the first or lowest line, as you see here.

And so haue I ended this worke and the sum appeareth to be the same which was the second summe of mine Addition, and therefore I perceiue I haue well done.

Another way of Addition.

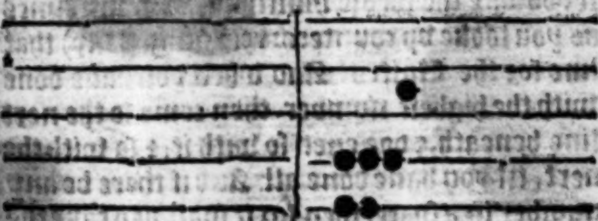
Maister. To stand longer about this, it is but follie; except that this you may also vnderstand, that many doe begin to subtract with counters, not at the highest summe as I haue taught you, but at the neathermost, as they do vse to adde: and then the summe to be abated in any line appeareth greater then the other, then do they borow one of ÿ next higher roome, as for example.

If I should abate 2846 from 2 5 9 8, they set the sums thus.



First

First they take 6 which is the lower line, and his space frõ 8 in the same roomes in the second summe, and yet there remaineth 2 counters in the lowest line. Then in the second line must 4 be subtracted from 7, and so remaineth there 3. Then 800 in the third line, and his space, from 200 of the second summe can not be, therefore do they bate it from a higher roome, that is from 1000: and because that 1000 is too much by 200, therefore must I set downe 200 in the third line, after I haue taken vp 1000 from the fourth line. Then is there yet 1000 in the fourth line of the first sum, which if I withdraw from the second summe, then do all the figures stand in order, thus.



So that (as you see) it differeth not greatly whether you begin Subtraction at the higher lines or at the lower.

Howbeit, as some men like that one way best, so some like the other: therefore you now knowing both, may vse which you list.

Multiplication.

But nowe touching Multiplication: you shall set your numbers in two roomes as you did in those other kindes) but so that the multiplyer bee sette in the first roome then shal you begin with the highest numbers of the second roome, and multiply them first, after this sort.

Take þ overmost line in your first working, as it were the lowest line, setting on it some moueable marke (as you list) & looke how many counters be in him, take them vp, & for them set downe the whole multiplier so many times as you tooke vp counters: reckning (I say) that line for the Unites. And when you haue done with the highest number, then come to the next line beneath, & doe euen so with it, & so with the next, til you haue done all. And if there be any number in a space, then for it shall you take the multiplier 5 times: and then must you recken that line for the vnites, which is next beneath that space. Or els after a shorter way, you shall take onely halfe the multiplier, but then shall you take the line next aboue the space for the line of Unites. But in such working, if by chance your multiplier be an odde number, so

that you can not take the halfe of it iustly, then must you take the greater halfe and set downe that, as if that it were the iust halfe: and further you shall set one counter in the space beneath that line, which you reckon for the line of vnites, or else onely remooue forward the same that is to be multiplied.

Scholer. If you set forth an example hereto I thinke I shall perceiue you.

Maister. Take this example: I would multiplie 1542 by 365, therefore I set my numbers thus.

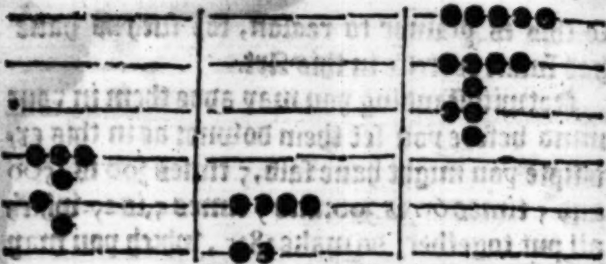
Then first I begin at the 1000 in the highest line, as if it were the first place, and I take it vp, setting downe for it so often (that is once) the multiplier, which is 365, thus as you see here: where, for the one counter taken vppe from the fourth line, I haue set downe other 6, which make the summe of the multiplier reckoning the fourth line as if it were the first, which thing I haue marked by the hand set at the beginning of the same.

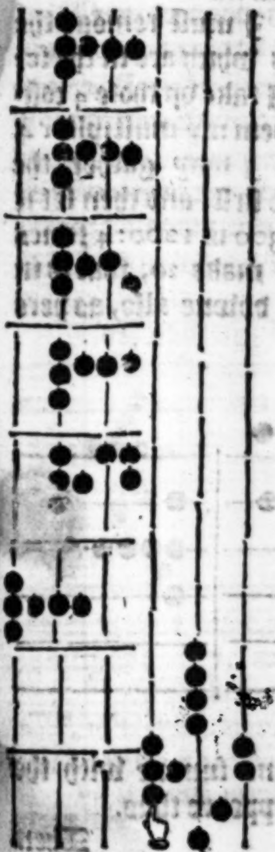
Scholer. I perceiue this well, for in deed this summe that you haue set down is 365000: for so much doth amount of 1000, multiplied by 365.

Mast. Well then go forth, in the next space I find one counter, which I remoue forward, but take it not vp, but do (as in such case I must) set down the greater half of my multiplier (seeing it is an odde number (which is 182, & here I doe still let that fourth place stand, as if it were the first: as in these examples you shall see.

Where I haue set this multiplcation with other, but for the ease of your vnderstanding, I haue set a litle line betweene them. Now should they both in one summe stand thus.

Now





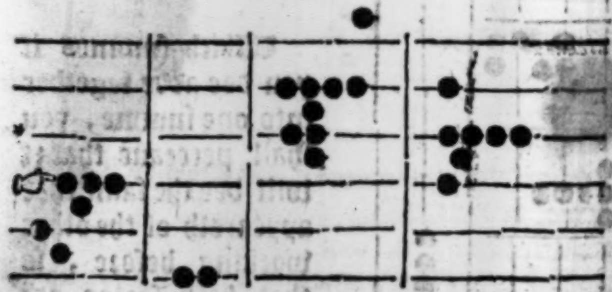
Howbeit, another forme to multiply ſuch Counters in ſpace is this: Firſt to remooue the finger to the next line beneath ẏ ſpace, and then to take vppe that Counter, and to ſet down the multiplier fiue times: as here you ſee.

Which ſummes if you doe adde together into one ſumme, you ſhall perceaue that it will bee the ſame that appeareth of the other working before, ſo that both ſortes are to one intent: but as the other is ſhorter,

ſo this is plainer to reaſon, for ſuch as haue had ſmall exercise in this Art.

Notwithſtanding you may adde them in your mind before you ſet them downe: as in this example you might haue ſaid, 5 times 300 is 1500 and 5 times 60 is 300: alſo 5 times 5, is 25. which all put together, do make 1825, which you may at one time ſet downe if you liſt.

But now to go foorth, I muſt remoue the hand to the next counters which are in the ſecond line, and there muſt I take vp thoſe 4 coũters ſetting downe for them my multiplier 4 times ſeuerally, or elſe I may gather the whole ſumme in my mind firſt, and then ſet it downe: as to ſay, 4 times 300 is 1200: 4 times 60 are 240: and 4 times 5 make 20, that is in all 1460, that ſhall I ſet downe alſo, as here you ſee.



Which if I ioyne in one ſumme with the former numbers, it will appeare thus.

Then

Then to ende this multiplication, I re-moue the finger to the loweſt line, where are only 2, them do I take vp, and in their ſtead do I ſet downe twiſe 365, that is 730, for which I ſet one in the ſpace aboue the third line for 500, and 2 more in the third line with that one that is there already and the reſt in their order, and ſo haue I ended the whole ſumme thus.

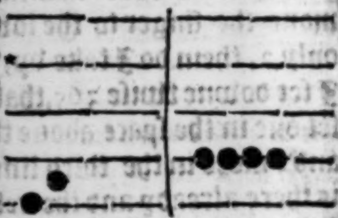
Whereby you ſée, that 1542 (which is the num-ber of yeares ſith Chriſt his incarnation) being multiplied by 365 (which is the number of daies in one yeare) doth amount vnto 562830, which declareth the nūber of daies ſith Chriſts incar-nation vnto the end of 1542 yeares. (beſide 385 daies and 12 houres for leape yeares.)

The ſum of the daies ſith Chriſts in-carnation.

Scholer. Now will I prooue by an other example, as this: 40 laborers (after 6 d the day for each man) haue wrought 28 daies: I would know what their wages doth amount vnto.

In this case must I worke doubly: first I must multiplie the number of the labourers by the wages of a man for one daye, so will the charge of euery one day amount.

Then secondarily shall I multiply the charge of one day by the whole number of dayes, and so wil the whole sum appeare: first therfore I shall set the sums thus.



Where in the first place is the multiplier (that is 1 daies wages for one man) and in the second space is sette the number of the workemen to be multiplied.

Then say I: 6 times 4 (reckoning that second line of the line of Unites) maketh 24, for which summe I should set 2 counters in the third line, and 4 in the second, therefore doe I set 2 in the third line, and let the 4 stand still in the second line thus.

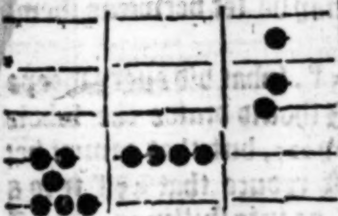
So appeareth the whole dayes wages to

be

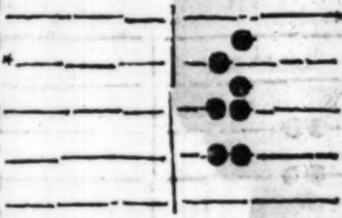
be 240 d, that is 20 s.

Thẽ do I multiply again the ſame ſum by the number of dayes, and firſt I ſette the numbers thus.

Then becauſe there are counters in diuerſe lines, I ſhall beginne with the higheſt, and take them vp, ſetting for them the multiplier ſo many times as I tooke vp counters, that is twiſe, then will the ſumme ſtand thus.



Then come I to the ſecond line, and take vp thoſe 4 counters, ſetting for them the multiplier foure times, ſo will the whole ſum appeare thus.



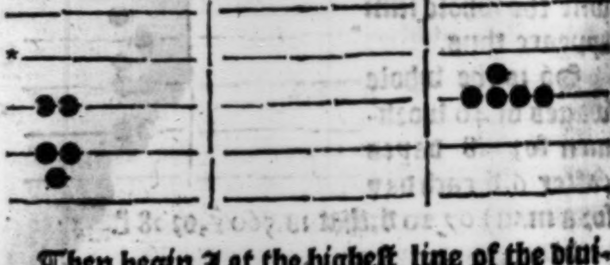
So is the whole wages of 40 workmen for 28 dayes (after 6 d each day for a man) 6720 d, that is 560 s, or 28 l.

M. Now if you would prooue Multiplication, the ſureſt way is by Diuiſiõ, therefore wil I ouerpaſſe it, till I haue taught you the Art of Diuiſion, which you ſhall worke thus.

Diuiſion.

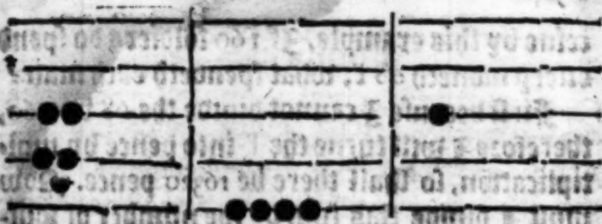
First ſet downe the diuiſor, for feare of forgetting, and then ſet the number that ſhall be diuided, at the right ſide, ſo farre from the diuiſor, that the quotient may be ſet betwéene them: as for example.

If 225 ſheepe coſt 45 l., what did euery ſheepe coſt? To know this, I ſhould diuide the whole ſumme, that is 45 l. by 225, but that cannot be: therefore muſt I firſt reduce that 45 l. into a leſſer denomination, as into ſhillings, then I multiplie 45 by 20, and it is 900: that ſum ſhall I diuide by the number of ſheepe, which is 225, theſe two numbers therefore I ſet thus.

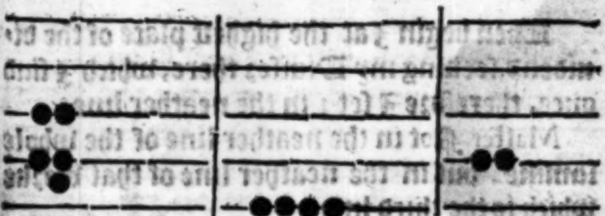


Then begin I at the higheſt line of the diuidend, and ſéeke how often I may haue the diuiſor therein, and that may I doe foure times: then ſay I foure times 2 are 8, which if I take from 9, there reſteth but 1, thus.

And



And because I found the diuisor 4 times in the diuidend, I haue set as you see, 4 in the middle roome, which is the place of the quotient: but now must I take the rest of the diuisor as often out of the remainer, therefore come I to the second line of the diuisor, saying: 2 times 4 make 8, take 8 from 10, and there resteth 2 thus.



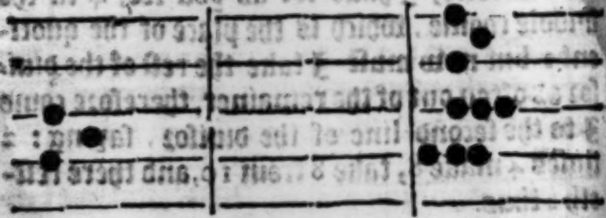
Then come I to the lowest number which is 5, and multiplie it 4 times, so is it 20, that take I from 20, & there remaineth nothing, so that I see my quotient to bee 4, which are in value shillings, for so was the diuidend: and thereby I know that if 225 sheepe did cost 45 li. euery sheepe cost 4 s.

Scholer. This can I doe, as you shall per-

Example of wages.

ceiue by this example. If 160 souldiers do spend euery moneth 68 ℓ, what spendeth each man?

First because I cannot diuide the 68 by 160, therefore I will turne the ℓ into pence by multiplication, so shall there be 16320 pence. Now must I diuide this sum by the number of souldiers, therefore I set them in order thus.

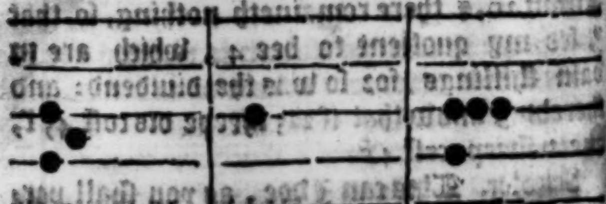


Then begin I at the highest place of the diuidend, seeking my Diuisor there, which I find once, therefore I set 1 in the neather line.

Master. Not in the neather line of the whole summe, but in the neather line of that worke which is the third line.

Scholer. So standeth it with reason.

Master. Then thus do they stand.



Then

Then seeke I againe the rest, how often I may find my diuisor: and I see that in 300 I might find 100 three times, but then the 60 will not be so often found in 20, therefore I take 2 for my quotient: then take I 100 twise from 300, and there resteth 100, out of which with the 20 (that maketh 120) I may take 60 also twice and then stand the numbers thus.

Where I haue set the quotient 2 in the lowest line: So is euerie Souldiers portion 102 d, that is 8 s, 6 d.

Master. But yet because you may iustly perceiue the reason of Diuision, it shall be good that you do set your Diuisor still against those numbers from which you doe take it, as by this example I will declare.

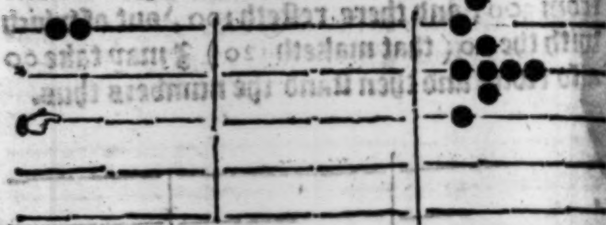
If the purchase of 200 acres of ground did cost 290 pound, what did one acre cost? Exãple of purchase.

First will I turne the poundes into pence, so will there be 69600 pence. Then in setting downe these numbers I shall doe thus.

First set the diuidend on the right hand as it

did

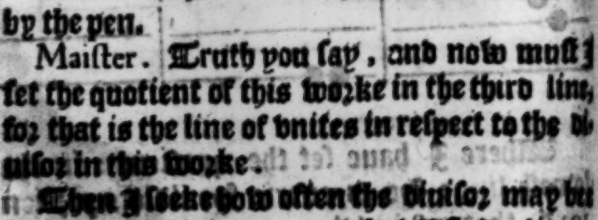
ought, & then the diuisor on the left hãd against those numbers from which I intend to take him first, as here you see, where I haue set the Diuisor two lines higher than his own place.



Scholer. This is like the order of Diuision by the pen.

Maister. Truth you say, and now must I set the quotient of this worke in the third line, for that is the line of vnites in respect to the diuisor in this worke.

Then I seeke how often the diuisor may bee found in the diuidend, and that I find 3 times, then set I 3 in the third line for the quotient, and take away that 60000 from the diuidend, and further I set the Diuisor one line lower, as you see here.



And

And then seeke I how often the Diuisor will be taken from the number against it, which wil be 4 times, and 1 remaining.

Scholer. But what if it chaunce that when the diuisor is so remoued, it cannot be once taken out of the diuidend against it?

Maister. Then must the Diuisor be set in an other line lower.

Scholer. So was it in Diuision by the pen, and therefore was there a cipher set in the quotient: but how shall that be noted here?

Master. Here needeth no token, for the lines do represent the places: onely looke that you set your quotient in that place which standeth for vnites in respect of the Diuisor. But now to returne to the example. I finde the Diuisor 4 times in the Diuidend, and one remaining: for foure times 2 make 8, which I take from 9, and there resteth 1, as this figure following sheweth: and in the middle space for the quotient I set 4 in the second line, which is in this worke the place of vnites.

Then remoue I the Diuisor to the next

lower line, and ſeeke how often I may haue it in the diuidend, which I may do here 8 times iuſt, and nothing remaine, as in this forme.

Where you may ſee that the whole quotient is 348 pence, that is 29 ſhillings, whereby I know that ſo much coſt the purchaſe of one acre.

Scholer. Now reſteth the proues of Multiplication, and alſo of Diuiſion.

Maſter. Their beſt proues are each one by the other: for multiplication is proued by Diuiſion, and Diuiſion by Multiplication, as in the worke by the pen you learned.

Scholer. If that be all, you ſhall not neede to repeate againe that which was ſufficiently taught alreadie: and except you will teach me any other feate, here may you make an end of this Art, I ſuppoſe.

Maſter. So will I do as touching whole number: and as for broken number, I will not trouble your wit with it, till you haue practiſed this ſo well, that you be full perfect, ſo that you neede not to doubt in any point that I haue taught you, and then may I boldly in-
ſtruct

struct you in the Art of Fraction or Broken number: wherein I will also shew you the reasons of all that you haue now learned. But yet before I make an end, I will shew you the order of common casting, wherein are both pence, shillings, and pounds, proceeding by no grounded reason, but onely by a receiued forme, and that diuersely of diuers men: for the Marchants vse one forme, and Auditors an other.

Marchants vse.

BUt first for marchantes forme, marke this example here, in which I haue expressed this summe 198 l, 19 s, 11 d. So that you may see that the lowest line serueth for pence, the next aboue for shillings, the third for pounds, and the fourth for scores of pounds.

And further you may see that the space betweene d and s, may receiue but one Counter (as all other spaces likewise doe) and that one standeth in that place for 6 d.

Likewise betweene the shillings and the pounds, one counter standeth for 10 s.

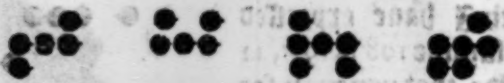
And betweene the poundes & 20 l, one counter standeth for 10 l.

But beside those you may see at the left side of shillings, that one counter standeth alone, and betokeneth 5 s.

So against the poundes, that one counter standeth for 5 l. And against the 20 poundes, the one counter standeth for 5 score poundes, that is 10 pound, so that euery side counter is 5 times so much as one of them against which he standeth.

Auditors account.

Now for the account of Auditors, take this example.



Where I haue expressed the same summe 198 l, 19 s, 11 d.

But here you see the pence stand towardes the right hand, and the other encreasing orderly toward the left hand.

Againe you may see that Auditors will make two lines (yea and more) for pence, shillings, and all other values, if their summes extend thereto. Also you see that they set one counter at the right end of each rowe, which so set there, standeth for 5 of that roome: and on

on the left corner of the row it standeth for 10 of the same row.

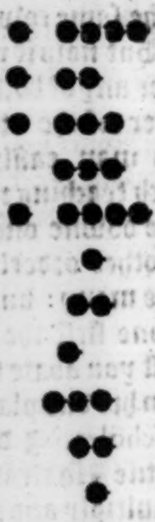
But now if you would adde, either subtract after any of both those sorts, if you marke the order of the other feate which I taught you, you may easily doe the same here without much teaching: for in Addition you must first sette downe one summe, and to the same set the other orderly, and in like manner if you haue many: but in Subtraction you must set downe first the greatest summe, and from it must you abate the other, euery denomination from his due place.

Scholer. I doe not doubt but with a little practise I shall attaine these both: but how shall I multiply and diuide after these formes?

Maister. You cannot duely do any of both by these sortes, therefore in such case you must resort to your other artes.

Scholer. They that vse such accountes that it exceede 200 in one summe, they set not 5 at the left hand of the scores of poundes, but they set all the hundreds in an other farther row, and 500 at the left hand thereof, and the thousandes they set in a farther row yet, and at the left side thereof they sette the 5000, and in the space ouer they set the 10000, and in a higher row 20000, which all I haue expressed in this example, which is 97869 l. 12 s, 9 d, ob, q. Ninety seuen thousand, eight hundred, three score and nine pound,

twelue shillings and nine-pence halfepeny farthing, for I had not told you before, where, neither how you should set downe farthings, which (as you see here) must be set in a voide space sideling beneath the pence: for farthing one counter, for ob. two counters: for ob. farthing three counters: and more there cannot be: for 4 farthings make 1 d, which must bee set in his due place.



And if you desire yͤ same summe after Auditors maner: Lo here it is.

But in this thing you shall take this for sufficient, and the rest you shall obserue as you may see by the working of each sort: for the diuerse wits of men haue inuented diuerse and sundry waies, almost vnnumerable.

The

The ſecond parte of Arithmeticke touching Fractions, briefly ſet forth.

Scholer.

Albeit I do perceiue your manifold buſineſſe doth ſo occupy, or rather oppreſſe you, that you cannot as yet completly ende the treatiſe of Fractions Arithmeticall, which you haue prepared, wherin not onely ſundrie works of Geometrie, Muſicke, and Aſtromomie be largely ſet forth, but alſo diuerſe concluſions and naturall workes, touching mixtures of metals and compoſitions of medicines, with other ſtrange examples, yet in the meane ſeaſon I cannot ſtay my earneſt deſire, but importunely craue of you ſome brief preparation toward the vſe of fractions, whereby at the leaſt I may be able to vnderſtand the common workes of them, and the vulgar vſe of thoſe rules, which without them cannot wel be wrought.

Maſter. If my leyſure were as great as my will is good, you ſhould not neede to vſe any importunate crauing, for the attaining of that thing, whereby I may be perſwaded that

I shall any wayes profite the common wealth, or helpe the honest studies of any good members in the same: wherefore, while mine attendance will permit me to walke and talke, I am well willing to helpe you as I may.

What a fraction is

Therefore first to begin with the explication of this name Fraction, what take you it to be?

Scholer. Marie sir, I thinke a Fraction (as I haue heard it often named) to be a broken number, that is to say, to be no whole number, but a part of a number.

Master. A Fraction indeed is a broken number, and so consequently, the part of another number: but that must be understood of such another number, as cannot be diuided into any other parts than fractions: for although I may take the third part of 60, or the fourth part of it: and so of other parts diuersly, yet these parts be not properly, nor ought not to be called fractions, because they may be expressed by whole numbers; for the third part of it is 20 the fourth part is 15: the twelfth part is 5, and so forth of other parts, which all be whole numbers.

What a fraction is properly.

Wherefore properly a Fraction expresseth the partes or part onely of an vnite, that is to say, that the number which is the whole or entire summe of any Fraction, may not be greater than one: and therefore it followeth, that no one Fraction alone can be so great, that it shall make 1, as by examples I will declare, as soone as I haue taught you to knowe the

forme

forme how a Fraction is expreſſed or repreſented in writing.

Numeration.

BUt firſt to beginne with expreſſing of a Fraction, which is the Numeration of it, you may vnderſtand that a fraction is repreſented by two numbers, ſet one ouer the other, and a line drawne betwéene them as thus. $\frac{1}{3}$. $\frac{3}{4}$ $\frac{2}{5}$. $\frac{10}{17}$. which foure fractions you muſt pronounce thus: $\frac{1}{3}$ one third part: $\frac{3}{4}$ thrée quarters: $\frac{2}{5}$ two fift partes: $\frac{10}{17}$ ten ſeuentéene partes.

Scholer. I vnderſtand the forme of their expreſſion and pronuntiation, but their meaning or valuation ſéemeth more obſcure: yet I thinke that by the two firſt Fractions I vnderſtand the valuation of the two latter fractions, and ſo conſequently of other.

Maſter. Value them then, that I may perceiue your taking of them.

Scholer. $\frac{2}{5}$ Betokeneth two fift partes, that is to ſay, if one be diuided into 5 partes. that Fraction doth expreſſe two of thoſe fift partes: $\frac{10}{17}$ doth ſignifie, that if one be diuided into xvij.

parts, I must take ten of them: And this I gather of the two first examples: for $\frac{1}{3}$ that is one third parte, doth easily declare, that if any one thing be divided into thrée partes, I must take but one of them: so $\frac{3}{4}$ that is, thrée quarters, doth declare that one being divided into foure quarters, I must take (for this Fraction) thrée of those quarters.

If there be no more difficultie in their Numeration, then I pray you go forward to their Addition and Subtraction, and so to the other kindes of workes, for I understand that the same kindes of workes be in Fractions, that be in whole numbers.

Master. There are the same kindes of workes in both, albeit the order of them is diverse, as I will anon declare: but yet more in Numeration before we leave it. You must understand, that those two numbers which expresse a Fraction, have severall names. The overmost which is above the line, is called the Numerator, and the other beneath the line is called the Denominator.

Numerator & Denominator.

Scholer. And what is the reason of their diverse names? For in mine opinion both be Numerators, séeing both they do expresse the Numeration of the Fraction.

Master. You are deceived: for one onelie (which is the overmost) doth expresse the Numeration: and the Denominator doth declare the number of partes into which the unite is

divided,

diuided, as in this example, when I say: Diuide a pound weight of gold betwéen four men, so that the first man shall haue $\frac{2}{15}$, the second $\frac{3}{15}$, the third $\frac{4}{15}$, and the fourth $\frac{6}{15}$.

Now doe you perceiue that by the Denominator (which is one in all four Fractions) it is intended, that the pound weight should be diuided into so many parts, I meane fiftéene, and by the foure seuerall numerators is limited the diuerse portion that each man should haue, that is, that when the whole is parted into fiftéene, the first man shall haue two of those fiftéene parts; the second man thrée of them: the third man foure: and the fourth man sixe. And so may you sée the seuerall offices (as it were of those two numbers, I meane of the Numerator and the Denominator.

And hereby you perceiue, that a man can haue no more partes of any thing than it was diuided into, neither yet aptly so many: so that it were vnaptly sayd: You shall haue $\frac{15}{15}$, that is 15 parts of any thing, séeing it were better said: You shall haue the whole thing.

Scholer. So doth it appeare reasonably, for the labour is vaine to diuide any thing, and then to apply the Diuision to no vse. And much lesse reasonable where it to say $\frac{16}{15}$: for if the whole be diuided into 15 partes onely, it is not possible to take 16 of them, that is to say, more than altogether.

Maister. This is true touching the proper

and apt use of the name of a Fraction: yet improperly, and after a vulgar acceptation (for easinesse in worke) both those formes bee called Fractions, because they bee written like Fractions, although they be none in deede for $\frac{11}{11}$. and generally all such other: where the Numerator and Denominator bee equall, are not Fractions: but the whole thing with all his parts. And so $\frac{16}{12}$ is not to bee called a Fraction, but a mixt number, of a whole number & a fraction: for it is as much, as $1\frac{4}{12}$. that is one whole one and 4 twelue parts, as shalbe declared in reduction. Therefore they doe abuse the names, that call them fractions, where the Numerator is eyther equal or greater than the denominator.

Scholer. But is there any needefull cause why they should so abuse the name?

Maister. There is cause why they shall sometimes for easinesse in worke, write some numbers after that sort, like fractions: but they needed not to call them fractions, but as they be, whole numbers or mixt numbers (that is whole numbers with fractiõs) expressed like fractions.

Now must you vnderstand, that as no fraction, properly can be greater than 1, so in smalnesse vnder one the nature of fractions doth extend infinitely: as the nature of whole numbers is to encrease aboue one infinitely, so that not onely one may be diuided into infinite frac-

fractions or parts, but alſo euery fraction may bee diuided into infinite fractions or partes, which commonly be called fraction of fractions, and they be expreſſed diuerſely: As for example, $\frac{3}{4}$ of $\frac{2}{3}$ $\frac{1}{2}$, that is thrée quarters of two third parts, of one halfe part. Whereby is ſignified, that if one be diuided into two halfes, and the one halfe into thrée partes, and two of thoſe thrée parts bee diuided iointly into foure quarters, this fraction of fractions doth repreſent thrée of thoſe quarters.

Scholer. I pray you let me prooue by an example in common money, whether I do rightly vnderſtand you or no. One Crowne, which I take for an vnit, doth cōtaine 60 pence, therfore the halfe of it is 30 pence: $\frac{2}{3}$ of that halfe is 20 pence, whereof $\frac{3}{4}$ is fiftéene pence, ſo then 15 pence is $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$ of a Crowne. And ſo thrée pence is $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$ of a ſhilling.

Maiſter. You perceiue this well inough: yet this note I giue you by the way, that the forme of expreſſing theſe Fractions is voluntary: and hath no other reaſon than the will of the Deuiſer, which forme many follow: for ſome expreſſe them thus $\frac{3\ 2\ 1}{4\ 3\ 2}$ without any figure of diſtinction betweene them, which forme many follow. Some other doe make lines betweene euery Fraction, and adde words of diſtinction, after this ſort, $\frac{3}{4}$ of $\frac{2}{3}$, of $\frac{1}{2}$ which forme is moſt beſt.

Some

Some other expresse them thus in slope formes, to distinct them from severall Fractions of one whole number. for if $\frac{3}{4}$ $\frac{2}{3}$ $\frac{1}{2}$ they were set in one right line thus, $\frac{3\ 2\ 1}{4\ 3\ 2}$: then ought it to be pronounced, three quarters, and two third parts, and an halfe, which maketh almost two whole unites, lacking but one xii. part. And so is it nothing agreeable with the other fraction of fractions, wherefore it is a great oversight in certaine learned men, which doe expresse them so confusedly with such severall fractions, that a man cannot know the one from the other.

Therefore some men (as Stifelius) doe expresse without a line numbers of proportion, being applied to Addition or Subtraction: because they must be taken as two, where the line in fractions maketh them to be taken for one: for of the Numerator and Denominator is made one number.

Three severall varieties.

Scholer. Then I perceive there be three severall varieties in fractions: First when one onely fraction is set for one number, as $\frac{4}{5}$, that is foure fifth parts. The second is, when there be sette two or more severall fractions of one number, as $\frac{4}{9}$ $\frac{2}{5}$, that is foure ninth partes, and two fift parts. The third sort is fractions of fractions, as $\frac{4}{9}$ of $\frac{2}{5}$, that is 4 ninth partes of two fifth partes.

Maister. You have sayd well, if you understand

derstand well your owne words.

Scholer. If it shall please you, I will by an example in the parts of an old English Angell expresse my meaning.

Master. Let me heare you.

Scholer. The old English Angell did containe 7 shillings 6 d, that is 90 d. Now $\frac{4}{5}$ of it, is 72 d. And of the same 90 pence, if I take $\frac{4}{9}$ and $\frac{2}{5}$, that is foure ninth partes, and two fifth partes, $\frac{4}{9}$ is 40, and $\frac{2}{5}$ is 36, which both make 76: but if I take $\frac{4}{9}$ of $\frac{2}{5}$, that is, 4 nine partes of two fifth partes, seeing $\frac{2}{5}$ is but 36, then $\frac{4}{9}$ of 36 will yeeld but 16, for $\frac{1}{9}$ is but 4, and that taken 4 times maketh 16.

Master. This is plainely expressed and truely, and hereby (I doubt not) but you do perceive, that as great a difference as is betweene 16 and 76, so much difference is betweene those 2 Fractions $\frac{4}{9}$ and $\frac{2}{5}$: and $\frac{4}{9}$ of $\frac{2}{5}$.

And now that you understand these varieties, I will proceed to the rest of the workes: first admonishing you, that there is another order to be followed in Fractions than there was in whole numbers, for in whole numbers this was the order: Numeration, Addition, Subtraction, Multiplication, Division, and Reduction, but in Fractions (to followe the same aptnesse in proceeding from the easiest workes to the harder) we must use this order of the workes: Numeration, Multiplication, Division, Reduction, Addition, and Subtra-

ction.

Scholer. That Multiplication and [illegible] tion should goe together, and [illegible] follow Addition, [illegible] swade: but [illegible] Multiplication [illegible] in order here next to Numeration, [illegible] duction in the middle, I desire to [illegible] the reason.

Master. As in the Arte of whole numbers order would reasonably begin with the easiest, and so goe forward by degrees to the hardest, euen so reason teacheth in Fractions the like order. And considering that Addition or Subtraction of fractions can verie seldome be wrought without Multiplication and Reduction: and contrariwise, Multiplication and Reduction may bee wrought without this forme of Addition or Subtraction: Therefore was it orderly required, that Multiplication and Reduction should goe before Addition and Subtraction. And the same reason serueth for the placing of Multiplication before Reduction.

Scholer. [illegible] Multiplication be the easiest, I pray you declare the forme of it first by rule, and then by example.

Master. Your example is good.

Multiplication.

THerefore when any two Fractions be proponed to be multiplied together, the Numerator of the one must be multiplied by the Numerator of the other: and the summe that amounteth thereof, must be set for a new Numerator: likewise the Denominator of the one must be multiplied by the Denominator of the other, and that that amounteth, shall be set for the common Denominator; and this new thirde fraction expresseth the product of the Multiplication of the two first fractions proponed, whereof take this example, $\frac{3}{5}$ multiplied by $\frac{5}{12}$, doth make $\frac{15}{60}$.

Scholer. I perceiue then, that 3 being the numerator of the first fraction, is multiplied by 5, being the Numerator of the second fraction, whereof amounteth 15, the Numerator of the third fraction. And so likewise, 5 being Denominator of the first Fraction, is multiplied by 12 the Denominator of the second Fraction, whereof amounteth 60 the new Denominator: So that I perceiue how the worke is doone, but I doe not perceiue howe $\frac{15}{60}$ is greater than $\frac{3}{5}$: For I shall not vse my former manner of examination by the partes of some Coine. I see that $\frac{3}{5}$ of a Crowne is 36 d, and $\frac{5}{12}$ of a Crowne, is 25 d, whereof the one

multiplied by the other, doth make 900 pence, which is 15 Crownes : but by your multiplication there amounteth $\frac{15}{60}$, which is but 15 d, and that is much lesse than any of both the first Fractions.

Master. That difference is betweene multiplication in whole numbers, and multiplication in broken numbers, that in whole numbers the summe that amounteth, is greater then both the other whereof it came : but in fractions it is contrariwise : for the summe that amounteth, is lesser then any of the other two fractions, whereof it came.

Scholer. I desire much to understand the reason thereof.

Master, Although I purposed to reserue the reasons of workes Arithmeticall for the perfect booke of Arithmeticke, yet I will shew you this, because of the strangenesse of the worke.

You see in whole numbers, that of two numbers being multiplied together, is made the thirde number : which third number doeth beare the same proportion to the number multiplied, that the multiplier doth beare to an unite. And so in Fractions, the third number which amounteth of multiplication, beareth the same proportion to each of the two first fractions, that the other of those two fractions doth beare to an unite.

Scholer. Syr, I understand your wordes thus :

thus: when 40 is multiplied by 12, there doeth amount 480, which 480 doeth containe 40 ſo many times in it, as twelue doth containe vnites, that is to ſay: twelue times. And ſo it appeareth that 480 doeth containe twelue ſo many times alſo, as 40 doth containe vnites, that is 40 times. But now I ſee not how the third number in this example of Fractions can containe any of the two former (as it happened in whole numbers) ſeeing it is leſſer than either of them.

Maiſter. No maruaile if you cannot ſee that thing which is not poſſible to be ſeene of any man, how the third number in multiplication of Fractions ſhould bee greater than any of the two former Fractions, but yet this may you ſee (which I ſayd) that the third number in Fractions ſo multiplied, doeth beare the ſame proportion to any of the two former fractions, that the other of thoſe two Fractions doth beare to an vnite, as in your example $\frac{3}{5}$ being multiplied by $\frac{5}{12}$, doth make $\frac{15}{60}$. Now ſay I, that $\frac{15}{60}$ doeth beare the ſame proportion to $\frac{3}{5}$, that $\frac{5}{12}$ doth beare to an vnite, as you may in your owne forme of examination by coyne trie it. For in an old Angell (which in times paſt was currant for 7 ſ. 6 d.) are 180 halfe pence, which I ſet for the entire vnite, whoſe partes (according to the fractions aforeſayd) are theſe, for $\frac{15}{60}$ ſette 45 ob. for $\frac{3}{5}$ take 180 ob. and for $\frac{5}{12}$ put 75 ob. Now doeth 45 beare the ſame pro-

portion to 108 that 75 doth beare to 180: for 45 is $\frac{5}{12}$ of 108, and so is 75 also $\frac{5}{12}$ of 180.

But these reasons may be better reserved till an other time, when the knowledge of proportions in due order shall be taught. Yet in the meane season I will shew you how it commeth to passe that in Fractions the third summe must needes be lesser then any of the other two.

Consider thus, that when a Fraction is proponed, as in the former example $\frac{3}{5}$, if it be multiplied by more than 1, it will make more than one entire number. As if I multiply $\frac{3}{5}$ by 5, that is to say, if I take it 5 times, it will make three entire unites: example in a crowne, $\frac{3}{5}$ of it maketh 3 s. which if I take fiue times, it will amount to 15 s, that is three entire Crownes: so if I take the same $\frac{3}{5}$ but twise, it will yeelde sixe shillings, that is one entire Crowne and $\frac{1}{5}$. Now if I take it but once, it cannot be more than it was before, that is fiue shillings. And if I take it lesse then once, it cannot be so much as it was before. Then seeing that a fraction is lesse then one, if I multiply a Fraction by an other Fraction, it followeth that I doe take that first fraction lesse than once, and therefore the summe that amounteth, must needes bee lesse than the first fraction.

Scholer. Sir, I thanke you much for this reason. And I trust I doe perceiue the thing, as by example of this same fraction $\frac{3}{5}$ I will expresse.

presse. If I take $\frac{3}{5}$ of a crowne once, that is to say, if I multiply $\frac{3}{5}$ by 1, it will bee as it was before, but 3 s: so if I doe multiply it by $\frac{1}{2}$, that is, if I take but halfe one time, then will it bee but halfe so much: likewise if I multiply it by $\frac{1}{3}$, that is, if I take but the third part once, it will yeeld but 12 pence, that is, the third part of the first fraction.

And so to make an end. If I take it but the twelfth part once, that is, if I do multiply it by $\frac{1}{12}$, it will yeeld but the twelfth part of the first fraction, which is but three pence. And it followeth that if $\frac{1}{12}$ make three pence, then $\frac{5}{12}$ must needs make five times so much, that is fifteene pence, which was the summe that hath given the occasion of all this doubt.

Maister. Then I perceive you have sufficient understanding in this sort of Multiplication for this time, wherefore I will omit that I might say more of Multiplication, till we come to Reduction, and wil passe to the other works, and first to Division, whose place followeth Multiplication, both by naturall order, and also in easinesse of worke.

Diuision.

Whensoeuer two Fractions bee proponed, that the one should be deuided by the other, I must set downe first the Fraction that shall be diuided (which is called the Diuidend) and then after it the other, which is the Diuisor. Then shall I multiply the numerator of the diuidend by the denominator of the diuisor, and that which amounteth, I must put for a new Numerator. Againe I shall multiply the denominator of the diuided by the numerator of the diuisor, and the number that amounteth thereof, I must put for the new denominator. And this third fraction is the quotient of the said Diuision.

Scholer. This seemeth easie in forme, as by example thus: If I would diuide $\frac{5}{8}$ by $\frac{2}{6}$, first I must multiply 5 (being the numerator of the diuidend) by 6, which is denominator of the diuisor, and thereof riseth 30: then I multiply 8 (being the denominator of the diuidend) by 2, being the numerator in the diuisor and so riseth 16, the which I must make in a third fraction thus $\frac{30}{16}$.

Maister. Me seemeth you are quicker in vnderstanding nowe, than you were when I taught you the Art of whole numbers: but

that

that is no maruell, for the more knowledge that any man getteth, the readier shall he find his wit, and quicker in vnderstanding: but yet of two things I will admonish you, which you might haue obserued heere for ease of worke, and lightnesse of vnderstanding the nature of the Quotient.

Whensoeuer you diuide one Fraction by an other, either they be both equall together: either else the one is greater than the other: if they be equall, their quotient shall be such that the Numerator and the Denominator of it shal be equall also. And if the two first Fractions be vnequall, their quotient shall declare the same by the vnequalitie of the Numerator and Dinominator, as in these examples following shall appeare.

First of equall Fractions: $\frac{4}{9}$ and $\frac{12}{27}$ be equall together: and if the one be diuided by the other, the quotient will be $\frac{108}{108}$, as you may perceiue by that rule aforesaid.

Nowe in the vnequall Fractions, as $\frac{4}{3}$ and $\frac{9}{10}$, the quotient will be $\frac{40}{27}$: where the Numerator is greater than the Denominator.

Scholer. I see it is so, but I see not the reason why it should be so.

Master. The reason is this. When any Fraction is diuided by another, the Quotient declareth what proportion the Diuidend beareth to the Diuisor. so $\frac{1}{2}$ diuided by $\frac{1}{4}$, maketh 2, which must be sounded, not two, but

Note how to know the proportiō betweene two numbers.

twice: declaring that $\frac{1}{4}$ is contained twice in $\frac{1}{2}$,

And note this, that the Numerator in the quotient, representeth the Diuidend, and the Denominator representeth the Diuisor. And this is alwaies true, whether the greater fraction be diuided by the lesser, or the lesser by the greater. But this proportion will not be exactly knowne, till you haue learned the Art of proportions: notwithstanding some what of it I will declare in the next Rule of Reduction. But now for the easie remembraunce of the Quotient in Diuision, as sone as you haue set downe your two Fractions, the one against the other, then make a straight line for the quotient: and as sone as you haue multiplied the Numerator of the Diuidend, by the Denominator of the diuisor, set ÿ number that amounteth ouer the said line, and then multiplie the other two numbers, and set their totall vnder the same line.

Scholer. I perceiue you woulde not haue me trust to memorie till I were better expert, least oftentimes I happen by missremembrãce to be abused. This example I take for that declaration.

If I would diuide $\frac{2}{3}$ by $\frac{3}{4}$, I must set the numbers one against the other, (as herre doeth appeare) and then make another line for the Quotient in some good distance, where I may set the numbers of the Quotient, as sone as any of

$$\frac{2}{3} \text{ by } \frac{3}{4}$$

them

them is multiplied: so then as sone as I haue multiplied 2 by 4, which maketh 8, I shall set that 8 ouer that line thus. And then multiplie 3 by 3, which yeeldeth 9: and that 9 must be set vnder the same line, and then will the whole quotient appeare thus $\frac{8}{9}$. Whereby appeareth (as I remember your wordes) that $\frac{2}{3}$ is in proportion to $\frac{3}{4}$, as 8 is to 9: but how may I perceiue that?

8
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Note.

Master. Although you shall better perceiue it by the rule of Reduction, yet this example may be declared in common Coyne, as in a common shilling of xii d, of which $\frac{2}{3}$ maketh 8 d, and $\frac{3}{4}$ doth make 9 d, and so you may easily see that their proportion do agree. And if you had taken this example before, when you tooke the example of $\frac{1}{8}$ and $\frac{1}{8}$, your quotient would appeare (as it doth) more easie to vnderstand, whereas that quotient being $\frac{12}{16}$, is not an easie proportion for you to perceiue, being yet litle acquainted with proportions: whereof to giue you some tast, I will enter to the rule of Reduction: in which also I will declare other workes, both of Multiplication, and also of diuision, which now I must for a time omit, as things that do neede the helpe of Reduction.

Reduction.

Fiue varieties of Reduction.

Therefore will I now declare the diuersities of Reduction of Fractions, which commonly haue fiue varieties.

1 First, when there be sundrie Fractions of one entire vnite, they must be reduced to one denomination, and also into one Fraction.

2 Secondarily, when there be proponed fractions of fractions, they must be reduced likewise into one fraction, for otherwise they cannot be brought into one Denomination.

3 Thirdly, when an improper Fraction is proponed, that is to say, a Fraction in forme, which indeede is greater than an vnite, it must be reduced into apt forme expressing the vnite or vnites of it, and the proper fraction distinctly. And sometimes also it shall be needefull to conuert such a mixt number of vnites, with fractions into the forme of a fraction, that is, into an improper fraction: which two formes I esteeme but as one, because they worke on one kinde of number.

4 Fourthly, there happeneth sometimes Fractions to bee written in great numbers, which might be written in lesser numbers, therfore is there a mean to reduce such great numbers into their smallest termes.

5 Fiftly

5 Fiftly, when any fraction betokeneth the partes of a whole thing, which hath by common partition certaine partes, but none of like Denomination with that Fraction, then may you reduce the sayd Fraction into an other, whose denomination shall expresse the common parts of that whole thing.

Scholer. This distinction in doctrine delighteth me much, but more with hope than present fruite, for as yet I doe not understand scarcely the varieties, and much lesse the practise and use of their workes.

Maister. Reduction is an orderly alteration of numbers out of one forme into another, which is never done orderly but for some needefull use, as in every of the sayd 5 severall varieties I will distinctly declare.

First therefore, when two or more severall Fractions of any unite be proponed, as for example $\frac{1}{16}$ and $\frac{4}{5}$: because it is hard to tell what proportion of the entire number those two fractions doe expresse, therefore was Reduction devised, to be a meane whereby these severall fractions might be brought into one denomination and fraction. The first sort of Reduction.

And in these Fractions this is the Art for bringing them to one denomination.

Multiply first the denominators together, and the totall thereof you shall set twise downe under two severall lines for two new denominators, or rather for one common denomi- How to reduce fractions of divers denominations into one denomination.

nator: Then multiply the numerator of the firſt fraction, by the denominator of the ſecond, and ſet the totall thereof for the numerator ouer the firſt line. Likewiſe multiply the numerator of the ſecond fraction by the denominator of the firſt, and ſet that totall ouer the ſecond line for the numerator of that fraction, and ſo are thoſe two firſt fractions of ſeuerall denominations, brought to one denomination.

Scholer. If I vnderſtand you, as I thinke I doe, my example ſhall declare the ſame. The fractions which you proponed, were theſe, $\frac{3}{16}$ and $\frac{4}{6}$, whoſe denominators (being 16 and 6) I multiply together, and there amounteth 96, which I ſet vnder two lines, thus. $\frac{}{96}$: $\frac{}{96}$.

Then I multiply the numerator of the firſt fraction by the denominator of the ſecond, ſaying: 3 into 6 maketh 18, that ſet I ouer the firſt line for a new numerator, and it will be thus. $\frac{18}{96}$.

The firſt ſort of Reduction.

Likewiſe I multiply the numerator of the ſecond fraction by the denominator of the firſt, ſaying: 4 times 16 maketh 64, that I ſet for the ſecond numerator, and the fraction will appeare thus. $\frac{64}{96}$.

So that both fractions brought to one denomination, muſt ſtand thus: $\frac{18}{96}$ and $\frac{64}{96}$.

How to reduce fractions of diuers denominations into one denomination.

Maiſter. You haue done well.

Scholer. I beſéech you, let me examine it after my accuſtomed forme, by common partes of

of coyne or other measure.

Maister. Goe to.

Scholer. I haue a péece of gold which is accounted worth 8 shillings, and containeth 96 pence, whereof $\frac{1}{16}$ that is the xvi part, is 6 pence, and $\frac{3}{16}$ is 18 pence, that is $\frac{18}{96}$. Againe $\frac{1}{6}$ of the same peece of gold, is 16 d, so that $\frac{4}{6}$ maketh 64 d, that is $\frac{64}{96}$. And so I finde the summes to agrée with the other before.

Maister. So haue you now the Art to bring such two fractions into one denomination. And if there be more than two, then must you multiply all the denominators together, and set the totall thereof so many times downe as there be fractions, and then to get for each one a new numerator, multiply the numerator of the first by the denominator of the second, and the totall thereof multiply by the denominator of the third, and so foorth if there bee more. Likewise multiply the numerator of the second, by the denominator of the first, and the totall thereof by the denominator of the third. And in the same sort multiply the numerator of the third into the denominator of the first: and the totall thereof into the denominator of the second, and so foorth, if there were moe. So these 3 fractions $\frac{1}{5}\,\frac{3}{4}\,\frac{2}{3}$ doth make by Reduction these other 3 Fractions of one denomination $\frac{24}{60}.\ \frac{45}{60}.\ \frac{40}{60}.$ All which you may bring into one fraction by adding the Numerators together, and putting that totall for

the common numerator, reseruing still that same common denominator. And those 3 fractions make one improper fraction thus $\frac{102}{60}$.

Scholer. All this I perceiue, and also that this last fraction is more than an vnite, and therefore you did call it an Improper Fraction.

Maister. There be certaine other formes of working in this Reduction, which I wil briefly touch also, to giue you an occasion to exercise your wit therein.

The first variety of this Reduction.

The first variety is this. When you haue made and written downe your common denominator (as I haue taught before) then to get a numerator for the first, doe thus. Deuide the common denominator by the denominator of the first fraction, and the quotient multiplied by the numerator of the same, yeeldeth a new numerator for the first new fraction. So likewise doe with the second and the third, and with all the residue, if there be more.

Scholer. That will I proone in your last example of these three fractions $\frac{2}{5}$: $\frac{3}{4}$: $\frac{2}{3}$. When the denominators be multiplied, they make 60, for 5 into 4 maketh 20, and 20 by 3 yeeldeth 60, that I set downe three times, thus. $\frac{\ }{60}\ \frac{\ }{60}\ \frac{\ }{60}$: then to haue a numerator for the first, I must diuide 60 by 5, (the denominator of the first) and the quotient is twelue, which I must multiply by two (the numerator of the first) and that maketh foure and twenty, and so haue I for the first frac-

fraction $\frac{24}{60}$.

Likewise for the second Fraction I divide 60 by 4, and there commeth 15: which I multiplie by 3, and so haue I 45 and the seconde Fraction $\frac{45}{60}$. Then for the third in like sort will come $\frac{50}{60}$.

The secōd varietie

Master. Another way is this. If it happen so that the lesser denominator can by any multiplication make the greater, then note the multiplier, and by it multiplie the Numerator ouer that lesser Denominator, and for the lesser Denominator put the greater, as thus in these two Fractions, $\frac{5}{12}$ and $\frac{2}{3}$ three being the lesser Denominator multiplied by 4, wil make 12, which is the greater Denominator: therefore by the same 4, I doe multiplie 2, which is the Numerator ouer 3, and that maketh 8: vnder which I doe put 12, being the greater Denominator, which is also made by multiplication of 4 into 3, and so haue I these 2 Fractions $\frac{5}{12}$, $\frac{8}{12}$: thus shortly reduced without altering the one Fraction.

Scholer. This I vnderstand.

The third varietie.

Master. Then marke this thirde way: If the denominators do not happen so, that one by multiplication may make the other, then looke whether they both may be partes of any other one number, as in $\frac{5}{12}$, and $\frac{7}{18}$, although the lesser taken but twise, be too great to make 18, yet they both may be parts vnto 36: therefore looke how many times twelue is in 36, and

that quotient being multiplyed by the Numerator ouer 12, the totall shall be put in stead of the Numerator ouer 12, and for 12 put 36, thus, $\frac{15}{36}$. So likewise looke how often is 18 in 36, and because it is twise, therefore by 2 multiplie 7 which is ouer 18, and it will be 14 set that for the Numerator, and in stead of 18 put 36, and then shall your fractions reduced stand thus, $\frac{15}{36}$ $\frac{14}{36}$ in stead of $\frac{5}{12}$ and $\frac{7}{18}$.

The second varietie.

And if you will proue whether you haue wrought well or no, that may be proued by Reduction of them againe to their former Denominations, which Arte shall be taught in the fourth kinde of Reduction, where greater termes of Fractions be reduced into smaller in number, but no smaller in proportion. And if in such Reduction the same termes or numbers come againe that were before, then is the worke good, else not.

Scholer. Sir, I heare your wordes, but I do not understand many of them, which it may please you to declare.

The third varietie.

Master. With a good will, when conuenient place serueth, but that must be in the said fourth kinde of Reduction. In the meane season I will declare the second forme of Reduction, which teacheth how to reduce fractions of fractions into one fraction, and so to one Denomination.

When fractions of fractions be proponed, you shall multiplie the Numerators of each into

into other, and set the totall for the new numerator, and then multiply all the denominators likewise, and take their totall for the new denominator, and so are they speedily reduced.

Reduction of fractions into one denomination.

Scholer. If that be all then I understand it already, as by this example I will declare. These be the fractions, $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{6}{7}$ of $\frac{7}{9}$ which I would reduce to one denomination.

Therefore beginne I with the numerators and multiply them altogether, saying: 3 into 2 maketh 6, and 6 by 6 maketh 36, which multiplied by 7, yeeldeth 252, that I set ouer a line for the numerator, thus

$$\frac{252}{}$$

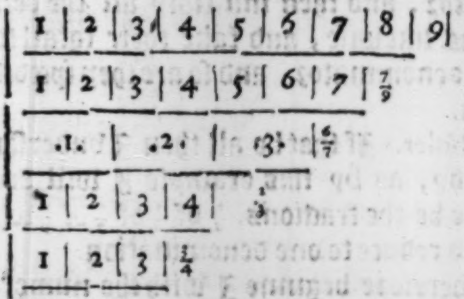
Then I multiply the denominators, 4 by 3 maketh 12, and that by 7 bringeth 84, which multiplied by 9, yeeldeth 756, the new denominator. And so the whole reduced fraction is this, which is too hard a fraction for me to understand.

$$\frac{252}{756}$$

Maister. You thinke so, and no maruell, but anon you shall learne to iudge it easily, for this fraction is no more in deede than $\frac{1}{3}$, although it be in greater termes, and therfore more stranger and more obscure.

And this sufficeth for this Reduction, saue that I will shew you by a figure of measure, the iust rate and reason of this kinde of fractions, and also the due understanding of the Reduction.

The entire meaſure parted into 9.



Here you ſee the longeſt meaſure, (which ſtandeth for the whole and entire quantity) firſt parted into Diviſions 9, whereof 7 are ſevered by the ſecond meaſure: and thereof againe are parted out 6. And that 6 being diſtinct into 3 parts, 2 of them are parted by the fourth meaſure, of which fourth meaſure, being divided into 4 parts, the loweſt meaſure doth containe $\frac{3}{4}$, ſo that the ſumme $\frac{3}{4}$ muſt bee named, not $\frac{3}{4}$ of the whole meaſure, but in deede is $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{6}{7}$ of $\frac{7}{9}$. $\frac{2}{3}$ of $\frac{6}{7}$ of $\frac{7}{9}$

Scholer. This example is ſo ſenſible, that I cannot chooſe but ſee it. And furthermore, I ſee alſo that the ſame fraction is equall to $\frac{3}{9}$ of the entire meaſure, as the lines which run up and downe doe expreſſely ſet forth. Also I ſee here, that $\frac{2}{3}$ of $\frac{6}{7}$ of $\frac{7}{9}$ is equall to $\frac{4}{9}$. And further yet that $\frac{2}{7}$ of $\frac{7}{9}$ is equall to $\frac{6}{9}$ or $\frac{2}{3}$. $\frac{2}{3}$ of $\frac{6}{7}$ of $\frac{7}{9}$

Maiſter. I am glad that you ſee it ſo well. not doubting but you will gather greater light of knowledge hereby.

But

But now it is time that we come to the third forme of Reduction, which teacheth of improper fractions, that is to say, mixt numbers of unites and fractions, although they appeare like fractions, as this, $\frac{26}{5}$, which doth include five units wholly, and $\frac{1}{5}$ over. Wherefore first you shall know them, by that the numerator is greater than the denominator. The third forme of Reduction of improper fractions.

Scholer. Indeede Sir, that appeareth reasonable, that if the numerator doe expresse more partes to be taken of any unite than the denominator doth signifie that unite to be divided into, it must needes follow that such a fraction importeth more than the whole, that is to say, the whole with certaine parts over. But what Reduction is there in it?

Master. There be two severall kindes of Reduction, concerning such fractions. Sometimes it shall be needfull to convert those fractions into the unites and the proper fraction that will remaine. And sometimes contrariwise, it shall be meete to reduce mixt numbers, that is unites, written with fractions into the forme of one simple fraction, and so be there two waies.

Scholer. What is the meane of the first way to turne improper fractions into unites with their proper fractions?

Master. That is thus. Your Numerator being greater than the Denominator, must be divided by the same Denominator, and the

Reductiō of improper fractions into vnites with their proper fractions.

quotient thereof expresseth the vnites: the remainer shall be put for the Numerator of the fraction that resteth, and the denominator must be the same that was before.

Scholer. For example, I take $\frac{17}{5}$. And dividing 17 by 5, the quotient will be thrée, and there will remaine 2.

Master. That must you write thus, $3\frac{2}{5}$ where (you sée) I haue written 3, without any line, as entire numbers ought to be written, and the 2 that remained I haue set ouer the former denominator with a line, as a proper fraction. And this number doth signifie nowe 3 vnites, and $\frac{2}{5}$ of one.

Scholer. Then if I woulde by vnites here vnderstand crownes, so it were thrée crownes, and $\frac{2}{5}$, that is 2 s.

Master. Euen so, and therefore $\frac{17}{5}$ did signifie the same. But this happeneth sometimes, that when the Reduction is so wrought, there remaineth nothing. And then it is not a mixed number, but a simple entire number, represented like a fraction.

Reductiō of whole numbers, either alone, or ioyned with fractions into fractions.

Scholer. As $\frac{15}{5}$ will make 3 iust, and $\frac{18}{3}$ will make euen 6. This I wil remember. But now, what is the second forme of Reduction, that you spake of for these sortes of fractions?

Master. Whensoeur you haue anie of these two sortes of numbers, that is to say, whole numbers without fractions, or whole numbers with fractions, and you would turne

them

them into the forme of a fraction, you must multiplie the whole number by that denominator, which you will haue to remaine still, and to the totall thereof adde the Numerator which you haue alreadie, and all that shall you set for the new numerator, keeping still the former denominator: as if you haue $6\frac{3}{4}$ which you would conuert into an improper fraction, you must multiplie 6 by 4, whereof commeth 24, and thereto adde the Numerator, which is 3, & so haue you 27 for the Numerator, & 4 still for the Denominator.

Scholer. Then is $\frac{27}{4}$ equall to $6\frac{3}{4}$.

Master. Euen iust, and so backeward (as appeareth by the former reduction) $6\frac{3}{4}$ maketh $\frac{27}{4}$. And thus one of these Reductions may be the proofe of the others worke.

Scholer. This I perceiue, but now if you would turne whole numbers without fractions into any fraction, I see not how that may be done, because there is no denominator to make the multiplication by.

Master. That was well marked: but this you know, that no man intendeth to turne any whole number into a fraction, but he hath in his minde that Denominator by which the multiplication must be made: for the proofe whereof I sette downe 7, which is a whole number. And if you will haue this number conuerted into anie certaine fraction, will me to do it.

Scholer. I pray you reduce seuen into a fraction.

Master. Then you care not what the fractions be, so it be some fraction.

Scholer. No, I passe not for the sort of the fraction.

Master. Then howe can you thinke that you require mee to doe anie thing certaine, when you leaue me to do as I list? And seeing you stande at that stay, whether thinke you that I must first intende in minde what fraction I will make of it, before I can doe it indeede?

Scholer. Else you should do ignorantly.

Master. Then will I limit my selfe (seing you will not) to turne it into quarters. And therefore I multiplie 7 by 4 (which is the denomination of quarters) and there amounteth 28 to be set for the Numerator, and 4 must be set for the denominator, and the fraction will be thus, $\frac{28}{4}$.

Scholer. Indeede I perceiue this to be reasonable, for without much trial I vnderstand that $\frac{28}{4}$ of any thing doth make 7. And so then if I would turne 8 into fiue partes, it will make $\frac{40}{5}$ which is all one with 8, for 8 crownes turned into fift partes (that is, into shillings) will make 40 shillings, that is $\frac{40}{5}$ of a crowne.

Master. Seing you vnderstand now these thrée kindes of Reduction, I will declare vnto you the fourth kinde, that is when fractions

be

bee written in greater termes than they neede, how they may bee brought to lesser termes.

Scholer. To write any thing in greater termes than needeth, seemeth to be a fault, and so this rule seemeth to amend that fault.

The forth kinde of Reduction

Maister. It were a fault to doe any thing without neede which after must be redressed: but in this case it is not so: neither did I say absolutely (as you doe) that it needeth not to expresse those Fractions in so great termes, but that the Fractions doe not neede, I meane for their value to be understanded: but yet it may be needfull for the ease of those workes, whereto they be applied, as for example: In the first kind of Reduction this was your owne example: $\frac{3}{16}$ and $\frac{2}{3}$, which when you would reduce, you were faine to turne them first into one denomination, and so appeared they thus, $\frac{18}{96}$, and $\frac{64}{96}$, where the fractions (for their owne understanding) needed not to be turned out of smaller termes into greater, but yet the easinesse of working needed it.

Scholer. Sir, I understand now, not onely the difference of this neede (for the Fractions might better be understanded as Fractions severall, each in his value, when they were in lesser termes, although they could not so well be reduced) but also I understand what you meane by greater termes and lesser termes, whereof before I was in doubt, for I see you

Termes of fractions.

call the numerator & denominator, the termes of the fraction.

Master. I am glad you understand it so wel. Now then when you would value any fractions (because that may best be done when the termes are smallest) you shall reduce them to the smallest that you can, which thing you may doe thus: Deuide the greatest of any such two termes by the lesser, and if any thing remaine, by that remainer diuide the last diuisor: and if any thing remaine now, by that diuide the last diuisor (which was before the remainer of the first Diuision) and so continue still, till nothing doe remaine in the Diuision: and then marke your last diuisor, for it is the number that will easily reduce your fraction, if you diuide both the numerator and the denominator by the same number, and put for the numerator the quotient of his diuision, and for the denominator also his quotient, that riseth by his Diuision.

Reduction of fractions into their smallest terms or least value.

Scholer. I take for example $\frac{18}{96}$, and because 96 is the greatest number, I diuide it by 18, and the quotient is 5, and there resteth 6: what shall I doe with this quotient?

Maister. Nothing in this worke, but now seeing there remaineth somewhat, by that remainer you must diuide the last diuisor.

Scholer. If I shall diuide 18 (which was the last diuisor) by 6, that was the remainer, so is the quotient 3, and nothing resteth.

Maister.

Master. As for the quotient I omit him yet: but because there doth remaine nothing, therefore is 6 (which was your last diuisor) that number by which you may reduce the fraction proponed.

Scholer. Then as you taught mee, I must diuide the numerator 18 by 6, and the quotient is 3, which I must put for the numerator ouer a line, thus. $\frac{3}{}$ And then by the sayd 6, must I diuide also the denominator 96, and the quotient will be 16, which I must take for the denominator, and so is the fraction $\frac{3}{16}$. And so me thinketh this rule doth proue the worke of the first Reduction.

Maister. That is true, if the first Reduction were made of fractions in their last terms, and else not, without some help, as the second number in that place will declare.

Scholer. The second number was $\frac{4}{6}$, which was turned into $\frac{64}{96}$ by that rule. Now if I shall by this rule reduce it againe into the least termes, I must diuide 96 by 64, and there remaineth nothing, wherefore I must take that 32 for the diuisor, to reduce the said fraction. Then doe I diuide 64 by 32, and the quotient is 2, which I set for my numerator. Againe, I diuide 96 by 32, and the quotient will be 3, and so haue I but $\frac{2}{3}$.

Maister. Muse not at the matter, for you haue done well enough but you thinke you

haue not the fraction that you looked for, that is $\frac{4}{6}$, yet haue you one equall to it, as by the parts of a ſhilling you may prooue.

Scholer. Trueth it is for each of them will bring forth 8 pence, ſo that $\frac{8}{12}$, and $\frac{4}{6}$ and $\frac{2}{3}$, be all three equall. And now I perceiue that becauſe $\frac{4}{6}$ was not written in the leaſt termes that it might bee, therefore this Reduction brought forth not it, but that other which is written in the leaſt termes. Now vnderſtand I this rule well. But is there any other way to worke this Reduction?

Another way for to worke Reduction.

Maiſter. Yes, but firſt note this, that if you finde no ſuch diuiſor to reduce the fraction till you come to 1, becauſe one doth make no diuiſion, therfore that fraction is already in his leaſt termes, as by $\frac{11}{100}$ you may prooue, and ſo of $\frac{81}{91}$ and many other like.

Note that to mediate any thing is to diuide the ſame by 2.

¶ But now for your better aide to finde the due proportion in leaſt termes, with more eaſe for a young learner, you ſhall mediate or take the halfe of the numerator, as alſo the denominator as long as you may vppon a line, alwaies parting them with a right downe daſh of your pen as your worke, which may eaſily be done, if the numbers bee euen: as 2, 4, 6, 8, or 10: But if they be odde (though it be but in one of them) then muſt you abbreuiate them by 3, 5, 7, or 9, &c.

And becauſe examples doe moſt inſtruct, I haue here ſette downe the manner of 2 or 3, whoſe

whose last number at the end of the line, sheweth the least terme or valuation of that fraction.

As for example, I would reduce $\frac{288}{576}$ into his least tearme or value: whereupon I set forth $\frac{288}{576}$, with a long line drawne frō it thus $\frac{288}{576}|$— And because both the Numerator and the Denominator ende in euen numbers, I see this may be abbreuiated by 2 or 4, or 6, &c. therefore on the other side of the right downe dashe toward the right hand, I first take the halfe of the numerator: saying the halfe of 2 is 1, the halfe of 8 is 4, and againe the halfe of 8 is 4: which 144 is nowe a new Numerator, and therefore I part it with a right downe dash as before.

Then do I also take the halfe of 576 in saying, the halfe of 5 is two, and the halfe 17 is 8, and the halfe of 16 is 8, and so haue I 288 for a new denominator.

Then beginning againe: saying the halfe of 144 is 72, and the halfe of 288 is 144: thus continuing the mediation or diuision by 2, vntill you come to this last worke as appeareth here in the example, where the same is reduced to $\frac{1}{2}$, $\frac{288}{576}$:

288	144	72	36	18	9	3	1
576	288	144	72	36	18	6	2

28	14	7	1
112	56	28	4.

Abbreuiated first by 5, and then by 293.

1465	293	1
4395	879	3

Scholer. Sir, I thanke you much, this is verie easie, and good for a young learner.*

Master. So it is, but yet notwithstanding, if you can without that diuision by memorie espie the greatest number that may diuide exactly both termes of your fraction proponed, then neede you not to vse that diuision, as in this fraction $\frac{60}{96}$ I see that 12 is the greatest number that can diuide them both: and therefore without any worke, by memorie onely, I turne that into $\frac{5}{8}$, but this abilitie in knowledge is gotten by exercise.

Yet one other way of easie Reduction in this kinde there is, when your fraction hath any Cyphers in the first places of both termes, then may you by casting away the Ciphers, make a briefe Reduction, as thus $\frac{300}{400}$, heere take away the Ciphers and it will be $\frac{3}{4}$, which is the same in value with $\frac{300}{400}$.

Scholer. And so if I haue $\frac{400}{850}$, it will be $\frac{4}{85}$

Master. You are deceiued, for you take away more Ciphers from the Numerator, than you do take from the denominator, which you may not do.

Scholer. I confesse my fault, which came of too much haste, I was more gladder of the

rule

rule than wise in vsing it: but nowe I vnderstand it I trust.

Master. Then may I goe in hand with the fifth or last kinde of Reduction, which teacheth how to turne any fraction proponed into anie other denomination that you list: or into any partes of common coynes, weightes, or measures, or such like. The fifth kind of Reductiō

For declaration whereof, first you shall marke whether your fraction be a simple fraction, eyther else a fraction of sundrie partes. I meane of more termes than 2. And if your fraction be a fraction of fractions, or otherwise compounde, you must reduce it to one simple fraction. And then marke well the Denomination of that other fraction, into which you would turne this, for by that denominator you must multiplie the numerator of your first fraction, and the totall product thereof shall you diuide by the denominator of your first fraction, and that quotient shall be the numerator to the denominator proponed: as for example, I haue this fraction $\frac{3}{5}$, which I would turne into ten partes, therefore I multiplie this 10 by 3, that is the Numerator of my fraction, and there riseth 30, which I diuide by 5, and the quotient is 6, which must be the Numerator to 10, and so $\frac{3}{5}$ will be $\frac{6}{10}$.

Scholer. This is easie enough to do.

Master. Then shall you see an other example of the same fraction that is not so easie:

as if I would turne $\frac{3}{5}$ into viii. partes, prove you that worke.

Scholer. I must multiplie 8 by 3, and there amounteth 24, which I diuide by 5, and the quotient is 4, then is the new fraction $\frac{4}{8}$.

Master. And see you nothing doubtfull in this worke?

Scholer, I see that when 24 was diuided by fiue, there remained 4, which I did not passe of, because ye speake nothing of any remainer, but onely of the quotient.

Maister. By likelyhoode you remember what I sayd to you in Diuision of whole numbers, that you should not passe of the remainer there, but onely note it as a summe that could not be diuided without knowledge of fractions. Wherefore nowe marke this, that in all Diuisions of whole numbers, when there is any remainer, you shall set it ouer a line as a numerator, and set the Diuisor for the Denominator, and that fraction doth make the Diuision complete, and is part of the quotient: as if I would diuide 48 by 5, the quotient will be $9\frac{3}{5}$; so in your former worke when 24 was diuided by fiue, the quotient should be $4\frac{4}{5}$, and so the new fraction should be thus: $\frac{4}{8}$ and $\frac{4}{5}$ of 8 that is $\frac{4}{8}$ of the entire number, and $\frac{4}{5}$ of $\frac{1}{8}$ part of any thing, which you may proue by example of some coyne.

Scholer. Then take a crowne, whose $\frac{3}{5}$ is 3 s. Now if I would proue whether that 3 s be $\frac{4}{8}$ and

$\frac{4}{8}$ and $\frac{4}{5}$ of $\frac{1}{8}$, I shall haue a combrous worke to do.

Maister. In deed for whole pence, your example is a little troublesome: yet turning the crowne into halfe pence, it is easie enough.

Scholer. That will I try.

¶ First I see that $\frac{3}{5}$ of a crowne is 3 s: which is 36 d, or 72 halfe pence. Now if I can finde that this Fraction $\frac{4}{8}$ and $\frac{4}{5}$ of $\frac{1}{8}$ be equall vnto 3 s: then am I fully aunswered.

Because I cannot take $\frac{4}{8}$ of a crown, I turne the crowne into halfe pence, as you willed me, which makes 120, which I diuide by 8, my quotient is 15, which taken 4 times, makes 60 ob. Now resteth me to haue $\frac{4}{5}$ of the $\frac{1}{8}$ part of a crowne, whereof $\frac{1}{8}$ part is 15 ob. that 15 being parted into 5 parts, the quotient is 3, which taken 4 times maketh 12 ob. which with my 60 before amounteth to 72, which are then equall to $\frac{3}{5}$ my desire.

Maister. I commend you for your diligence, you might haue wrought it thus either: $\frac{4}{8}$ being abbreuied as before I taught is $\frac{1}{2}$. Now the halfe of a Crowne is 2 s, 6 d. Then $\frac{4}{5}$ of $\frac{1}{8}$ is a fraction of fractions, which if you doe reduce into one entire Fraction as before you haue learned, in saying 5 times 8 is 40, for a new denominator and once 4 is 4, for a new numerator, it maketh $\frac{4}{40}$, and abbreuiated also make but $\frac{1}{10}$. Now the tenth part of a Crowne is 6 d, which put to 2 s, 6 d make also 3 s your desire. *

But now one example more for this rule, & then shal we end it. If I haue $\frac{7}{15}$ of a soueraigne (accounting the Soueraigne 20 shillings) how many shillings is that $\frac{7}{15}$?

Scholer. I must multiply 7 by 20, and that maketh 140, which I shall deuide by 15, and the quotient will be $9\frac{5}{15}$: or else in lesser termes, $\frac{1}{3}$.

Maister. That is 9 s, and one third part of a shilling, that is 4 d, as by the same rule you may prooue. And this for this time shall suffice for Reduction, saue that I must now repeate a little touching Multiplication and Diuision, and so go forward.

I Multiplication

j Multiplication

Multiplication.

In Multiplication it happeneth sometime that there bee whole numbers to bee multiplied with fractions: And may be in two sortes, for either the whole number is severall frō the fraction, and is the multiplier, or else, the whole nūber is ioyned with one, or both of the Fractions, and so maketh a mixt number therof. If it be in the first sort, then néedeth there no Reduction, but onely multiply the numerator of the fraction by that whole number, and the totall thereof set for the new numerator.

Reduction of whole numbers into fractions.

Scholer.

Scholer. I vnderstand you thus. If I haue $\frac{4}{23}$ to be multiplied by 16, then must I multiply that 16 with 6, which is the Numerator, whereof commeth 96, and that must I set for the new Numerator, keeping still 23 for the Denominator, and so the fraction will be $\frac{96}{23}$ that is $4\frac{4}{23}$.

Maister. And in this sort of worke you may abridge the labour, thus. If it happen the Denominator to be such a number as may euenly be diuided by the sayd whole number proposed, then diuide it thereby, and sette the quotient of that diuision for the former denominator: but reserue still the Numerator, and so is the Multiplication ended.

Scholer. Then I faine this example, $\frac{7}{20}$ to be multiplied by 5. And because 5 will iustly diuide 20, therefore I take the quotient of that diuision, which is 4, and set in stead of 20, and so the Fraction will be $\frac{7}{4}$, that is $1\frac{3}{4}$.

Maister. Which is all one with $\frac{35}{20}$, that would haue followed of the other sort of worke.

Scholer. I perceiue it very well.

Master. Now then for the other sort where the number is mixt, take this way: first to reduce the sayd whole number, and fraction into one fraction improper (as I shewed you in Reduction) and then multiply them together, as if they were proper fractions. How to multiply mixt nũbers.

Scholer. $13\frac{1}{3}$ being sette to bee multiplied by $\frac{1}{5}$, first I must reduce the mixt number, as

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$13\frac{3}{5}$ appeareth in the margent, by multiplying 13 by 5, and that maketh 65, whereto I must adde

$\frac{68}{5}$ the Numerator 3, and so the fraction will be $\frac{68}{5}$, which two fractions now I shall multiply after the accustomed forme, and it will be $\frac{340}{40}$.

Master. You haue done well: and so may you see, that although most part of the formes of Multiplication may be wrought without Reduction, yet some cannot, as namely mixed numbers.

And yet one note more will I tell you of Multiplication, before we leaue it: That is, whensoeuer you would multiply any Fraction by 2, which commonly is called Duplation, you may doe it not onely by doubling the Numerator, but also by parting the Denominator into halfe, if it be euen.

Scholer. Then if I would double $\frac{5}{12}$, I may choose whether I will make it, $\frac{10}{12}$, or else $\frac{5}{6}$. And in deede I see that all is one, but that the diuiding of the Denominator seemeth the better way to make smaller termes of the Fraction, and so they shall neede the lesser reduction.

Master. It is so: and now I shall not neede to tell you that Multiplication is prooued by Diuision, and Diuision likewise by Multiplication, but the like workes that I shewed you in Multiplication will I shew you in Diuision also.

Diuision.

When any whole number shall be diuided by a fraction, you must multiply the sayd whole number with the Denominator of the fraction, and set the totall thereof for the new Numerator, and for the Denominator set the Numerator of the fraction.

Diuision to diuide a whole nũber by a Fraction.

Scholer. Then 20 diuided by $\frac{3}{4}$ will make $\frac{80}{3}$, as here appeareth:

$\frac{20}{1}$ by $\frac{3}{4}$
$\frac{80}{3}$

Maister. Euen so. But if you would diuide the Fraction by the whole number, then multiply the Denominator by the same whole number, and sette the totall for the Denominator, without changing the Numerator.

To diuide a Fraction by a whole number.

Scholer. Then to diuide $\frac{20}{23}$ by 4, it will be $\frac{20}{92}$. As here appeareth in this example.

$\frac{20}{23}$ by $\frac{4}{1}$
$\frac{20}{92}$.

Maister. You say well. And by the same example you giue me occasion to remember an other briefe way to doe the same: for if you had diuided the sayd Numerator by 4, & set the quotient for the Numerator, keeping still the old denominator, it would haue beene not onely as well done, but also in a fraction of lesser terms.

Another brief way.

Scholer. I gesse it to be euen so, by a like worke that you taught me in Multiplication. And for proofe thereof $\frac{20}{23}$ being the diuidend, & 4 the Diuisor, I diuide the Numerator 20

by 4, and the quotient is 5, which I set for 20 ouer 23, thus: $\frac{5}{23}$. And I see that it is all one with $\frac{20}{92}$, as by diuiding or abreuying both these terms by 4, and so reducing them to their least Denomination, I may easily prooue: as appeareth in this example: $\frac{20}{92} \big| \frac{5}{23}$

Maister. You conceiue it well. And if there be mixt numbers (either one or both) you must first reduce that mixt number into an improper fraction. And then worke as you haue learned.

Scholer. That was sufficiently taught in Multiplication. Therefore I pray you go forward to some other thing.

Maister. Then take this note yet for diuision. If the Denominators be like, then diuide the Numerators as if it were in whole numbers, and the quotient, whether it be fraction, whole number, or mixt, is a good quotient for that Diuision. And generally if one of the Numerators may iustly diuide the other, by that quotient, multiply the Denominator of the lesser Numerator, and set it that doth amount in the roome of the same Denominator, and then for a Numerator to it, set the Denominator of the other fraction.

Scholer. Then if I would diuide $\frac{3}{4}$ by $\frac{12}{17}$ I see that 3 will diuide 12, and the quotient will be 4, by which I must multiply the other 4 that is the Denominator vnder 3, and then it is 16, which I set for the Denominator 4, and

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ouer it in ſtead of 3, I muſt ſet 17, the other Denominator, and ſo is it thus. $\frac{17}{18}$.

Maſter. And ſo is $\frac{17}{16}$ in ſtead of $\frac{51}{48}$, which would haue riſen by the common $\frac{3}{4}$ by $\frac{12}{17}$ worke: as here appeareth: $\frac{51}{48}$

And now for Mediation (which is to diuide by 2) marke this: If the numerator be euen, ſet the halfe of it in his place without the Diuiſor, and ſo haue you done: and if the Numerator be not euen, then double the Denominator.

Scholer. That is if I would mediate $\frac{6}{11}$, I may make the quotient $\frac{3}{11}$. And if I would mediate $\frac{7}{11}$, I muſt make it $\frac{7}{22}$.

Maſter. Now truſt I that you haue ſufficient knowledge in Reduction, Multiplication, and Diuiſion: and therefore will I go in hand with Addition and Subtraction, which now will appeare eaſie enough.

Addition.

Whenſoeuer you haue any fractions to be added, you muſt conſider whether they bee of one Denomination or not. And if they be of one Denomination then adde the numerators together, and ſet that that amounteth, for the Numerator ouer the common Denominator, and ſo haue you done. The reaſon is, becauſe that ſuch do differ litle in Addition or

To adde Fractions of one denomination.

Subtraction from the worke of vulgar deno-mination, where the Denominators bee of no numbers: as 3 d and 5 d make 8 d, where the de-nomination is not altered. But and if the frac-tions be not of one denomination, or any of the be mixt of whole numbers and fractions, then must you first reduce them to one denominati-on, and after adde them. And if they bee many, then adde first two of them, & to the summe that doth amount of the Addition, adde the third, and then the fourth, & so forth, if you haue so many.

To adde fractions of diuerse denominations.

Scholer. This seemeth easie inough, now that I haue already learned to multiply and to re-duce, without which two, I could neuer haue wrought this. And therefore now I see good reason, why you did place Multiplication and Reduction before Addition.

M. It is wel considered, but yet refuse not to expresse your understanding of it, by an exãple.

Scholer. Then would I adde first $\frac{7}{18}$ with $\frac{5}{18}$, and because the Denominators are like (and so needeth no Reduction) I adde 7 to 5 which ma-keth 12, and then is my summe $\frac{12}{18}$, that is, in smaller numbers being abreuiated $\frac{2}{3}$.

And if I haue many nũbers to be added, as here $\frac{3}{8}$ $\frac{4}{5}$ $\frac{9}{10}$. (first I must reduce them, because they haue diuerse Denominators) into one denomi-nation, and then will they be thus: $\frac{150}{400}$ $\frac{320}{400}$ $\frac{360}{400}$ or in lesse termes $\frac{15}{40}$ $\frac{32}{40}$ $\frac{36}{40}$, which by Addition doe make $\frac{83}{40}$, that is $2\frac{3}{40}$.

Maister. Now may we go to Subtraction.

Sub-

Subtraction.

Subtraction hath the same precepts that Addition had, for if the denominators be like, then must you subtract the one Numerator from the other, and the rest is to be set ouer the common denominator, and so your Subtraction is ended: but and if you haue many fractions to be subtracted out of many, then must you reduce them to one denomination, and into two severall fractions, that is, all that must be subtracted into one fraction, and the residue into an other Fraction, and then worke as I sayd before.

Scholer. For the first example I take $\frac{15}{12}$ to be subtracted out of $\frac{17}{12}$, and the rest will be $\frac{2}{12}$ or $\frac{1}{6}$.

For an other example I take $\frac{3}{4}$ to be subtracted out of $\frac{7}{8}$, which I must reduce, and it will be thus, $\frac{24}{32}$ and $\frac{28}{32}$.

Then doe I subtract 24 out of 28, and there resteth 4, which I set ouer the common denominator for a Remainer, thus, $\frac{4}{32}$, that is $\frac{1}{8}$.

Now for the third example, I take $\frac{3}{4}$ and $\frac{5}{6}$ to be subtracted from $\frac{7}{8}$ & $\frac{9}{10}$. And because their denominators be diuerse, I doe reduce them into one denomination thus: $\frac{1440}{1920}$, $\frac{1600}{1920}$, $\frac{1680}{1920}$, $\frac{1728}{1920}$.

Then do I adde the two first, and they make $\frac{3040}{1920}$. Also I adde the two last, and they yeeld $\frac{3408}{1920}$. Then do I subtract 3040 out of 3408, and there resteth 368, so is the remainer $\frac{368}{1920}$, that is in smaller termes, $\frac{23}{120}$. And thus haue I done with Subtraction, except you haue anie more to teach me.

Master. Proue one example more of two Fractions of diuerse denominations.

Scholer. I take these two Fractions $\frac{7}{8}$ to be subtracted from $\frac{9}{24}$, which being reduced, will stand thus, $\frac{168}{192}$ and $\frac{72}{192}$. Nowe would I subtract 168 out of 72, but I cannot.

Master. Then may you perceiue that you mistooke the Fractions: for you can neuer subtract the greater out of the lesser, although you may adde, multiplie or deuide the greater with the lesser. And albeit that $\frac{7}{8}$ hath both his tearmes lesser than $\frac{9}{24}$, yet is $\frac{9}{24}$ the lesser Fraction: for generally if you multiplie the Numerators and denominators of two fractions crossewayes, the fraction is the greatest, of whose Numerator commeth the greatest summe, as in this example; 7 multiplied by 24, maketh 168: and 9 being multiplied by 8, yeeldeth but 72, therefore is the first Fraction $\frac{7}{8}$ the greatest of these two, so can you not subtract it out of a lesser Fraction.

The greatest of two Fractions.

But if you should subtract a Fraction out of a whole number what would you do?

Scholer. Marrie I would reduce the whole number

number into a Fraction of the same Denomination that my Fraction is, and then worke by Subtraction.

Master. So may you doe, but it is easier much, if your fraction be a proper fraction, that is to say, lesse then an Unite, to take an Unite from the whole number, and then turne it into an Improper Fraction, and so worke your subtraction. As if I would subtract $\frac{3}{5}$ from 4, I may take one from 4, and turne it into $\frac{5}{5}$, from which if I abate $\frac{3}{5}$ there will remaine $3\frac{2}{5}$. And if the first Fraction be an improper Fraction, then may I take so many Unites from the whole number, that they may make an improper Fraction greater than that first, and then worke by Subtraction: As if there bee proponed $\frac{10}{3}$ to be subtracted from 6, because $\frac{10}{3}$ is more than 3, and not so much as 4. I must take foure from 6, and turne them into thirds thus, $\frac{12}{3}$ then abate $\frac{10}{3}$, and there resteth $\frac{2}{3}$, so the whole remainer is $2\frac{2}{3}$. or else you may at your pleasure take $3\frac{1}{3}$ which is $\frac{10}{3}$ from 6 whole: Then set one under 6 as thus $\frac{6}{1}$: And then to reduce those two fractions into one Denomination as here appeareth: $\frac{10}{3}$ from $\frac{6}{1}$.

Then $\frac{10}{3}$ from $\frac{18}{3}$ resteth $\frac{8}{3}$: which maketh $2\frac{2}{3}$ your desire

$$\begin{array}{ccc} & 8 & \\ \frac{10}{3} & \times & \frac{18}{3} \\ & 3 & \end{array}$$

And thus will I make an end of the workes of common Fractions for this time, not doubting, but you can applie them both unto the

rule of Progreſſion, and alſo vnto the Golden rule, without any other teaching then you haue learned before, which might ſeeme tedious to repeat, ſaue that in ſome ſpeciall diuerſities, which be peculiar to fractions. I cannot ouerpaſſe, but inſtruct you ſomewhat by the way.

The Golden Rule.

Therefore as touching the Golden Rule for the placing of the three numbers propoſed in the queſtion, whereby to finde the third, and for the forme of their worke, with other like notes, I referre you to that which you haue already learned.

But this eaſie forme of working by Fractions ſhall you note, that if your thrée numbers be Fractions, for an apt worke and certaine, multiply the Numerator of the firſt number in the queſtion, by the Denominator of the ſecond: And all that againe multiply by the Denominator of the third number, and the totall thereof ſhall you kéepe for to be the Diuiſor. Then multiply the Denominator of the firſt number by the Numerator of the ſecond, and the whole thereof by the Numerator of the third, and the totall thereof ſhall bee your diuidend. Now diuide this diuidend by the Diuiſor which you found out before, and that number

ſhall

ſhall bee the fourth number of the queſtion which you ſeeke for: As in this example. If $\frac{3}{4}$ of a yard of veluet coſt $\frac{2}{3}$ of a Soueraigne (eſteemed at 20 ſhillings,) what ſhall $\frac{5}{6}$ coſt? A queſtion of Veluet.

Scholer. If it pleaſe you to let me make the anſwere, I would firſt place theſe three numbers, as I learned in whole numbers thus:

$\frac{3}{4}$ —— $\frac{2}{3}$
$\frac{5}{6}$

And then according to your new rule, I muſt multiply three being Numerator in the firſt number, by 3 the Denominator of the ſecond, and thereof commeth 9, which I multiply againe by 6, the Denominator of the third number, and ſo haue I 54, which I keepe for the diuiſor, then multiply I 4, the denominator of the firſt, by 2, the Numerator of the ſecond, and there riſeth 8, which againe I multiply by 5 the Numerator of the third, and it maketh 40: then muſt I diuide 40, by 54, and it will be $\frac{40}{54}$ that is $\frac{20}{27}$ in leſſer termes, and then the figure will ſtand thus.

$\frac{3}{4}$ —— $\frac{2}{3}$
$\frac{5}{6}$ —— $\frac{20}{27}$

But what that is in money, I can not tell, except I ſhall worke it by Reduction, as you taught me.

Maiſter. It forceth not now, you may reduce it when you liſt, but it were diſorderly done here to mingle diuerſe workes together, where we do not ſeeke the valne of the thing in common money, but in an apt nũber, which you haue well done. And therefore will I yet ſhew

you an other like way of easinesse in worke, how you may change your 3 Fractions into 3 whole numbers, by which you shall worke as if the question were proponed in whole numbers. The first number you shall finde as I taught you: now to finde the diuisor of the second number, take the Numerator for the second fraction: and for the third number take that that riseth of Multiplication of the Denominator of the first, by the Numerator of the third, and then worke your question.

A question of filuer.

Scholer. For example hereof, I put this question. If $\frac{11}{12}$ of 1 ℔ pound weight of Siluer, be worth $\frac{12}{4}$ of a Soueraigne, what is $\frac{1}{2}$ of 1 pound weigth worth? For the answere, first I place the fractions in order thus.

$\frac{11}{12}$ —— $\frac{12}{4}$
$\frac{1}{2}$ ——

Then to turne these Fractions into whole numbers, I multiply 11 which is the Numerator of the first, by 4 (the Denominator of the second) and there commeth 44, which I multiply by 2 the Denominator of the third, and so amounteth 88, which I set for the Diuisor, in the first place. Then in the second place I sette 12, which is the Numerator in the second Fraction, and in the third place I sette the summe that amounteth of 12, being the Denominator in the first number, multiplied by 1, being Numerator in the third number, and so the figure will stand as here you see.

88 —— 12
12 ——

Then

Then to worke it foorth, I multiply 12 by 12, and there amounteth 144, which I diuide by 88, and the quotient will be $1\frac{56}{88}$, or in lesser termes, $1\frac{7}{11}$, and then the figure will stand thus.

$$\begin{array}{ccc} \frac{11}{12} & & \frac{12}{4} \\ \frac{1}{3} & & 1\frac{7}{11} \end{array}$$

Maister. These two formes now you understand well inough. And as for any other, at this time I will not repeate, onely this shall you marke for the proofe of this rule, whether your worke be well wrought or no. Multiply the first number by the fourth, and note what amounteth: then multiply the second by the third, and marke what amounteth also. Now if those two numbers so amounting bee equall, then is your worke well done, else you haue erred. And this shall suffice for the former rule: but in the backer rule, this shall you note for ease of worke, that you multiply the Numerator of the first by the Numerator of the second, and the whole thereof by the Denominator of the third, and that amounteth thereof, shall bee the Diuidend. Then multiply the Denominator of the first by the Denominator of the second, and that whole by the Numerator of the third, and that riseth thereof shall be the Diuisor. Example of this: I did lend my friend $\frac{3}{4}$ of a Porteguise seuen monethes, vppon promise that he should doe as much for me againe: and when I should borrow of him, he could lend me but $\frac{5}{7}$ of a Portеguise, now I demaund how long time must

The proofe of the golden rule.

The Backer rule.

A questiō of loane.

I

I kéepe his money, in iust recompence of my loane, accompting 13 moneths in the yeare?

Scholer. The first number must be the first money borrowed, that is $\frac{3}{4}$ of the Portegnise: the second number the 7 moneths, that is $\frac{7}{13}$ of a yeare: and the third number the money that was lent in recompence, that is $\frac{5}{12}$ of a Portegnise: then I set the number thus:

$$\frac{3}{4} \quad \frac{7}{13}$$
$$\frac{5}{12}$$

Then (as you taught mee) I multiply thrée (being Numerator in the first number) by 7 the Numerator of the second number, and it maketh 21, which I multiply by 12, the Denominator of the third, and so haue I 252 for the diuidend: then I multiply 4 the denominator of the first, by 13 the Denominator of the second, and it yéeldeth 52 which I multiply againe by 5, the Numerator of the third, and it will make 260, that is the diuisor. Then must I diuide 252, by 260, so it will be in the smallest fraction, $\frac{63}{65}$ of a yeare.

Statute of Assise of bread and Ale.

Maister. And thus doe you see some ease in working, better than to multiply and diuide tediously so many Fractions. Another question yet will I propose, to the intent you may sée thereby the reason of the Statute of Assise of Bread and Ale, which in all Statute bookes in French, Latine and English, is much corrupted for want of knowledge in this Art: for the right understanding whereof I propone this question.

When

When the pꝛice of a quarter of Wheate is 2 s̄, the farthing white loafe ſhal weigh 68 s̄, then I demaund, what ſhall ſuch a loafe weigh, when a quarter of Wheat is ſold foꝛ 3 s̄? 2 Queſtion.

Scholer. The Queſtion muſt bee wꝛought as it is pꝛoponed in whole numbers, and not in Fractions.

Maiſter. You ſéeme to ſay reaſonably, howbe it, in that ſtatute of Aſſiſe, the rate is made by the pꝛopoꝛtion of partes in a pound weight Troy, elſe could it not be a Statute of any long continuance, ſéeing the ſhillings doe change often, as all other moneys doe: but this Statute being well underſtanded, is a continuall rule foꝛ euer, as I will anon declare by a new Table of Aſſiſe, conuerting the ſhillings into ounces and parts of ounces,

Therefoꝛe here by a ſhilling you muſt underſtand $\frac{1}{20}$ of a pound weight, and ſo by pence $\frac{1}{20}$ of an ounce, wherefoꝛe although yee might woꝛke this queſtion pꝛoponed by whole numbers well inough, foꝛ that time when the Statute was made, yet to apply it to our time, and to make it ſerue foꝛ all times generally: it is beſt to woꝛke it by fractions, ſetting foꝛ a ſhilling $\frac{1}{20}$: and foꝛ 68 ſhillings, $\frac{68}{20}$, and ſo foꝛ thꝛee ſhillings $\frac{3}{20}$, and then will the figure of the queſtion ſtand thus.

$$\frac{1}{20} \qquad \frac{68}{20}$$
$$\frac{3}{20}$$

In which queſtion becauſe all the denominatoꝛs be like, you ſhal woꝛke only with the Numeratoꝛs.

Scholer. Then I shall multiply 68 by 2, whereof commeth 136, which if I diuide by 3, the quotient will bee $45\frac{1}{3}$: but how shall I make a Fraction of that to stand with the other?

Maister. Haue you so soone forgotten what was taught you so lately? This is his forme.

Scholer. I remember it now and then it signifieth 45 twenty parts, & the third deale of one twenty part.

$$\frac{45\frac{1}{3}}{20}$$

Maister. So is it, and that maketh in shillings 45 s 4 d: whereby you may note one greater errour in the Statute bookes, which haue constantly 48 s in that Assise: And by this rule, if you examine the Statute you shall find many summes false, wherefore for the true understanding of that Statute and such like as I haue made mention of it, and somewhat recognised it. so doe I wish that all Gentlemen and other students of the Lawes, would not neglect this Art of Arithmeticke, as unneedfull to their studies. Wherefore, to encourage them thereto, and to gratifie both them and all other in generall, I will exhibite a Table of that part of the Statute in two columes, and in a third colume I will adde the correction of those errors which haue crept into it.

Here followeth the Table.

The

Edward Jones

his book

this 11 of Julij 1669

Edward Johnson

The price of a quarter of wheate.		The weight of a farthing white loafe by the statute bookes.			The correction by iust Assise.		
s	d	l	s	d	l	s	d
1	0	6	16	0	6	16.	0
1	6	4	10	8	4	10	8
2	0	3	8	0	3	8	0
2	6	2	14	4 $\frac{1}{7}$	2	14	4 $\frac{2}{5}$
3	0	2	8	0	2	5	4
3	6	2	2	0	1	18	10 $\frac{2}{7}$
4	0	1	16	0	1	14	0
4	6	1	10	0	1	10	2 $\frac{2}{3}$
5	0	1	8	2 $\frac{1}{3}$	1	7	2 $\frac{2}{5}$
5	6	1	4	8 $\frac{1}{4}$	1	4	8 $\frac{8}{11}$
6	0	1	2	8	1	2	8
6	6	0	19	11	1	0	11 $\frac{1}{13}$
7	0	0	19	1	0	19	5 $\frac{1}{7}$
7	6	0	18	1 $\frac{1}{2}$	0	18	1 $\frac{5}{7}$
8	0	0	17	0	0	17	0
8	6	0	16	0	0	16	0
9	0	0	15	0 $\frac{1}{4}$	0	15	1 $\frac{1}{3}$
9	6	0	14	4 $\frac{1}{2}$	0	14	3 $\frac{15}{19}$
10	0	0	13	7 $\frac{1}{2}$	0	13	7 $\frac{1}{5}$
10	6	0	12	11 $\frac{1}{4}$	0	12	11 $\frac{1}{7}$
11	0	0	12	4 $\frac{1}{4}$	0	12	4 $\frac{4}{11}$
11	6	0	11	10	0	11	9 $\frac{21}{23}$
12	0	0	11	4	0	11	4

In the cõmon bookes there is no further rate of Assise made, thã vnto 12 s̃ ẏ quarter of wheat: but in an ancient copie of 200 yeares old (which I haue) there is added the rate of Assise vnto 20 s̃ the quarter, but yet was that assise also either wrõg cast at the first penning, or els corrupt sith that time, for lack of iust knowledge in the rule of proportiõ, which I wil ad here also, to gratifie such as be desirous to vnderstand truth exactly.

The price of a quarter of wheate.		The weight of a farthing white loafe by the statute bookes.		The correction of the errours.	
s̃	d̃	s̃	d̃	s̃	d̃
12	6	11	0	10	10 $\frac{14}{25}$
13	0	15	0 $\frac{1}{3}$	10	5 $\frac{7}{26}$
13	6	10	1 $\frac{1}{2}$	10	0 $\frac{8}{9}$
14	0	9	7	9	8 $\frac{4}{7}$
14	6	9	2 $\frac{1}{3}$	9	4 $\frac{16}{29}$
15	0	9	1 $\frac{1}{3}$	9	0 $\frac{4}{5}$
15	6	9	1 $\frac{1}{4}$	8	9 $\frac{9}{31}$
16	0	9	0	8	6 0
16	6	8	6	8	2 $\frac{10}{33}$
17	0	8	3	8	0 0
17	6	7	10	7	10 $\frac{3}{7}$
18	0	7	6	7	6 $\frac{1}{3}$
18	6	7	3	7	4 $\frac{8}{37}$
19	0	7	2	7	1 $\frac{17}{19}$
19	6	5	10	6	11 $\frac{2}{13}$
20	0	5	6	6	9

These two tables I haue set seuerall, because no man should think that I would either adde or take away from any law, those partes which might of right seeme either superfluous, eyther diminute, but yet I may not be so curious as to neglect manifest errours, which is not onely my part, but euerie good subiects dutie with sobrietie to correct. And for auoyding of offence I haue rather done it in this priuate booke, rather than in any booke of the Statutes selfe, trusting that all men will take it in good part.

Scholer. I would wish so. but I dare not hope so sith neuer good mā that would reforme errour, could escape the venemous tongues of enuious detractors, which because they either cannot or list not to do any good themselues do delight to barke at the doinges of other, but I beseech you to stay nothing for their peruerse behauiour.

Master. I consider many things that some may obiect, whereunto I am not vnprouided of iust answeres, but I will not seeme so hastie to make the answeres before I heare their obiections, but as I trust that men are of a better nature, and more gratefull now than some haue beene in time passed, as I haue done in the Statute of Assise for bread in rate of s̄, so wil I set forth the like table in pounds and ounces and the parts thereof, that it may be easily applied to all times: but I meane not

by this to alter any word of the statute (being so good an ordinaunce, and of so great continuance) but onely to make it as a kind of exposition and declaration of the said Statute, trusting that thereby the Statute may be better understood, and consequently better put in execution. And here you shall note, that I haue accounted the shillings after the rate of lx. s, to the pound waight, because I esteeme it the most apt rate for our time. Wherefore if in the first columne you find the price of wheate directly against it in the second columne, you may find the waight of the farthing white loafe, in this our time: and if you double the number (as I haue done in the third columne) then haue you the waight of the halfe penie white loafe, and so in the fourth columne is set the waight of the penie white loafe. It needeth not to tell you that, that the sight doth testifie, how that euery columne is parted into 3 smaller pillers, whereof the first columne hath these 3 titles, pounds, shillings, and pence: the other 3 columnes haue ech of them these 3 titles, pounds, ounces, and penie waights. And as in the first columne xii d, maketh a s, and 20 s make a pound, so in the other 3 columnes 20 pence waight maketh an ounce, and 12 ounces do make a pound.

A pound waight.

Scholer.

Scholer. Sir, I do thanke you most heartily for this, not onely in mine owne name and in the name of all Students, but also in the name of the whole Commons, to whom the restitution of this Assise (I trust) shall bring restitution of the weight in bread which long time hath beene abused. And if you know any like thinges more, wherein you would vouchsafe to declare the errours, and set foorth the truth, you cannot but obtaine great thanks of all good hearted men that loue the common wealth.

Master. I haue sundrie things to declare, but I haue reserued them for a priuate booke by it selfe, yet notwithstanding because the Statute of the rate of measuring of ground is so common that it toucheth all men, and yet no more common then needfull, but so much corrupt, that it is too farre out of all good rate, not onely in the English bookes of Statutes commonly printed, but also in the Latine bookes, and in the French also, for I haue read of each sorte, and conferred them diligently, I will giue you a Table for the restitution of those errours, as may suffice for this present time. And first I will propose one question to you touching the vse of that Statute, whereby you may perceiue the order how to examine the whole statute, and euery parcell thereof, & the question is this.

When the Acre of ground doth containe

A question of measure of ground.

foure perches in breadth, then must it containe 40 perches in length: then doe I demaund of you, how much shall the length of an Acre be when there is in the breadth of it 13 perches? but before you shall answere to this question, I will declare unto you another Statute, which is the ground of the former Statute. And that Statute is this. It is ordained that 3 Barlie cornes, drie and round, shall make up the measure of an inch: 12 inches shall make a foote, and 3 foote shall make a yard (the common English bookes haue an nine) fiue yards and a halfe shall make a perch, and 40 perches in length, and 4 in breadth, shall make an Acre. This is that Statute: whereby you may perceiue, that the intent of the Statute is, that one Acre should containe 160 square perches. Now let me heare your answere to the question.

A statute of measure.

An Acre.

Scholer. As I perceiue by the wordes of that Statute, a perch to be $\frac{1}{160}$ of an Acre, so could I make those numbers all in fractions, and so worke the question: but seeing I may do it also in whole numbers I take that forme for the most easie, therefore thus I set the question in forme. Then do I multiply 40 by 4, and it maketh 160, which I diuide by 13, and the quotient is 12 $\frac{4}{13}$.

4 — 40
13 — 1

Maister. Now turne that $\frac{4}{13}$ into the common partes of a perch, as they bee named in the

the former statute: howbeit, it shall be best to take one of the least parts in Denomination for auoyding of much labour, as feete, whereof the perch containeth $16\frac{1}{2}$.

Scholer. Then to turne $\frac{1}{13}$ into feete, I multiplie $16\frac{1}{2}$ by 4, and it maketh 66, which I must diuide by 13, and the quotient is $5\frac{1}{13}$.

Maister. So I find that if the acre hold in breadth xiii. perches, it shall containe in length 12 perches, 5 foote and $\frac{1}{13}$ of a foote, which is not fully an ynch, for the ynch is $\frac{1}{12}$ of a foote. But here all the Statute bookes in Latine and English (that I haue seene) do note it to be 13 perches, 5 foote and 1 ynch: which maketh aboue 13 perches too many in the acre, so that I would haue thought the error to haue crept into the printed bookes by the great negligence that Printers in our time doe vse, saue that in written Copies of great antiquitie, I doe finde the same. Yet haue I one French Copie, which hath 12 perches $\frac{1}{2}$, and one foote, and that misseth verie little of the truth. Note this errour.

Scholer. Then I see it is true that I haue often heard say, that the truest copies of the statutes be the French copies.

Maister. That is often true, but not generally, as I haue by conference tried diuersly: but in this statute the French booke is most corrupt in all other places lightly.

But now to performe my promise, I will

set forth the Table for measuring of an Acre of ground onely by such parts as the statute doth mention, because at this time I doe of purpose write it for the better understanding of the statute, and hereafter with other things I intend to set forth this same more at large.

In this Table following, I haue not done as in the other statute before compared by restitution with the faultes crept into the statute, but onely haue written that true measure: which the equity of the statute doth pretend. For it were vile to iudge of so noble Princes and worthy Counsellers, as haue authorised and set forth this statute, that they would make an Acre in any forme greater then an other, but euery one to be iust and equall with each other, which is the ground also of my worke: and hereby may all men perceiue how needefull Arithmeticke is to the students of the Law. But now I thinke best to make an end of these matters for this present time, sith the Table hath in it no obscurity, that I should neede to declare.

The

The breadth.	The length of the acre.			
Perches.	Perches.	Feete.	Ynches.	Parts of an ynch.
10	16	0	0	0
11	14	9	0	0
12	13	5	6	0
13	12	5	0	12/13
14	11	7	0	6/7
15	10	11	0	0
16	10	0	0	0
17	9	6	9	6/17
18	8	14	8	0
19	8	6	11	7/19
20	8	0	0	0
21	7	10	2	4/7
22	7	4	6	0
23	6	15	9	[illegible]
24	6	11	0	0
25	6	6	7	1/5
26	6	2	6	6/13
27	5	15	3	1/3

The breadth.	The length of the acre.			
Perches.	Perches.	Feete.	Ynches.	Parts of an ynch.
28	5	11	9	$\frac{3}{7}$
29	5	8	6	$\frac{12}{29}$
30	5	5	6	0
31	5	2	7	$\frac{29}{31}$
32	5	0	0	0
33	4	14	0	0
34	4	11	7	$\frac{13}{17}$
35	4	9	5	$\frac{1}{7}$
36	4	7	4	0
37	4	5	4	$\frac{8}{37}$
38	4	3	5	$\frac{13}{19}$
39	4	1	8	$\frac{4}{13}$
40	4	0	0	0
41	3	14	10	$\frac{18}{41}$
42	3	13	4	$\frac{2}{7}$
43	3	11	10	$\frac{10}{43}$
44	3	10	6	0
45	3	9	2	0

Scholer. In deede Sir, I understand the Table (as I thinke) by those other which you set forth before. For in the first Columne is set the perches of the breadth of any Acre, and then in the 2 Columnes following appeareth how many perches & how many foote the same Acre must haue for his length.

Master. You take it well: howbeit to speake exactly of breadth and length, the first Columne doth sometime betoken the breadth, and sometime the length: for properly the longest side of any square doth limite his length, and the shorter side doth betoken the breadth, yet it is no greater abuse in such tables, where a man cannot well change the title, to let the name remaine although the proportions of the numbers doe change: for still by the first Columne, is expressed the measure of the one side, and by the two other pillers in one Columne, is set foorth the measure of the other side. And this shalbe sufficient now for the use of the Golden Rule.

The rule of Felowship.

Now somewhat will I touch certaine other rules, which for their severall names may seeme diverse rules and distinct from this, but in deede they are but branches of it: yet be-

cause they haue not onely seuerall workings in appearance, but also pleasant in vse, I will giue you a taste of each of them. As for the rule of Felowship, both single and double with time and without time, I shall neede to say little more then I haue alreadie saide in teaching the workes of whole numbers, yet an example or two will wee haue to refresh the remembrance of the same, and to declare certaine proper vses and applications of it, as this for one.

A questoin of vnequal societie.

Foure men got a bootie or prise in time of warre, the prise is in value of mony 8190 £ and because the men be not of like degree, therefore their shares may not be equall, but the chiefest person will haue of the bootie the third part, and the tenth part ouer: the second will haue a quarter and the tenth part ouer: the third wil haue the fift part: and so there is left for the fourth man a verie small portion but such is his lot (whether he be pleased or wroth) he must be content with one xx. part of the pray. Now I demaund of you, what shall euery man haue to his share?

Scholer. You must be faine to answere to your owne question, else is it not like to be answered at this time.

Maister. The forme to vnderstand the solution of this question and all such like, is thus: Reduce all the denominators into one number by Multiplication, except that any of them

them be parts of ſome other of them, for all ſuch partes you may ouerpaſſe, and take for them: all thoſe numbers, whoſe partes they be: as in this example the ſhares be theſe $\frac{1}{3}\frac{1}{10}$. $\frac{1}{4}\frac{1}{10}$. $\frac{1}{6}\frac{1}{20}$. if I multiply all the denominators together, beginning with 3 and ſo go on vnto 20, it will make 144000: but conſidering that 3 is a part of 6, I ſhall omit that 3; and likewiſe 10, which is a part of 20, I may ouerpaſſe alſo, and then is there but 3 denominators to multiplie, that is 4, 6, and 20, which make 480, which ſumme I take for my worke, becauſe all the denominators will be found in it. Then I take ſuch partes of it, as the queſtion importeth, that is for the firſt man $\frac{1}{3}\frac{1}{10}$, the $\frac{1}{3}$ is 160, the $\frac{1}{10}$. is 48: which I put in one ſumme for the firſt mans ſhare, and it maketh 208. Then for the ſecond mans ſhare, I take $\frac{1}{4}$, which is 120, and $\frac{1}{10}$. which is 48, and that maketh in the whole 168. Now for the third man which muſt haue $\frac{1}{6}$, I take 80. And for the fourth man there remaineth but 24, which is $\frac{1}{20}$. of the whole ſumme: ſo that if the whole pray had beene but 480 l, then were the queſtion anſwered: but becauſe the ſumme was of greater value, by this meanes now ſhall I know the partition of it: I muſt ſet my numbers by the order of the Golden Rule, putting in the firſt place the number that I found by multiplying the denominators, and in the ſecond place the ſumme of the

the booke. And looke what proportion is be-tweene the first number and that second, the same proportion shall be betweene the partes of that first number and the partes of the second, comparing each to his like. Therefore I must put in the third place, one of the parts or shares, and then worke by the former rule of proportion or Goulden Rule. And because I haue foure seuerall partes of the first number, by which I would find out foure like partes of the second number, therefore must I make foure seuerall figures.

The reason of this rule.

Scholer. Now I trust I can answere to your question, as by your fauour I will prooue.

A		B	
480	8190	480	8190
208		160	

C		D	
480	8190	480	8190
80		24	

And to trie it, I set the 4 figures thus, marked with A, B, C, D, to shewe their order. And then in each of them I multiplie the second numbers by the third, and diuide their totall by the first, and so amounteth the fourth summe which I seeke for, if I doe multiplie 8190 by 208, it maketh 1703520, which being diuided by 480, maketh in the quotient 3549 for the first mans portion.

And

And ſo working with the other three figures, I find for the ſecond man $2866\frac{1}{2}$, and for the third man 1365: and then for the fourth man $409\frac{1}{2}$. And ſo is euery mans ſhare ſet forth in the figure here annexed.

A

480 — 8190
208 — 3549

B

480 — 8190
160 — $2866\frac{1}{2}$

C

480 — 8190
80 — 1365

D

480 — 8190
24 — $409\frac{1}{2}$

And thus I thinke I haue done well.

Maiſter. If you miſdoubt your working, and liſt to prooue it, adde all the ſhares together, and if they make the totall, then ſeemeth it well done.

Scholer. I may ſet them thus: and then by Addition the iuſt ſumme doth amount, that is 8190, and therefore (as you ſay) it ſeemeth to bee well wrought.

The proof by Addition.

$$\begin{array}{r} 3549 \\ 2866\frac{1}{2} \\ 1365 \\ 409\frac{1}{2} \\ \hline 8190 \end{array}$$

But I beſeech you, is there any doubt in this triall, that you vſe that worde, Seemeth?

The iuſt proofe.

Maiſter. You may eaſily coniecture, that if you did aſſigne the firſt mans ſhare to the laſt, and ſo change all the reſt, that one had an others ſhare, yet would the Addition appeare

all one, and therefore is not the proofe exact.

But if you will make a iust proofe for the first mans part, take $\frac{1}{3}$ & $\frac{1}{10}$. of the whole summe and if it agree with the number in the figure, then it is well done. And so do for the second, third, and fourth summes, and this proofe fayleth not. Now will I propound certaine other questions which haue beene set foorth by certaine learned men, albeit not without some ouersight, which questions I protest heartily, I doe not repeate to depraue those good men, whose labours and studies I much prayse and greatly delight in, but onely according to my profession, to seeke out truth in all thinges, and to remoue all occasions of errour, as much as in me lieth: and for that cause I will onely name the questions without hurting the Authors name. The first question is this.

A question of building

Foure men did builde a house which coste them 3000 Crownes, their shares were such, that one man should pay $\frac{1}{2}$ of the summe, and 6 crownes ouer: the second should pay $\frac{1}{3}$, and 12 crownes ouer: the third man must lay out $\frac{2}{3}$ abating 8 crownes, & the fourth man should pay $\frac{1}{4}$ and 20 crownes more: can you answere to this question?

Scholer. No in good faith sir, and that you know best of any man, for I know no more than you haue taught me.

Master. Then I dare say you cannot doe

it

it, neither yet the beſt learned man that euer did propoſe it, for the queſtion is impoſſible: for declaration whereof I will bee bold to vſe firſt the repreſentation of the numbers in their apteſt forme, (although I haue not yet taught that manner of worke) becauſe it may appeare plainly that the queſtion is not poſſible, for here I haue ſet the parts, and added them, and they make the whole ſumme and $\frac{3}{4}$ and 30 more. Now how is it poſſible to diuide truely eyther gaynes, either charges, ſo that the particulars ſhall be more than the totall? An impoſſible queſtion.

$$\left\{\begin{array}{ll} \frac{1}{2} + & 6 \\ \frac{1}{3} + & 12 \\ \frac{2}{3} - & 8 \\ \frac{1}{4} + & 20 \end{array}\right\}$$

$$1\tfrac{3}{4} + \quad 30$$

Scholer. It is againſt the forme of proofe by Addition of the particulars.

Maiſter. You ſay truth. And becauſe you ſhall perceiue it the better, I will try it after the vulgar forme, as in this figure you ſee whether the $\frac{1}{2}$ with 6 ouer is 1506: for the totall is, as you heard before 3000: the $\frac{1}{3}$ and the 12 more, is 1012: the $\frac{2}{3}$ would bee 2000, but then abating 8, it is but 1992, and then laſt of all, the $\frac{1}{4}$ is 750, and the 20 more maketh 770: which all beeing added in one ſumme, doe make 5280, where the totall ſumme ſhould be but 3000, which ſumme of 3000 if you diuide by $\frac{3}{4}$, ſo ſhall

1506
1012
1992
770
5280

you haue ¾ of it, that is 2250, and thereto adde 30 more, then will those thrée summes make 5280: whereby you may sée how this forme as well as the other, doth declare that the particulars in that question would make more than the whole summe by ¼, and 30 more: and therefore can that questiō not be accepted as a possible thing, but yet doe certaine learned men propound such questions, and answere to them. Therefore somewhat to say to their excuse, rather of their good meaning, than for their doing, I will anon declare what may be sayd for their defence: but in the meane season I will propound the question as it may bee wrought by góod possibilitie. As if foure men build a house together, and it cost them 2000 crownes, and then for the partition they agrée thus: that as often as the first man doth pay 6 crownes, so often shall the second man pay 4, the third man 8, and the fourth man 3. Or else thus: that the first man shall pay double so much as the fourth, and the second man shall pay ⅔ of the first mans charge: the thirde man shall pay double so much as the second: (and these two wayes are to one ende) but further for their agréement it is appointed also, that the first man shall haue 6 crownes ouerplus, and the second 12, and the fourth shall giue 20, but the third man shall giue no ouerplus,

3000
2250
30
5280

plus, but shall haue 8 Crownes abated of his charge. Now is the question possible to be assoiled, and this is the way to doe it. Marke the proportion of the seuerall charges, and set out small numbers in that rate, by which you may reduce the worke to the Golden Rule, as here in the first forme, the numbers are already named, 6, 4, 8, 3: and in the second forme, although they be not plainely named, yet they may be the same numbers: for 6 is double to 3, and 4 is $\frac{2}{3}$ of 6: and againe 8 is double to 4. Now adde these together, and they make 21, which 21 must be set in the first number in the Golden Rule: for if it with the ouerplus of ech mans charge, would make the totall summe of the charges, then were those seuerall summes the charges of each man, besides his ouerplus: but now it is not so.

But yet this is true, (so excellent are conclusions Arithmeticall,) that looke what proportion each of these seuerall summes doth beare to 21, the same proportion doth the iust charges of euery man (besides his ouerplus) beare to the totall of the charges, the ouerplus being deducted: Wherefore this may you note, that before you doe apply the totall of the charges to the Golden Rule: you must deduct the ouerplus which is 6, 12, and 20, that is in the whole 38: but then 8 must bee restored for the abatement of the third man, and then remaineth to be deducted 30. Take 30 there- The Rule.

fore out of 3000 and there will rest 2970, which I must set in the Golden rule for the second summe: and for the third summe I must put each of the small numbers before mentioned, which although they be not the seuerall charges, yet they represent them in proportion. And so making for euery mans charge a seuerall question, the figures will be 4, which I marke with foure letters, A, B, C, D, thus.

A			B		
21		2970	21		2970
6		$848\frac{4}{7}$	4		$565\frac{5}{7}$

C			D		
21		2970	21		2970
8		$1131\frac{3}{7}$	3		$424\frac{2}{7}$

Where I haue set for briefnesse the summe of euery mans charge in the fourth place presupposing that you can tell how to try out that fourth summe by so many examples as ye haue had.

Scholer. As I trust that I vnderstand this fourme, so I desire much to know what may be sayd for them that mistooke this question.

Maister. You seeme so desirous to know this errour, that you haue forgotten to examine whether this worke be without fault.

Scholer. Me seemeth this worke to bee well done, because the Addition of the 4 seuerall num-

numbers doth make the totall ſumme of 2970, which was to bee diuided into ſuch foure partes.

Maſter. But then haue you forgotten that the firſt man muſt pay ſixe Crownes more beſides this ſhare, and the ſecond man twelue Crownes more: the third man 8 Crownes leſſe: and the fourth man 20 Crownes more, for without theſe, your firſt total of 3000 crownes will not be made.

Scholer. Then muſt I adde to the firſt mans ſumme 6 more, and it will be $854\frac{4}{7}$: and to the ſecond ſumme I muſt adde 12, and it will be $577\frac{5}{7}$: from the third ſumme I muſt abate 8, and then will the ſumme bee $1123\frac{3}{7}$: then adding vnto the fourth ſumme 20, it will bee $444\frac{2}{7}$: and theſe foure ſummes will make 3000, which is the whole charge, as in this example it may appeare, where firſt I gather the $\frac{14}{7}$, that maketh 2, and ſo proceede I in the Addition to the end.

$854\frac{4}{7}$
$577\frac{5}{7}$
$1123\frac{3}{7}$
$444\frac{2}{7}$
3000

Maiſter. Now haue you well done, and this worke in the ſame ſummes is brought of other learned men for the true ſolution of the queſtion as it was firſt proponed, which as (I ſayd) was impoſſible: and now examine it by theſe ſeuerall ſummes, and ſee whether it doe agree with the ſummes in the queſtion proponed.

The first man must pay ½ and 6 ouer of the totall summe: how thinke you, is 845 ½ the halfe and 6 more of 3000?

Scholer. No that it is not, for it would be 1506: and for the second man 1012: and for the third man 1992: and for the fourth man 770, whereof not one summe agreeth to this worke. But I marvell that so wise men could be so much ouerseene.

Maister. It is commonly seene, that when men will receiue things from elder writers, & will not examine the thing, they seeme rather willing to erre with their auncients for company, than to be bold to examine their workes or writings, which scrupulosity hath ingendered infinite errours in all kinds of knowledge, and in all ciuill administration, and in euery kinde of Art: but these learned men did not meane any other thing by this question, than to finde such numbers as should beare the same proportion together, as those numbers in the question proponed did beare one to an other: which thing you shall perceiue more plainely by another question of theirs, that is this.

A question of a testament.

A man lying vpon his death bedde, bequeatheth his goods (which were worth 3600 Crownes) in this sort. Because his wife was great with childe, and he yet vncertaine whether the childe were male or female, hee made his bequest conditionally, that if his wife bare a daughter, then should the wife haue halfe his goods

goods, and the daughter $\frac{1}{3}$, but if she were deliuered of a sonne, then that sonne should haue $\frac{2}{3}$ of the goods, and his wife but $\frac{1}{3}$. Now it chaunced her to bring forth both a sonne & a daughter, the question is: How shall they part the goods agreeably to the testator his will.

Scholer. If some cunning Lawyers had this matter in scanning, they would determine this Testament to be quite voyde, and so the man to die intestate, because the Testament was made insufficient, sith this condition was not expressed in it, and also it might haue chanced that she should haue brought forth neither Sonne nor Daughter, as often hath beene seene: so is the will unsufficient in that point also.

Master. Such scanners should seeme too cunning, and yet not so cunning as cruell: for the minde of the Testator is to be taken fauorably, for the ayd of the legatories, when there riseth such doubts. But let us trie this worke, not by force of law, but by proportion Geometricall, seeing the testator did minde to prouide for each sort of them.

Scholer. If the sonne shall haue $\frac{2}{3}$ by force of the Testament, so must the mother haue $\frac{1}{3}$. Againe, because she hath a daughter also, therefore ought she to haue $\frac{2}{3}$, & the daughter $\frac{1}{3}$: that is both wayes $\frac{2}{3} : \frac{1}{3}$, and $\frac{2}{3} : \frac{1}{3}$, which commeth to the whole goods, and $\frac{1}{3}$ more. Wherefore it seemeth also impossible.

Maister. In this matter the mind of the Testator is to bee vnderstood, that such proportion should bee betweene the portion of the wife and the sonne, as is beeweene $\frac{1}{3}$ and $\frac{1}{2}$, that is, the sonne must haue $\frac{3}{6}$ for $\frac{2}{6}$ to his mother so shal he haue 3 to 2, that is as much as his mother, and halfe as much more: and the mother must haue the like rate in comparison to her daughter. Then must I finde out 3 numbers in such proportion, that the first may be as much as the second, and halfe as much more (that is) in proportion sesquialtera, and the second to the third in the same proportion, such numbers be 6, 9, 4.

Scholer. I pray you Sir, how shall I finde out those numbers?

Maister. That will I gladly tell you.

To finde three nũbers in any proportiõ.

Whatsoeuer the proportion bee of any three numbers, multiply the Termes of that proportion together and the number that amounteth, shall be the middle number of the 3: then multiply that middle number by the lesser terme, and diuide the totall by the greater, and the least number of the three will amount. So if you multiply that middle number by the greater extreame, and diuide that totall by the lesser extreame, then will the greatest number of that progression amount.

To find the proportion betweene 2 numbers

Scholer. Then in this example, to finde the proportion of $\frac{1}{2}$ to $\frac{1}{3}$, I must diuide (as you taught me in Diuision) $\frac{1}{2}$ by $\frac{1}{3}$ and the quotient will be $\frac{3}{2}$, that is $1\frac{1}{2}$, whereby I perceiue that the

the proportion in this question is, as 3 to 2. Therfore (as you taught me euen now) I multiply 3 by 2, and the summe is 6, which must be the middle number: then I multiply the middle number 6 by 2, which is the least tearme, and the summe is 12, that I doe diuide by 3, being of the greater Terme, and the quotient is 4, so is 4 the least number of the 3. Then I multiply 6 by 3, whereof commeth 18, and that I diuide by 2, and so haue I 9, which is the greatest number of the 3.

Maister. An other way yet may you finde the third number in any progression, if you haue two of them: for if the middle number be one of them which you haue, then multiply it by it selfe (as in this example 6 by 6 maketh 36) and that totall diuide by the other number which you haue, and the third number will bee the quotient.

Scholer. Then if I diuide 36 (which commeth of 6 multiplied by it selfe) by 4, the quotient will be 9, and if I diuide 36 by 9, the quotient will be 4. But what if I know the first number and the third, and would haue the middle number?

Maister. Multiply the two numbers together, and in their totall you must seeke the roote of that number, & it shal be the middle number: but because as yet you haue not learned how to extract rootes, therefore vse the first forme which I haue taught you, till I teach you to

O. l. E. ff

extract rootes. And now goe forward with the answer to the same question.

Scholer. I perceiue then that the sonne must not haue $\frac{1}{2}$ of the goods, neither the mother $\frac{1}{3}$, nor yet the daughter $\frac{1}{3}$, but yet must the goods be diuided into such portion, that the sonne shall haue 9 crownes for 6 to his mother: and the mother shall haue 6 crownes for euery 4 to the daughter. Then I apply it to the Golden rule in 3 examples thus: where the first number is the Addition of those 3 numbers, 9, 6, 4, and the third is one of them seuerally: the secōd is the totall of the goods in the testament: and then by the worke of ẏ Golden rule I finde out the fourth number in euery work, that is, for the sonne $1705\frac{5}{19}$: for the mother $1136\frac{16}{19}$: and for the daughter $757\frac{17}{19}$, which three summes added together, doe make ẏ sum of the whole goods as may be seene by this exāple.

19 — 3600
9

19 — 3600
6

19 — 3600
4

1705 | $\frac{5}{19}$
1136 | $\frac{16}{19}$
757 | $\frac{17}{19}$
3600

And this (me thinketh) I do perceiue, that because in this case there is a necessarie remedie deuised against an vrgent inconuenience, therfore those learned men thought they might vse the like libertie in that other question.

Master. Your guesse is good, but they had so good reason for them in the one, as they haue

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in the other: as in an other example of theirs, it may better appeare, that is this.

A man left vnto his three sonnes 7851 crownes to be parted in this sort, that the first sonne should haue $\frac{1}{2}$, the second sonne $\frac{1}{3}$, and the third sonne $\frac{1}{4}$, which is not possible, for $\frac{1}{2}$, and $\frac{1}{3}$, & $\frac{1}{4}$, doth make $\frac{26}{24}$: or $\frac{13}{12}$, that is $1\frac{1}{12}$, so is it more than the whole: but reduce these fractions into one Denomination, the least that they will come to, and they will be $\frac{6}{12}$, $\frac{4}{12}$, $\frac{3}{12}$, and so may you part the goods in such proportion as these 3 Numerators beare together: that is, the first to haue 6 for euery 4 to the second: and the second to haue 4 as often as the third hath 3: and so their portions will be for the first, 2623 $\frac{7}{13}$: for the second 2415 $\frac{9}{13}$: and for the third 1811 $\frac{10}{13}$, and those thrée shares added together, wil make the totall summe of the whole goods, as you may easily see in this example. An other questiō of a testamēt

	3623 $\frac{7}{13}$
	2415 $\frac{9}{13}$
	1811 $\frac{10}{13}$
	7851

Another question is there proponed thus.

There is 450 Crownes to bee diuided betwéene 3 men, so that the first man must haue $\frac{1}{2}$ and $\frac{1}{3}$, the second man $\frac{1}{3}$ and $\frac{1}{4}$, the third man shall haue $\frac{1}{4}$ and $\frac{1}{5}$.

Master. I maruell that any man should be so ouerseene to propound that question as a thing possible, sith $\frac{50}{60}$, $\frac{35}{60}$, $\frac{27}{60}$, doe make $1\frac{52}{60}$, that is almost double the whole summe.

But I perceiue it might bee thus propo-

ned, that as often as the first man did receiue 50 crownes, so often the second man should receiue 35, and the third man 27, for $\frac{1}{2}$ and $\frac{1}{3}$, is equall to $\frac{50}{60}$, and so is $\frac{1}{3}$ and $\frac{1}{4}$ equall to $\frac{35}{60}$. and $\frac{1}{4}$ and $\frac{1}{5}$, is $\frac{27}{60}$, and so working the question, the 3 figures wil appeare in this forme: whereby the first mans portion is found to bee 200 $\frac{50}{56}$: the second mans part is 140 $\frac{15}{56}$: the third mans share is 180 $\frac{24}{56}$: which in the whole doth make 450 Crownes that was ẏ whole summe to bee diuided betweene them.

112 —— 450
50 —— 200 $\frac{50}{56}$

112 —— 450
35 —— 140 $\frac{15}{56}$

112 —— 450
27 —— 180 $\frac{24}{56}$

Maister. And thus you are (I thinke) sufficiently instructed in the Rule of Felowship.

The Rule of Alligation.

NOw will I goe in hand with the Rule of Alligation, which hath his name, for that by it there are diuers parcels of sundry prices, and sundry quantities alligate, bound or mixed together, whereby also it might be well called the rule

rule of mixture, and it hath great vse in composition of medicines, and also in mixtures of mettals, and some vse it hath in mixtures of wines, but I wish it were lesse vsed therein than it is now a dayes. The order of this Rule is this: When any summes are proponed to be mixed, sette them in order one ouer an other, and the common number whereunto you will reduce them, set on the left hand, then marke what summes be lesser than that common number, and which be greater, and with a draught of your penne, euermore linke two numbers together, so that one be lesser than the common number: and the other greater than he, for two greater or two smaller cannot well be linked together, and the reason is this, that one greater and one smaller may be so mixed, that they will make the meane or common number very well, but two lesse can neuer make so many as the common number, being taken orderly: no more can two summes greater than the mean, neuer make the meane in due order, as it shall appeare better to you hereafter. And as it is of necessity to linke euery smaller (once at the least) with one greater, and euery greater with one smaller: so it is at liberty to linke them oftner then once, and so may there bee to one question many solutions. When you haue so linked them, then marke how much each of the lesser numbers is smaller then the meane or common number, and that difference

The reason of this Rule.

let

set against the greater numbers which be linked with those smaller, each with his match still on the right hand, and likewise the excesse of the greater numbers aboue the meane, you shall set before the lesser numbers which bee combined with them. Then shall you by Addition bring all these differences into one summe, which shall be the first number in the Golden Rule: and the second number shall bee the whole masse that you shall haue of all those particulars: the third summe shall be each difference by it selfe, and then by them shall be found the fourth number, declaring the iust portion of euery particular in that mixture. As now by these examples I will make it plaine.

The reason of this Rule.

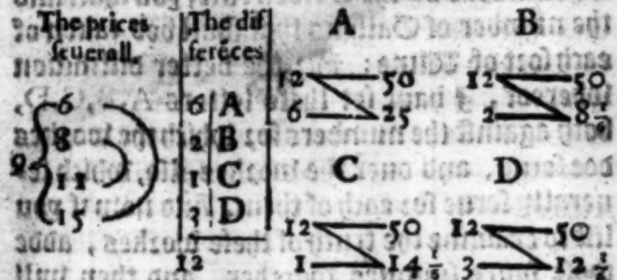
There are foure sortes of Wine of seuerall prices, one of 6 d a gallon, an other of 8 d, the third of 11 d, and the fourth of 15 d the gallon; of all these Wines would I haue a mixture made to the summe of fifty gallons, and so that the price of each gallon may be 9 d. Now demaund I how much must be taken of euery sort of Wine?

a question of mixing of wine.

Scholer. If it shall please you to worke the first example, that I may marke the applying of it to the Rule, then I trust I shall bee able not onely to doe the like, but also to see the reason in the order of the worke?

Maister. Marke then this forme and the placing of euery kind of number in it.

The



The common price

Here (you ſee) I haue ſet downe the ſeuerall prices, which be 6, 8, 11, 15, and haue linked together 6 with 15, and 8 with 11. The common price 9, I haue ſet on the left ſide: And the difference betweene it, and euery particular price, I haue ſet on the right hand not againſt the ſumme whoſe difference it is, but againſt the ſumme that it is linked withall: ſo the difference of 15 aboue 9, is 6, which I haue ſet not againſt 15, but againſt 6, that is linked with 15, and the difference betweene 6 and 9 that is 3, I haue ſet againſt 15. So likewiſe the difference betweene 8 and 9 is but 1, that haue I ſette againſt 11, and the difference of 11 aboue 9 (which is two) I haue ſet againſt 8. Then adde I all thoſe foure differences, and they make 12, which I ſet for the firſt number in the Golden Rule: the ſecond number I make 50, which is the ſum of Gallons that I would haue, and the third ſumme is euery particular difference. Now

if you worke by the Golden rule, you shall find the number of Gallons that shall bee taken of each sort of Wine: For the better distinction whereof, I haue set these letters A, B, C, D, both against the numbers for which the workes doe serue, and ouer the workes also, which seuerally serue for each of them. And now if you list to examine the truth of these workes, adde those foure summes together, and they will make fifty, that is the totall which I would haue, as by this example you may easily perceiue. And for to proue how the prices doe agrée, do this. Multiply this totall summe 50, by the common price 9, and it will make 450: then keepe that summe by it self, and afterward multiply euery seuerall summe of Gallons, by the price belonging to the same gallons, and if that summe doe agrée with this which you haue kept first, then is your worke well done. As here, 25 is the number of Gallons of 6 pence price, multiply then 25 by 6, and it maketh 150, which you shall set downe: then multiply 8 2/6 by 88 which is the price for the number of Gallons, and it will make 66 4/6: so againe 4 1/6 multiplied by 11, doth make 45 5/6. And last of all 12 3/6 multiplied by 15, maketh 187 3/6. And these added together

The proof of this Rule.

25
8 2/6
4 1/6
12 3/6
50

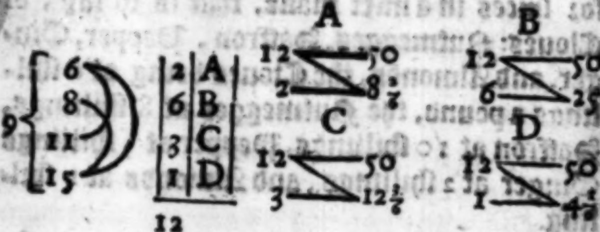
150
66 4/6
45 5/6
187 3/6
450

doe

bee make 450, as in the example annexed you may see: wherfore seeing it doth agree with the former sum of 50, multiplied by 9, I may iustly affirme this worke to be good and wel done.

And now to prooue how you can do the like, I propound the same question, onely willing you to vse some other forme of combining or linking the summes.

Scholer. That shall I prooue with your fauour, and therefore I combine 8 with 15, and 6 with 11, and then the forme will bee as followeth.



whereby amounteth the same summe in totall of the differences, as did before: and yet now the differences bee altered, as the combination is changed, whereof I vnderstand the reason by your former worke. And therefore here appeareth no strange thing, but that now I must haue 8 2/7 gallons of sixe pence, and 25 gallons of 8 d, and 12 gallons and 1/6 of 11 d, and so consequently 4 gallons & 1/7 of 15 d, so that multiplying 8 2/7 by 6, it maketh 50, &

50
200
137 1/2
62
450

if you worke by the Golden rule, you shall find the number of Gallons that shall bee taken of each sort of Wine: For the better distinction whereof, I haue set these letters A, B, C, D, both against the numbers for which the workes doe serue, and ouer the workes also, which seuerally serue for each of them. And now if you list to examine the truth of these workes, adde those foure summes together, and they will make fifty, that is the totall which I would haue, as by this example you may easily perceiue. And for to proue how the prices doe agrée, do this. Multiply this totall summe 50, by the common price 9, and it will make 450: then keepe that summe by it self, and afterward multiply euery severall summe of Gallons, by the price belonging to the same gallons, and if that summe doe agrée with this which you haue kept first, then is your worke well done. As here, 25 is the number of Gallons of 6 pence price, multiply then 25 by 6, and it maketh 150, which you shall set downe: then multiply $8\frac{2}{6}$ by 88 which is the price for the number of Gallons, and it will make $66\frac{4}{6}$: so againe $4\frac{1}{6}$ multiplied by 11, doth make $45\frac{5}{6}$. And last of all $12\frac{3}{6}$ multiplied by 15, maketh $187\frac{3}{6}$. And these added together doe

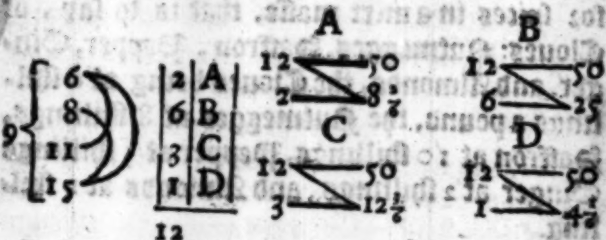
The proof of this Rule.

25	150
$8\frac{2}{6}$	$66\frac{4}{6}$
$4\frac{1}{6}$	$45\frac{5}{6}$
$12\frac{3}{6}$	$187\frac{3}{6}$
50	450

doe make 450, as in the example annexed you may see: wherfore seing it doth agree with the former sum of 50, multiplied by 9, I may iustly affirme this worke to be good and wel done.

And now to proove how you can do the like, I propound the same question, onely willing you to vse some other forme of combining or linking the summes.

Scholer. That shall I proove with your favour, and therefore I combine 8 with 15, and 6 with 11, and then the forme will bee as followeth.



whereby amounteth the same summe in totall of the differences, as did before: and yet now the differences bee altered, as the combination is changed, whereof I vnderstand the reason by your former worke. And therefore here appeareth no strange thing, but that now I must haue 8 1/3 gallons of sixe pence, and 25 gallons of 8 d, and 12 gallons and 1/2 of 11 d, and so consequently 4 gallons & 1/6 of 15 d, so that multiplying 8 1/3 by 6, it maketh 50, &

50
200
137 1/2
62
450

then 25 multiplyed by 8, maketh 200: likewise $12\frac{1}{2}$ multiplyed by 11, yeeldeth $137\frac{1}{2}$, and $4\frac{1}{6}$ multiplyed by 15, maketh $62\frac{1}{2}$, which 4 summs added in one, will yeeld in the totall 450, which agreeth with the multiplication of 50 (being the totall summe of Gallons) by 9 the common or meane price.

A question of spices.

Master. Seeing you conceiue this worke so well, I will propound an other example unto you of more varietie in the Alligations or combining, as thus.

¶ A marchant being minded to make a bargain for spices in a mixt masse, that is to say, of Cloues: Nutmegges, Saffron, Pepper, Ginger, and Almonds, the Cloues being at 6 shillings a pound, the Nutmegges at 8 shillings, Saffron at 10 shillings, Pepper at 3 shillings Ginger at 2 shillings, and Almonds at 1 shilling.

Now would he haue of each sort some to the value of 300 poundes in the whole, and each pound one with another to beare in price 5 shillings, how much shall be haue of each sort?

Scholer. That will I trie thus.

First I set downe those sixe seuerall prices and at the left hand I set the common price 5 s. Then I linke them thus, one with 10, two with 6, and three with 8, as in the example following.

Master.

Master. I had minded to haue combined them in more varietie, but I am content to see your owne worke first, and then more varieties in combination may follow anon.

Scholer. Then to continue as I began, I seeke the difference betweene 1 and 5 (which is 4) and that I set agaynst 10: then agaynst 1 I set 5, which is the excesse of 10 aboue 5: so I gather the difference betweene 2 and 5, which is three, and that I set against 6, because it is combined with 2: and likewise the difference of 6 aboue fiue (which is 1) I set against 2. Then take I the difference of three from 5, which is two, and that I set against 8, and before that three I set the difference of 8 aboue 5, which is three. Then gather I all these differences by Addition, and they make 18, which I set for my first number in the Golden rule, and so appeareth by those workes, that of Almonds I must take 83 l. 1/3, of Ginger 16 pounds 2/3, of Pepper 50 poundes, of Cloues 50 pounds, of Nut-

megges 33 poundes $\frac{1}{3}$, and of Saffron 66 pounds $\frac{2}{3}$. Then for triall hereof, I multiply euery parcell by his seuerall price, as 83 $\frac{1}{3}$, which is the summe of Almondes. I multiply by 1, which is their price.

83 $\frac{1}{3}$
33 $\frac{1}{3}$
150
300
266 $\frac{2}{3}$
666 $\frac{2}{3}$
1500

Also 16 $\frac{2}{3}$ the summe of Ginger I multiply by two which is the price of it. And so each other in his kinde, as this Table annexed doth represent, and then adding them al together, I find the totall to be 1500, which also will amount by the multiplication of the grosse masse of 300 by the common price 5, wherefore it appeareth well wrought.

Maister. Now will I make the Alligation to proue your cunning somwhat better: but because you shall not thinke your selfe pressed so much, I will also note the differences, as in this example you may see where I haue Alligate

5	1	18	4
	2	35	8
	3	5	5
	6	4	4
	8	43	7
	10	32	5
		33	

a
33 — 300
4 — 36 $\frac{4}{11}$

b
33 — 300
8 — 72 $\frac{8}{11}$

c
33 — 300
5 — 45 $\frac{5}{11}$

d
33 — 300
4 — 36 $\frac{4}{11}$

e
33 — 300
7 — 63 $\frac{2}{11}$

f
33 — 300
5 — 45 $\frac{5}{11}$

gate 1 with 6 and 8, and therefore haue I sette against 1 both their differences, that is 1 and 3. Likewise because two is combined with 8 and 10, I set before him their differences, 3 and 5. Against 3 I haue set onely 5, which is the difference of 10, with whom 3 is combined onely. Likewise 6 is onely alligate to 1, and therefore is the difference of 1 from 5: which is 4 onely set against it: 8 is linked with 1 and 2, and therefore hath hee against him both their differences, 4 and 3; and 10 is ioyned with 2 and 3, therefore hath hee their differences 3 and 2.

And because of ease for you, in another Columne I haue set the differences reduced into one number, for euery seuerall sort, and haue also added them together, whereby appeareth that they make 32, and so consequently you see the workes of the Golden Rule set forth for the sixe seuerall drugges, I haue added letters, a, b, c, &c. as before. But I would not wish you to cleaue still to these elementary aydes, but accustome memory to trust her selfe, so shall occasion of negligence bee best auoyded. And as for the proofe, try it at more leysure, because the time now is short, and you sufficiently instructed in that proofe. And there resteth diuerse things behinde yet, of which I woulde gladly giue you some taste before your departure.

Scholer. But if it may please you to let mee

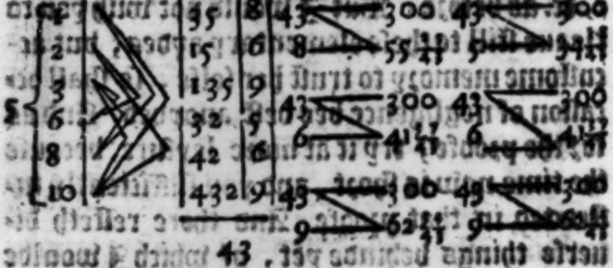
see all the variations of this question, before you goe from it, for me thinketh I could vary it two or three wayes more yet.

Maister. I am content to see you make two or three variations, but I would be loth to stay to see all the variations, for it may be varied above 300 wayes, although many of them would not serue well to this purpose.

Scholer. I thought it impossible to make so many variations.

Maister. Marvell not thereat, for some questions of this rule may bee varied aboue 1000 wayes, but I would haue you forget such fantasies, till a time of more leysure. And now go forward with some variation of this question.

Scholer. For the first variation I linke the first number 1 with 8 and 10, and 2 I combine with 6 and 10, then ioyne I 3 with 6, 8, and 10, as in this forme.



And so doeth there appeare the portion of weight for every kinde of drugge in this mixture. Now for the triall.

Maister.

Maister. Nay stay there, you shall not neede to make triall in one example so often, or if you list to doe it by your selfe, I am content. But now set forth (for declaration that you conceiue the rule) two or three examples of seuerall combinations, and then will we passe to some other example, and so end this Rule.

Scholer. As it pleaseth you, so will I doe. And these bee the varieties, in which as the

combinations are seuerall, so doth it plainely appeare, that the differences by which the proportion of each seuerall kinde is taken, are also seuerall. And yet I see in the three first of these

fiue varieties and in one other before, the totall summe of the differences to bee one, that is to say 18, whereby I perceiue that the varietie of their mixture doeth depend of the varietie of their differences seuerall and not of the varietie of their totall summe.

Master. So is it. And seeing you conceiue it so well, I will make an end of this Rule, onely exhibiting to you one question or two of the mixture of mettals, that by it you may deuise other like, and exercise your selfe therein also, because the vse of it serueth often in businesse of charge, not so much for Goldsmithes as for coynage in Mintes. First I demaund you this question. If a Mint-master haue gold of 22 karectes, and some of 23 karects, some of 24. Againe, some of 15, some 16, and some of 18 karectes, and would mixe them so that hee might haue 100 ounces of 20 karectes, howe much shall he take of euerie sort?

Scholer. To know that I answere in order thus.

20	15	2
	16	3
	18	4
	22	5
	23	4
	24	2
		20

20	100	20	100
2	10	5	25
20	100	20	100
3	15	4	20
20	100	20	100
4	20	2	10

Maister.

Maiſter. You haue wrought the queſtion well, but how chanced it you made no doubt of that new name Karect?

Scholer. Becauſe I thought it out of time to demaund ſuch queſtions now, ſeeing you make ſo much haſt to end: and againe in this caſe the proportion of the number is ſufficient for my purpoſe in this worke, truſting that an other time you will inſtruct mee as well of this, as of ſundry other things which I haue heard you talke of, ſo I haue a great deſire to know them.

Maiſter. Your annſwere is reaſonable, and your requeſt and truſt, with Gods helpe, I intend to ſatisfie. And to goe forward with this matter, let me ſee your examination of this laſt worke.

Scholer. Firſt for the one part I adde together all the particular ſummes as they appeare in the worke, and they make 100, as here by their Addition doth appeare.

And ſo it ſeemeth, that the ſums are well gathered, but for the further triall of them, I multiply firſt

10
15
20
25
20
10
100

20, which is the common or meane ſumme of the Karects by 100, which is the ſum of the whole maſſe which I would haue, and it maketh two thouſand. Then I multiply euerie particular ſumme by the Ka-

150
240
360
550
460
240
2000

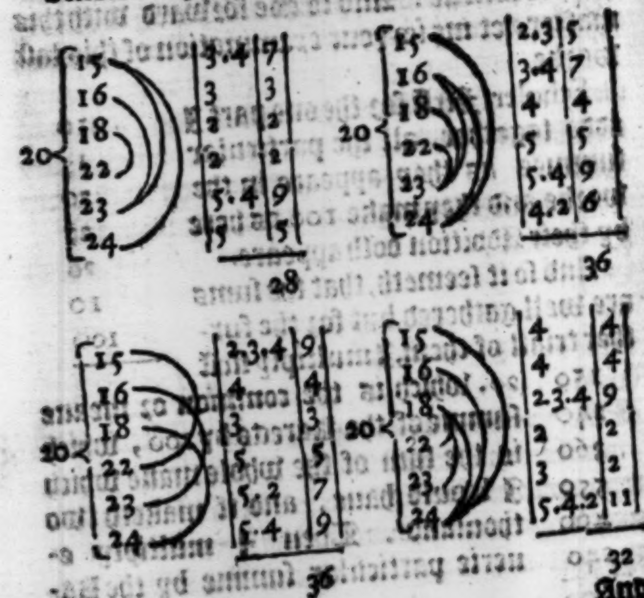
rects

rects that it doth containe, as 10 by 15, and that maketh 150.

Likewise I multiply 15 by 16, and it yieldeth 240: so 20 by 18 maketh 360. And 25 by 22 yeeldeth 550: likewise 20 by 23 bringeth forth 460, and last of all 10 multiplied by 24 yeeldeth 240: which summes all ioyned together make 2000, that doth agree with the like summe before, wherefore I may well say, that the worke is good. And nowe if it please you I would set forth some varieties of this question, to proue my wit.

Maister. Go to, let me see.

Scholer. Here be foure varieties.



And more yet I could make, but not like to the number that you spake of in the variation of the other question.

Maister. That will I teach you at more leysure, seeing it is a thing rather of pleasure than of any necessitie.

But now for your exercise in this Rule, one other question I will propose. A Mintmaister hath sixe ingottes of siluer of sundry finenesse, some of 4 ounces fine, and some of 5 ounces, some of 6, and other of 8, some of 11, and other of 12, and his desire is to mixe 500 pounds weight, so that in the whole masse euery pound weight shal beare 9 ounces of fine siluer, how much shall he take (say you) of euery sort of siluer? A question of mixing of siluer.

Scholer. To finde out that, I sette the numbers thus in order.

And gathering the differences, it will appeare, that of the first sort there must be 43 $\frac{11}{23}$ of the second like much: of the third sort 65 $\frac{5}{23}$: and of the fourth sort as much: of the fifth sort 195 $\frac{15}{23}$: and of the sixth sort 86 $\frac{22}{23}$, which in the whole will make 500 poundes weight: and in ounces after 9 ounces fine 4500, that is of the first sort 173 $\frac{21}{23}$: and of the second sort,

217 $\frac{9}{23}$: of the third sort 391 $\frac{7}{23}$: of the fourth sort 521 $\frac{17}{23}$: of the fift sort 2152 $\frac{4}{23}$, and of the sixt sort 1043 $\frac{11}{23}$, which altogether doe make 4500 ounces, agreeable to the multiplication of 9 by 500.

Maister. This is well done of you, therefore now marke three or foure varieties, and so an end of this rule.

A question of mixing of silver.

Scholer. These 4 varieties I set for example.

Maister. And by these it appeareth, that you can finde out more, with which I will not now meddle, save onely for to shew you an easie helpe in drawing the lines of Combination, I will set forth two varieties here.

And

	4	2		4	3
	5	23		5	23
	6	23		6	23
9	8	3	9	8	23
	11	543		11	431
	12	431		12	5431
		35			39

And this shall suffice now for the Rule of Alligation or mixture: for by these examples may you easily coniecture such other as doe appertaine to it, as well for the due working, as for variety of drawing the lines of combination.

Scholer. Sir, albeit it pleased you ere while, to put me from my musing at the manifold varietie, that may fall in these combinations, and termed them fantasies, yet my fantasie giueth mee, that the consideration of this should in many other examples and cases of importance be very needfull, and the knowledge of it most profitable. Therefore ye may well thinke, that at an other time conuenient I will request you to ayde me herein.

Maister. Truth it is that this consideration may fall in practise as well Politike as Philosophicall, and sundry wayes in them bee applyed, therefore when time shall fall fit for the discussing of this consideration, you shall not want my helping hand.

The

The Rule of Falshood.

The occasion of the name.

NOw will I briefly also teach you somewhat of the rule of Falshood, which beareth his name, not for that it teacheth any fraud or falsehood, but for that by false numbers taken at all aduentures, it teacheth how to finde those true numbers that you seeke for.

Scholer. So might any other Rule be called the Rule of Falshoode, for they worke by wrong numbers, and by them finde out the right numbers, so doth the rule of Alligation, the rule of Felowship, & the Golde rule partly.

Maister. In the Golden rule, the rule of Felowship, & the rule of Alligation, although the numbers that you worke by, bee not the true numbers that you seeke for, yet are they numbers in iust proportion, & are found by orderly worke: whereas in this rule the numbers are not taken in any proportion, nor found by orderly worke, but taken at all aduentures.

And therefore I sometimes being merrie with my friends, and talking of such questions, haue caused them that proponed such questions, to call vnto them such children or Idiots as

happened to be in the place, and to take their answere, declaring that I would make them solue those questions, that seemed so doubtfull.

And in deed I did answer to the question and worke the triall thereof also by those answeres which they hapned at all aduentures to make: which numbers seeing they be taken as manifest false, therefore is this rule called the rule of false positions, and for briefnes, the rule of Falshood, which rule for readinesse of remembrance, I haue comprised in these few Verses following in forme of an obscure Riddle.

Gesse at this worke as hap doth leede,
By chance to truth you may proceede.
And first worke by the question,
Although no truth there in be done.
Such falshood is so good a ground,
That truth by it will soone be found.
From many bate too many moe,
From too fewe take too fewe also.
With too much ioyne too fewe againe,
To too fewe adde too many plaine.
In crossewise multiplie contrary kind,
All truth by falshood for to find.

The sense of these verses, and the summe of this Rule, is this.

When any question is proponed appertaining to this Rule, first imagine any number that you list, which you shall name the first

first position, and put it in steade of the true number, and then worke with it as the question importeth: if you haue missed, then is the last number of that worke either too great or too little: that shall you note as hereafter shall bee taught you, and you shall call it the first errour.

Then beginne againe, and take an other number, which shall bee called the second position, and worke by the question: if you haue missed againe, note the excesse or default as it is, and call that the second errour. Then multiplie crosse-wise the first position by the second errour, and againe the second position by the first errour, and note their totals seuerally by the names of totals. Then marke whether the two errours were both alike, that is to say, both too much, or both too little: or whether they be vnlike, that is, the one too much, and the other too little: for if they bee like, then you shall subtract the one totall from the other (I meane the lesser from the greater) and the Remayner shall bee your diuidend, so must you abate the lesser errour out of the greater, and the residue shall bee the Diuisor. Now diuide the diuidend by that Diuisor and the quotient will shew you the true number that you seeke for. But and if the errours bee vnlike, then must you adde both those totals (which you noted) togither, and take that whole number for the diuidend, so

shall

shall you adde both the errors together, and that whole number shall be the Diuisor, & the quotient of that diuision shall giue you the true number that the question seeketh for, and this is the whole rule.

Scholer. This rule seemeth so vnlike any other, that without some example I shall not easily vnderstand it.

Master. With a good will propose halfe a score sundry questions & examples of varietie, for the better vnderstanding of the worke hereof: and for the first take this example. A Mason was bound to build a wall in 40 daies, and it was couenanted so with him, that euerie day that he wrought, he should haue for his wages 2 s 1 d, and euerie day that he wrought not he should be amerced 2 s 6 d, so that when the wal was made, and the reckning taken of the daies that he wrought, and the other that he wrought not, the Mason had clearely but 5 shillings, 5 d, for his worke. Now do I demaund, how many daies did he worke of those 40, and how many did he not worke?

A question of Masonrie. The first example.

Scholer. I pray you expresse the order of the worke, that I may partly by imitation, & partly by comparing it with the rule, be able againe to do the like.

Master. This order shall you keepe in the worke of this rule: first take some number (as you list) at aduenture, as for example, I say he played 12 dayes and wrought 28 dayes. Now

cast you the wages of every day, and see whether it will agree with the summe of 5 s, 5 d.

Scholer. The 28 dayes that he wrought after 25 pence the day, receiveth 700 pence. Then the 12 daies that hee wrought not, at 30 pence ech day, doth amount to 360 pence, which if I abate out of 700 pence, there resteth 340: but you say he had not so much.

Maister. He had but 65 pence, and by this supposition he should have had 540: therefore is this summe too much by 275, which summe I must set downe after this sort, as you see here, where first I have made a crosse (commonly called S. Andrewes crosse) and at the over corner on the left hand I have set the first position 12, and at the other corner under it, I have set 275 which is the first errour, with this figure +, which betokeneth too much, as this line, — plaine without a crosse line, betokeneth too little. On the right hand of the crosse I have left two like rowmes for the second position and his errour. Therfore to prosecute the worke, I suppose he played 16 dayes, and wrought 24.

Scholer. I was a while in doubt why you named the daies of his working, seeing they hee not set in the figure: and I doubted how you knowe them, or else whether that you did suppose them at all adventures, as you did the dayes that hee played: but now I gather

ther, that seeing 40 dayes is the whole time limited, then the dayes that hee played being supposed, the rest of 40 must needes bee the dayes that hee wrought, and therefore 28 followed 12 of necessity, and 24 followeth 16 also of necessity: but yet I scarce perceiue why you set not in the figures as well 28 as 12.

Maister. It forceth not which of them I take, so that in the second position I take the numbers of the same nature, that is here both of working daies, or both of idle: but now examine you this second position.

Scholer. If he played 16 daies, then abating 16 times 30 d, the summe will bee 480 d. And for 24 dayes that hee wrought, euery day yeelding 25 d, the totall is 600 pence: so that abating 480 out of 600, there resteth 120; and as you say it should be but 55, therefore it is too much by 55, that must be set on the right hand of the figure at the neather part, and ouer it on the same side 16, which is the second position, thus.

12 16

275+ 55+

And as I gather by your words it were all one if I did set 28 in stead of 12, and 24 in stead of 16.

Maister. So were it. But this shall you marke, that of what nature soeuer the two positions bee, of the same nature is the quotient. Therefore when the positions in this

Note.

question are 12 and 16, which both being num-bers of the playing dayes, the quotient shall declare the true numbers of playing dayes, whereas if the positions had beene 28 and 24, which are supposed to bee the working dayes, then would the quotient declare the true number of the working dayes, and not of playing dayes, as it will doe now. And therefore to continue the worke of this question, and to find the true number of playing dayes, I must multiply crosswise the first position 12 by 55, that is the second errour, and the totall will bee 660. then I multiply 275 by 16, and it yeeldeth 4400. Now because the errours are like, that is to say, both too much, I must subtract 660, out of 4400, and so remaineth 3740, which is the dividend. Againe I must subtract the lesser errour 55 of 275, that is the greater errour, and there will remaine 220, which shall be the Divisor, then dividing 3740 by 22, the quotient will bee 17. Wherefore I say now constantly, that 17 is the true number of daies that the Mason plaid: and then it followeth, that he wrought 23 daies, and so is the question answered.

The proof of this Rule.

Now for the order of triall of this worke there needeth none other proofe but only this, to worke with this number according to the question, and if it agree, then appeareth the number to bee it that you would have. As here

here now seeing he wrought 23 dayes, and must have for every day 25 pence, the whole summe commeth to 575. Then againe seeing he plaied 17 daies, and must abate 30 pence for every day, the whole summe of the abatement will be 510: therefore I subtract 510 out of 575, and there will remaine 65. which maketh 5 s. 5 d, the cleare wages of the Mason for his worke, according to the question.

Scholer. Now I trust I understand the worke and the rule so well (and the better by this proofe) that I can bee able to doe the like. And for a proofe I take the same question all save the last number, where I wil suppose that he had 10 s for his wages cleare. And now to gesse at the nüber of the daies that he wrought, I suppose first that he wrought 20 dayes, then say I, if he wrought 20 daies, his wages must be 500 d, then did he play other 20 daies, for which must be abated 600 d, and then he looseth 100 d. And so am I at a stay, for it is not like unto your former worke.

Maister. You should have required of mee some question, and not have taken a question of your owne fantasying, untill you were more expert in this Art: for so might you as well happen on an impossible question as on a possible: but now to go forward, consider that this number is too little by 220, seeing he should gaine by your supposition 120 d, and in this position hee leeseth 100, those both make 220

which you shall set downe for the first errour with this signe—, betokening too litle, as here in this forme following doth appeare.

And now for the rest go forward your selfe once againe.

20

220—

Scholer. As my errour hath vttered my folly, so it hath procured me better vnderstanding. Now therefore considering this position not to solue the question, I take an other, supposing that he wrought 30 daies, then for his wages he must be allowed 750 d, and for the 10 daies which he wrought not, hee must abate 300 d, and so remayneth cleare 450 d: but it should be only 120 d, therefore is it too much by 330, which I set downe in the figure with the former position, and his error; and the figure appeareth thus.

20 30

220— 330+

Now must I multiply in crosse wayes 220, by 30, and it will bee 6600, Then againe I multiply 330 by 20, and it will bee also 6600. Wherefore if I shall subtract the one out of the other, there will remaine nothing to bee the Diuidend.

Maister. In this you forget your selfe againe: for in as much as the signes in the errours be vnlike, therefore must you worke by Addition, adding together those two totals to

make

make the diuidend, and also adding the two errors to make the diuisor. And because you shall no more forget this part of the rule, take this briefe remembrance:

Unlike require Addition,
And like desire Subtraction.

Scholer. You meane, that if the errours haue like signes, then must the diuidend and the Diuisor be made by Subtraction, as is taught before: and if those signes bee vnlike (as in this last example they be) then must I by addition gather the Diuidend and the Diuisor. Therefore must I adde 6600 to 6600, and it will be 13200, which shall be the diuidend. Then againe I adde 220 to 330 and it will be 550, which must be the Diuisor: wherfore diuiding 13200 by 550, the quotient will be 24, whereby I knowe that the Mason wrought 24 dayes, and then it followeth that he plaied 16 daies.

Master. Examine your worke whether it be agreeable to the question or no.

Scholer. For 24 dayes worke, the wages must be 600 d, and for 16 dayes which the Mason wrought not, there must be abated 480, and then remaineth cleare to the Mason 120 pence, as the question importeth: wherefore it is euident, that 24 is the true number of daies that he wrought.

Master. Although you seeme nowe to understand this worke, yet to acquaint your mind the better with the new trade of this rule, I thinke it good to propone to you fiue or sixe examples more, before I make an end of it.

Scholer. Sir I thanke you that you doe so consider my commoditie and profite in knowledge, for vndoubtedly it is practise and exercise that maketh men prompt and expert in euerie kinde of knowledge.

Master. You say well so that they followe some certaine preceptes to gouerne and rule their practise by, else may practise procure custome or error, and a repugnance to exactnesse of knowledge. namely as long as the error is not plainely knowne to the vulgar sort. But to returne to our worke.

A question of wares. There is a seruant that hath bought of veluet and damaske for his master 40 yardes, the veluet at 20 s a yard, and the Damaske at 12 s, and when he commeth home, his Maister demaundeth of him how much he hath bought of each sort: I cannot tell (saith he) exactly, but this I know, that I paide for Damaske forty eight shillings more then I payed for veluet, now must you gesse howe many yardes there is of each sort.

Scholer. Although the gesse seemeth difficult, yet I will proue what I can do: for I remember your saying, that it forceth not how fonde or false the gesse be, so it be somewhat

what to the question, and not to answer of a contrarie matter.

Therefore first I imagine that he bought 20 yardes of Damaske, for which hee should pay after the former price 240 shillings: then must he needes haue of veluet other 20 yardes (to make vp the 40 yardes) and that would cost 400 s. So that the totall of the price of the Damaske is lesse then the summe paide for veluet 160 s, and should be more by 48. Therefore the first errour is 208 too litle. Then begin I againe, and suppose hee bought of Damaske 30 yardes that cost 360 s., then had hee but 10 yardes of veluet, which cost 200 s: and now the price of Damaske is greater than the price of the Veluet by 160 shillings, and should be but 48, therefore is the second errour 112 too much, which I set in forme of figure as here doth appeare.

Then do I multiply in crosse waies 208 by 30, and the summe will bee 6240. Also I multiply 112 by 20, and there will amount 2240.

20   30

208—   112+

And in as much as the signes of the errours be vnlike, I know I must worke by Addition, therefore adde I those two totals together, and they make 8480, which is the diuidend: then adde I also the two errours together, 208 and 112, and they make 320, which is the diuisor. Where-

for dividing 8480, by 320, the quotient will be $26\frac{1}{2}$, which is the true summe of yardes of Damaske that he bought: and in Veluet 13 yardes $\frac{1}{2}$, and that appeared by examination thus: $26\frac{1}{2}$ yardes of Damaske at 12 shillings the yard maketh 318 shillings, then in veluet he had but 13 yardes and $\frac{1}{2}$ that cost 270 s at 20 shillings the yard. Now subtract 270 out of 318 and there will remaine 48, which is the number of shillings that the Damaske did cost more than the Veluet.

Master. Now shall you haue a question of an other kind.

There are thrée men that do owe money to me, and I haue forgotten what the totall sum is, and what the particulars be.

Scholer. Why, then is it possible to knowe the debt?

Master. Peace, you are too hastie: there is more helpe in it than you yet sée: I haue thrée seuerall notes, whereby it appeareth that I did conferre their debtes together, and found the debt of the first and the second to amount to 47 pound, the debt of the first man and the thirde man did make 71 pound, and the second man his debt with the third, did rise to 88 li. Nowe can you tell what euerie man did owe, & what was the whole totall?

A question of debt. The third example.

Scholer. Nay in good faith: but as I perceiue that it must be found by coniecture, so will I gesse at it, supposing that the first man did

did owe 20 ł, and the second man 30, and the third.

Maister. Nay stay there, for you are too far gone already, you may not suppose a severall summe for every man, for it is inough to suppose one summe for the first man, and let the other rise as the question importeth. Therefore seeing you set the first man his debt to be 20 ł, the second man cannot owe 30 ł, for the declaration is, that their debts added together did make 47 ł, so must the second man his debt bee but 27 ł. Now this second debt with the third must make 88, therefore subtract 27 out of 88, and there will remaine 61, as the third man his debt. Then saith the declaration, that the first and third mens debts doe make 71: but by this supposition they make 81, that is 10 too much: which I must set for the first error. Now worke you the second supposition.

Scholer. I suppose the first mans debt to bee 24 ł, then must the second mans debt (by your declaration) bee but 23 ł, seeing both they make but 47 ł. And the second man his debt with the third, doe make 88 ł, and the second man oweth but 23, therefore the third man must owe 65 ł. Now the third mans debt with the first, should make by the declaration 71 ł, and they doe make 89 ł: that is 18 ł too much: and that is the second errour, which I set downe with the first, and their position in this forme, and then doe I multi-

ply in crosse waies 20 by 18, and it is 360. Also 10 by 24 maketh 240. And because the signes of the errours be like, I must worke by Subtraction: therefore I subtract 240 out of 360, and there rest 120, which is the diuidend, then doe I subtract 10 out of 18 by the same reason, and so is the diuisor 8, which is found 15 times in 120, therefore I say that the first man did owe 15 l, and then the second man must owe 32 l, for those 2 doe make 47 l, and the third man his debt is 56, for so much remaineth if I bate 15 out of 71, or if I take 32 out of 88.

20 24
X
10+ 18+

A questiõ of Friendship. The third question.

Maister. For the fourth example take this easie question for the variety in worke. Two men hauing seuerall summes which I know not, do thus talke together: the first saith to the second, If you giue me 2 s of your money, then shall I haue 3 times so much money as you: the second answereth, It were more reason, that our summes were made equal, and so will it be, if you giue me 3 s of your money. Now gesse what each of them had.

Scholer. I imagine that the first had 9 s.

Note.

Maister. Consider euermore in your imaginations that you take a likely summe, as in this question take such a summe that hauing 2 added vnto it, may bee diuided into 3 parts euen.

Scholer.

Scholer. Why? I remember you sayd before, it forced not howe fondly soeuer I gessed.

Maister. As for the possibility of the solution it is truth, but for easinesse in worke, the aptest numbers are most conuenient.

Scholer. I thought no lesse, and therefore I tooke 9 as an apt number to bee parted into three: but I perceiue I should haue considered the aptnesse of that partition after the addition of two vnto it, and then 7 had bin more meeter.

Maister. What is truth, and then should the second man his summe bee 5: for although hee haue now but the third part of 9, that is 3, yet you must remember that hee lent the first man 2, and so had he 5.

Scholer. Then to goe forward: if the second man had three of the first man, then should hee haue 8, and the first man but 4, so hath hee double to the first man: yet he sayd in the question they should haue equall: wherefore it appeareth that hee hath foure too much. Therefore I note that errour with his supposition, and gesse againe that hee hath 10 s: whereunto I adde 2 shillings borrowed of the second man, and then hath hee 12 shillings, so the second man hath remaining but foure: whereunto I adde the 2 that hee lent to the first man, so had he but 6 s at the beginning. Then take 5 shillings from the first man, and

giue to the ſecond, then hath the firſt man but 7, and the ſecond hath 9, which are not e-quall, but there are two too many, wherefore I ſet downe both the poſitions with their errors, as here you ſee, and mulply a croſſe, ſo commeth

7 10

4+ 2+

there 40 and 14: and becauſe the ſignes bee like, I take 14 out of 40, and ſo reſteth 26 to be the diuidend, then likewiſe I take 2 out of 4, and there reſteth 2, by which I diuide 26, and the quotient will be 13, which is the ſumme that the firſt man had. And ſo appeareth that two being added thereto, the ſumme will bee 15, ſo hath the ſecond man now but 5, and before hee had 7: then take three from the firſt, and put to his ſeuen, ſo haue each of them 10, and that is equall, as the queſtion would.

A queſtiō of lambes. The fift example.

Maiſter. For the fift example take this queſtion. One man ſayd to another: I thinke you had this yeare two thouſand Lambes: ſo had I ſayd the other: but what with paying the tithe of them, and then three ſeuerall loſſes, they are much abated: for at one time I loſt halfe as many as I haue now left: and at an other time the third part of ſo many: and the third time $\frac{1}{4}$ ſo many. Now geſſe you how many are left.

Scholer. Becauſe here is mention made of certaine partes, I muſt take a number that may

may haue all those parts: that is to say $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$, which will be 24, howbeit 12 hath the same partes. Therefore, first I take 12 to be the number that doth remaine, so hath he lost 6, 4, and 3, that is 13, and in the whole 25, but it should be 2000.

Master. Ye are deceiued yet still: you haue forgotten the 10 part, which must be defalked, that is 200, so there remaineth but 1800, and now go on againe.

Scholer. Then to finde the errour, I take 25 out of 1800, and there remaineth 1775 to fewe, which I set in the first errour. Then for the second position I take 24, whose halfe is 12, the thirde parte 8, and the quarter 6, whereby riseth 50, which is to litle by 1750, therfore I set downe both ŷ positions with their errours thus. — A question of sheepe and tillage. The sixth example.

12   24

1775—   1750—

And multiplie in crosse wayes 1775 by 24, whereof commeth 42600. Also I multiplie 1750 by 12, and there ariseth 21000. And because the signes are like, I doe subtract the one from the other, and so remaineth the Diuidend 21600: then doe I subtract 1750, out of 1775, and there resteth 25: by which I diuide 21600, and the Quotient is 864, whereof the halfe is 432, and the third part is 288, the quarter is 216, which all being

These

added together will make 1800. And if you adde thereto the tenth which was abated before, then wil the whole summe be 2000. And now doth there come a question to my memorie which was demanded of me, but I was not able to answer to it, and now me thinketh I could solue it.

864
432
288
216
1800

Master. Propone your question.

A question of sheepe and tillage. The sixth example.

Scholer. There is supposed a lawe made, that (for furthering of tillage) euerie man that doth keepe sheepe, shall for euerie tenne sheepe eare and sowe an acre of ground: and for his allowance in sheepe pasture, there is appointed for euerie foure sheepe one acre of pasture. Now is there a rich sheepe-master which hath 7000 acres of grounde, and would gladly keepe as many sheepe as he might by that Statute: I demaund how many sheepe shall hee keepe?

Master. Aunswer unto the question your selfe.

Scholer. First I suppose he may keepe 500 sheepe, and for them he shall haue in pasture after the rate of foure sheepe to an acre, 125 acres, and in arable ground 50 acres, that is 175 in all: but this errour is to litle by 6825. Therefore I gesse againe, that he may keepe 1000 sheepe, that is in pasture 250 acres: and in tillage 100 acres, which maketh 350: that is to litle by 6650.

These

500 1000

6825 — 6650 —

These both errours with their positions I set downe as you see, and multiply in crosse 6825 by 1000, and it maketh 6825000. Then I multiplie 6650 by 500, and it doeth amount to 3325000, which summe I doe subtracte out of the first, and there remaineth 3500000 as the Dividend. Also I doe subtract the lesser errour out of the greater, and so remaineth 175 by which I divide the same dividend, and the quotient will be 20000, so that I see that by this rate he that hath 7000 acres of ground, may keepe 20000 sheepe: and thereby I conjecture, that many men may keepe so many sheepe, for many men (as the common talke is) have so many acres of ground.

Master. That talke is not likely, for so much ground is in compasse above 48 $\frac{3}{4}$ miles: leave this talke and returne to your questions, least your positing be scarce well taken.

Scholer. Indeede I do remember, that the Egyptians did grudge so much against sheepheards, till at length they smarted for it, and yet they were but small sheepmasters to some men that be now, and the sheepe are waxen so fierce now and so mightie, that none can withstand them but the Lion.

Master. I perceive you talke as you heare

An other way of working, vpon the 6. example

some other: but to the other worke of your question; both this last question, and the next before might be wrought without the second position, by the rule of proportion, as this. When in this question ye found in the first errour, that for 500 sheepe, there must be 175 acres, then might you reduce it to the golden rule, thus.

175 — 500
7000 — 20000

If 175 Acres will admit in allowance 500 sheepe, then 7000 will haue 20000. And so by one position with the helpe of the Golden Rule may you answer that question. Likewise for the question of Lambs, when you had found that 12 came of 25, you might haue set the figures thus as you see, and haue sayd:

If 25 do leaue but 12, what shall 1800 leaue? and it would appeare to be 864.

25 — 12
1800 — 864

Scholer. Sir, I thanke you for this order, for it doeth much shorten the worke of this Rule.

An other way yet of working vpon the 6. question

Master. Yet againe I will shewe you an other way, to answere to this last question without this Rule of False position, and that by the Rule of Felowship, for it appeareth in the proponing of the question, that 100 sheepe must haue in pasture two acres and ½, and for them there must be eared but one acre: so it fol-

followeth, that for two acres eared, there must be 5 set to pasture. And if you put them both into one summe, they will make 7. Therefore looke what proportion 7 being this totall, doth beare to 5 and to 2, such proportions shall any totall in this question beare to the pasture ground, and to the eared ground.

Scholer. This serueth wonderous aptly. Therefore to proue it, I demaund this by the former supposition: If a man haue 300 Acres, how much shall he leaue in pasture, and how much shall he turne to tillage? You say that as 7 is to 5, so shall three hundred bee to the Acres of pasture: and as 7 is to 2, so is 300 to the Acres of tillage, whereof for both I haue sette examples here following, whereby appeareth that of Pasture there shal be $214\frac{2}{7}$ Acres and of tillage $85\frac{5}{7}$, which both summes added together, doe make 300.

7 — 5
300 — $214\frac{2}{7}$

7 — 2
300 — $85\frac{5}{7}$

Maister. Now take an other example: A man hath three siluer cups with one couer, the couer weigheth eighteene ounces, the second cup weigheth euen halfe the weight of the first and the third. Now if the couer bee put to the first cuppe, they weigh iust as much as all the three cups doe weigh: and if the couer bee ioyned with the second cup, they weigh as much as the second twise, and the third; and if the

An other question, of the proportion of the weight of 2. cups, the 7 example.

couer be put to the third cup, they will make twise as much as the first and the second cup. Now try you what was the iust waight of euery cup.

Scholer. I doe set the weight of the first cup to be nine ounces: then in as much as these two (that is to say, the couer and the first cup) doe weigh the weight of the three cuppes, I see that the three cups must weigh 27 ounces, for so much is 18 and 9. Also because the first and the third doe weigh double so much as the second, therfore is it the third part of that weight, that is nine, and then would it follow, that the third cup also should weigh nine ounces, but then the question saith, that the couer being ioyned to the second cuppe, they weigh as much as the second twise, and the third once, they should bee seuen and twenty, and so it doth: then being ioyned with the third cup, that should weigh twise as much as the first and the second, that should be thirty sixe, and they weigh but 27, so is that errour nine to little. Then begin I againe and say, that the first cup doth weigh twelue ounces, which I ioyne with the couer, & they make 30 ounces: then seeing the second is $\frac{1}{3}$ of that weight, it must needes weigh 10 ounces, and the third must weigh 8 ounces, seeing the first and the third must weigh 20 ounces. Now put I the couer to the second cup, and they weigh 28 ounces, which should bee euen so: then ioyne I the couer with the third cup, and so should it

it weigh twise the first, and the second, that is 44 ounces, and they do weigh but 26, that is 18 too little: those errours

9 12

9— 18—

with their positions I set downe, and multiply in crosse waies 9 by 12, whereof commeth 108. Also 9 by 18, and it yeeldeth 162: and in as much as the signes bee like, I abate the lesser out of the greater, and there doth remaine 54. Then do I also abate the lesser errour from the greater, and so remaineth 9, by which I diuide 54, and the quotient is 6: which I take for the true weight of the first cup: which being ioyned with the couer must weigh as much as the three cups, so doe they weigh but foure and twenty ounces. Then seeing the second cup is the third part of that weight, for the other two cuppes (you say) must weigh double his weight, the weight of the second cup is eight ounces, and so the weight of the third must be 10 ounces. Now put the couer to the second cuppe, and it will make sixe and twenty ounces: that must bee the weight of the second twise, and the third ounce, that is twise eight, and once 10, and so it is. Againe, put the couer to the third cuppe of tenne ounces, and they must weigh twise as much as the first and the second, that is 28; and so is all agreeable.

A question of water. The eight example.

Maister. Then answere to this question.

Three is a Cesterne with foure cockes,

containing 72 barrels of water: and if the greatest cocke be opened, the water will avoyd cleane in sixe houres: at the second cocke it will aske eight houres: at the third cocke it will avoide in no lesse than nine houres: and at the smallest it will require twelue houres: Now I demaund, in what space will it avoide, all the cockes being set open?

Scholer. First I imagine that it will auoide in two houres.

Maister. Then must there auoide by the first cocke $\frac{1}{3}$ of the water, that is 24 barrels, and by the second cocke $\frac{1}{4}$ that is 18, and by the third cocke $\frac{2}{9}$ that is 16 barrels, and by the smallest cocke $\frac{1}{6}$, and this 12 barrels, all which summes put together doe make 70, as by their addition it doth appeare, but it should be 72, therfore the errour is 2 too few.

Scholer. Then I beginne againe by your fauour, because I thinke I vnderstand the worke, and put three houres for the due time: so shall there runne out at the greatest cocke $\frac{1}{2}$, that is 36 barrels, and at the second hole $\frac{3}{8}$,	24 18 16 12 70

that is 27, and at the third cocke $\frac{1}{3}$ that is foure and twenty, and at the smallest hole $\frac{1}{4}$, that is 18 barrels which altogether doe make 105, and should be but 72, so is it too much by 33, therefore doe I set the errors in order of the figure with their positions, and worke by multiplication,

in crosse, saying: 2 times 3 is 6, and 2 times 33 maketh 66: and because the signes are unlike, I must adde those two totals together, which make 72: also I adde the two errours, and they make 35, by which I divide 72, and the quotient riseth $2\frac{2}{35}$, whereby I see that all the Cockes being set open, the water will avoid in two houres, and $\frac{2}{35}$ of an houre.

2 3
X
2—33+

Master. This exercise maketh you to grow expert in the rule. Therefore I will inure you somewhat more with a question or two.

There were two men which had bene partners, and had in account betweene them 300 duckets: whereof the one should have for his part 180, and the other 120: but in the parting of them they fell at variance, so that each of them catched as many as he could: yet afterward being reconciled, they agreed that hee which had gotten most part of them should lay downe $\frac{1}{2}$ of them againe, and he that had gotten least should lay downe $\frac{1}{3}$ of those which he had taken, and then parting them unto two equall partes, each man to have halfe thereof, and so had they their iust portions, as they ought: now I demaund of you what each of them had gotten by the scambling. A question of partners. The ninth example.

Scholer. I suppose he that had least, got

108 duckets, then the other had 192: wherefore in laying downe againe of the 192, there was put downe $\frac{3}{4}$ that is 144, and so had hee left but 48. Also of the 180 there was laied downe 36, that is $\frac{1}{5}$, and so hath he left 72. Then I put together 144, and 36, and it maketh 180 which I part into two partes euen, and so commeth 90 to be giuen to each of them: which sum put to 72, maketh 162, and ioyned to 48, it maketh 138: and now I doubt how I shall go forward.

Note. Maister. You néede not to take but one of them which you list, the greater or the smaller, for all commeth to one purpose; and so may you compare it that you take to any of the other summes, remembring that you make comparison to the same in the second worke, as for example of the first parte, if you compare 138 with the lesser summe due, that is 120, so is it 18 too much: and if you compare it with the greater summe, then is it 42 too little. Againe, if you compare 162 to the greater summe, the error will be 18, as it was in the other: but it will haue a contrarie signe: and if you compare it with the lesser summe, it will be 42 too much: so that the errour both waies is either 18 or 42: and as for the signes it litle forceth, for in them is nothing considered here, but likenesse and vnlikenesse, which in this case doth neither further nor hinder. But now go on with the worke.

Scholer.

Scholer. If it be so, then am I out of my greatest doubt. Then I ioyne that 90 (which I founde as the halfe of the latter partition) vnto 48, which is left with the one man, and so hath hee 138, which (I may say) is 18 to many, for the least should be but 120: that errour do I note, and then make a new position, supposing the one man to haue 204, and the other to haue 96, wherefore of the 204 there must be layed downe 153, and so remaineth with him 51. Also of the 96 there must bee layed downe $\frac{1}{3}$, that is 32, and so resteth with that man 64. Now of the 153 and 32 I make one summe as 185, which I must diuide into two equall partes, and so each man shall haue $92\frac{1}{2}$, whereunto if I adde their former portions reserued, then the one shall haue $156\frac{1}{2}$, and the other hath $143\frac{1}{2}$. Wherefore I take the lesser summe now againe, as I did before, that is $143\frac{1}{2}$, and finde that he hath to many by $23\frac{1}{2}$, for he should haue but 120, so haue I for my two positions two errors, which I set downe, as here may be seene, each error vnder his position, and then by the Rule I doe multiply in crosse waies 108 by $23\frac{1}{2}$ and there riseth 2538 which I note, then againe I multiplie 96 by 18, and thereof amounteth 1728.

108 96

18+ $23\frac{1}{2}$+

Now because the signes are both like, that is, both too many, I must worke by Subtraction, and so abating 1728 out of 2538, there will rest for the Dividend 810: then for the Diuisor I subtract 18 out of 23 $\frac{2}{3}$, and there remaineth 5 $\frac{2}{3}$, by which I diuide 810, and the quotient will be 147 $\frac{1}{11}$, which is the iust portion of him that had the least summe. And if I doe subtract it out of 300 being the totall summe, then will there remaine 152 $\frac{10}{11}$ as the portion that the other did get.

Master. For the profe of this worke, you may choose whether you will examine those numbers according to the forme of the question, or else worke by other two positions for to finde the second number: and if those positions bring the same numbers that did amount by the two first positions, then doth each worke confirme other.

Scholer. By your patience, I will proue both waies, not onely to see their agreement, but also to accustome my minde to those works: for I perceiue it is exercise that must bee the chiefe engrauer of these rules in my memorie.

Master. You consider it well: then goe to.

Scholer. First I will by two other positions trie to finde the portion of him which had most.

Master. Although you may do it with any positions, yet to see the agreement of your

worke

worke the better, take the same positions that you did before, comparing them now to the greater, as you did before unto the lesser.

Scholer. Then I suppose, that hee that had most, had 192, so had the other 108. Now if I take $\frac{1}{4}$ out of 192, that will be 144, and there will rest to that man but 48. And from the second which had 108, if I take $\frac{1}{3}$, that is fire and thirty, there will remaine to him 72: then joyning 144 with fire and thirty, it will make 180, the halfe whereof being 90, if I adde to each of those two mens portions remaining with them, the one shall have 138, and the other 162, of which two I take the greater (that is 162) and see it to be eighteen too few, for it should be 180 that errour I note under his position. Then for the second position I take (as I did before) 204 for the one, and so resteth 96 for the other: then take I $\frac{3}{4}$ of 204, and it will be 153, and there resteth to him one and fifty. Also of the 96 I take $\frac{1}{3}$, that is 32, and there remayneth to him 64. Now put I that 32 to 153, and it yeeldeth 185: which being parted in equall values, maketh $92\frac{1}{2}$ to be added to each mans remainder, and so the one hath $143\frac{1}{2}$, and the other $156\frac{1}{2}$: wherfore I take the greatest summe and it is three and twenty $\frac{1}{2}$ too little, that doe I note also, and set both these errours under their positions, as in this example following doth appeare.

And then multiplying 192 by $23\frac{1}{2}$, there

doth arise 4512. Againe, I multiply 204 by 18, and it maketh 3672, which I do subtract out of 4512,

192	204
18 —	$23\frac{1}{2}$

because the signes be like, and there resteth 840 for the diuidend: then subtracting 18 out of $23\frac{1}{2}$, there will remaine $5\frac{1}{2}$, which I must take for the diuisor. And so diuiding 840 by $5\frac{1}{2}$, the quotient will be $152\frac{8}{11}$, whereby I haue found an agreable summe to that which I found by the former positions, for him that had most, which if I doe subtract out of 300, that is the totall, there will rest $147\frac{3}{11}$, which was the portion of him that had the least part.

Maister. So by diuerse positions you see that one doth confirme the worke of the other. Now examine those two numbers by the forme of the question, and so shall you prooue your work good also.

Scholer. If that hee which got most, hadde $152\frac{8}{11}$, then must he lay downe $\frac{3}{4}$ of his sum, that is $114\frac{6}{11}$, and so shall remaine with him but onely $38\frac{2}{11}$. The other that had least, that is $147\frac{3}{11}$ must put downe of his sum $\frac{1}{3}$, that is $49\frac{1}{11}$, and so doth there remaine with him yet $98\frac{2}{11}$. Then do I adde together $114\frac{6}{11}$ and $49\frac{1}{11}$, and it will make $163\frac{7}{11}$, which I must part into two equall partes, and that wil be $81\frac{2}{11}$, to be giuen to each of them: so putting 81 [illegible] unto 38 [illegible], there doth amount 120 iust, which

which is the true portion of him that should haue the lesser summe: and adding 81 $\frac{2}{11}$ to 98 $\frac{9}{11}$, the totall will bee 180, the true portion of the other. And so is the worke by this proofe also tried to bee good. And this I marke by the way, that in their scambling hee got most (as it chanceth often) that ought to haue had least by iust partition.

Maister. Let your study be to learne truth and iust Art of Proportion, and to distribute and part according thereunto, as often as occasion shall be ministred. And here would I make an end of this rule, saue that I remember one pleasaunt question which I can not ouerpasse, which I will declare somewhat largely, because you shall as wel vnderstand some reason in the pleasant inuention, as apt proceeding in the witty working thereof.

An example of mixture of gold and siluer. The ninth example.

Hiero king of the Syracusans in Sicilia, had caused to be made a Crowne of Gold of a wonderfull weight, to bee offered for his good successe in warres: in making whereof, the Goldsmith fraudulently tooke out a certain portion of Gold and put in Siluer for it, so that there was nothing abated of the full weight, although there was much of the value diminished. Which thing at length being vttered, (as no euill can alwaies lie hid) the King was sore mooued, and being desirous to try the trueth without breaking of the Crowne, proponed the doubt to Archimedes, vnto whose

witte nothing seemed vnpossible, which although presently hee could not aunswere vnto, yet hee had good hope to deuise some policy for that inuention. And so musing thereon, as hee chaunced to enter into a baine full of water to wash him, he obserued that as his body entred into the baine, the water did runne ouer the tubbe: whereby his ready wit of such small effectes coniecturing greater workes, conceiued by and by a reason of solution to the Kings question, and therefore reioycing exceedingly more than if he had gotten the Crowne it selfe, forgate that he was naked, and so ran home, crying as he ran, εὕρηκα, εὕρηκα: I haue found, I haue found. And thereuppon caused two masse peeces, one of Gold, and an other of Siluer to be prepared of the same weight that the said Crowne was of: and considering that Gold is heauier of nature than Siluer, and therefore Gold of like weight with Siluer must needes occupy lesse rowme, by reason it is more compact and sound in substance, hee was assured, that putting the masse of Gold into a vessell brimme full of water, there woulde not so much water runne ouer, as when he should put in the Siluer masse of the like weight. Wherefore he tried both, and noted not only the quantities of the water at each time, but also the difference or excesse of the one aboue the other, whereby he learned what proportion in quantity is betweene Gold and Siluer of equall weight.

weight. And then putting the Crowne it selfe into the vessell of water brimme ful (as before) marked how much water did runne out then, and comparing it with the water that ranne out when the Golde was put in, noted how much it did exceede that: and likewise comparing it to the water that ranne out of the Silver, marked how much it was lesse than that: and by those proportions found out the iust quantity of Gold that was taken out of the Crowne, and how much Silver was put in stead of it. But seeing Vitruuius which writeth this History, doeth not declare the particular worke of this triall, it shall bee no inconvenience to suppose an example for declarations sake, wherein although the true and iust proportions be not expressed, yet the forme of triall shall be truely set foorth. And for an example, I suppose the weight of the Crowne to be 8 pound, and so of each the other two masses. And when the masse of Gold was put into the water, I imagine that there ranne out 2 pound of water: and when the masse of Silver was put in, I suppose there ranne out three pound $\frac{1}{2}$. Againe when the Crowne was put in, there ran out 2 pound $\frac{1}{2}$. Now to know what quantity of silver was in the Crowne, worke by the rule of false position, and imagine that there was two pound of Silver: then must there be 6 pound of gold, then say thus by the rule of proportion: If 8 pound of gold do expell 2 pound of water, what

shall 6 pound expell? and it will be 1 pound $\frac{1}{2}$. Againe of the siluer: if 8 pound of siluer expell 3 pound $\frac{1}{2}$ of water, what shall 2 pound of siluer put out? it will be $\frac{7}{8}$. Now adde those two weights of water together, and they will make 2 pound $\frac{3}{8}$, and it should be by the supposition two pound $\frac{1}{4}$, so is it to much by $\frac{1}{8}$.

Scholer. Now do I vnderstand the worke as I thinke, therefore I pray you let me worke the rest of the question. And because this first supposition did erre, I note that position, and his errour, and take a new position, esteeming the siluer to be but one pound, so must there be in Golde 7 pound. Then say I: If 8 pound of Golde do yeelde two pound of water, what shal 7 pound yeeld? and it will be 1 pound $\frac{3}{4}$. Againe, if 8 pound of siluer expell 3 pound $\frac{1}{2}$ of water, what shall one pound expell? and it will be $\frac{7}{16}$. Now must I adde those two summes together, and they make 2 pound $\frac{3}{16}$, and they should make 2 pound $\frac{1}{4}$, so is it to litle by $\frac{1}{16}$. Therefore I set the positions with their errours in order as heere followeth. And then I multiply in crosse wise two by $\frac{1}{16}$, and it maketh $\frac{1}{8}$: likewise one multiplied by $\frac{1}{8}$ maketh $\frac{1}{8}$. And because the signes be vnlike, I must adde those two summes which make $\frac{1}{4}$, and that in the diuidend. Againe I must adde $\frac{1}{8}$ to $\frac{1}{16}$, and it will be

$$\begin{array}{ccc} 2 & & 1 \\ & \times & \\ \frac{1}{8} & + \; - & \frac{1}{16} \end{array}$$

be $\frac{1}{16}$ that is the Diuisor. Now I shall diuide $\frac{1}{12}$ by $\frac{1}{16}$, and the quotient will bee $\frac{16}{12}$, that is 1 $\frac{1}{3}$, whereby I know that there was put 1 l. and $\frac{1}{3}$ of siluer into the Crowne, and so much gold taken out for it.

Master. Proue it now by examination according to the question.

Scholer. If there were 1 pound $\frac{1}{3}$ of siluer, then was there of Gold 6 pounds $\frac{2}{3}$. Now say I by the Rule of proportion:

If eight pounds of Gold expell two poundes of water, what shall 6 pounds $\frac{2}{3}$ expell?

8 — 2
6 $\frac{2}{3}$ — 1 $\frac{2}{3}$

It will be 1 pound $\frac{2}{3}$. Again, If eight poundes of siluer expel 3 poundes $\frac{1}{2}$ of water, what shall 1 $\frac{1}{3}$ expell? It will be $\frac{7}{12}$.

8 — 3 $\frac{1}{2}$
1 $\frac{1}{3}$ — $\frac{7}{12}$

Now must I adde together 1 pound $\frac{2}{3}$, and $\frac{7}{12}$, and they will make 2 poundes $\frac{3}{12}$, that is two pounds $\frac{1}{4}$, according to the supposition of the question, whereby I perceiue the worke to be well done. And as I cannot but much reioyce of this excellent inuention, so my desire is kindled vehemently to bee perfectly instructed in euery part thereof, and namely in this point, whether the porportion betwéene water & gold be such, that for 8 poundes of gold into a vessell full of water there shall runne out two pounds of water: and for as much siluer, whether thrée pounds $\frac{1}{2}$ of water would auoide? Note.

A question of the proportion of gold, siluer and quicksiluer into water.

Master. I perceyue your meaning, and

coniecture your imagination to be thus: that if you knew the exact proportion betweene gold and siluer and water, both in their weight and quantities, then could you easily finde out the mixtures of them, which thing I haue reserued for another worke that entreateth of such matters especially. And at this time you must consider, that you learne Arithmeticke, which entreateth of the manner to solue doubtfull questions touching number, without regard what matter is signified by that number, else were it necessary in Arithmeticke to teach all Artes, seeing in it may bee moued questions of all Arts. But seeing you are so desirous to know this thing, I will tell it you in such a sort, that you shall practise your Art in finding it, and propound it in forme of a question. Gold beareth greater proportion to water than siluer doth, and their two proportions be in proportion together as $\frac{41}{31}$. But to helpe you somewhat in this riddle, you shall note that the proportion of quicke siluer vnto water, is the iust middle number proportionall in Progression Geometricall, betweene the proportions of Gold and siluer into water. And this proportion is $\frac{190}{31}$. Now if you will know the iust numbers of these two proportions, then must you finde out 3 numbers in Progression Geometricall, whereof the middlemost must be $\frac{190}{31}$, and the first must be vnto the last, as 25 to 48. And thus I will leaue you to finde those numbers when

Note.

1 question of the proportion of gold, siluer and quicksiluer into water.

when you be at leysure.

Scholer. Yet sir, I thanke you heartily for thus much, for now I see the possibility to find them out. Howbeit, because this question seemeth strange, if it might please you to instruct mee somewhat in the order of working for it, I should the more easily finde the true working.

Maister. You desire too much ease if you will study for nothing: therefore to occasion you to study the better, I will leaue this doubt wholly to your owne search. But as touching the generality of the rule. Archimedes needed not to take two masses of Gold and Siluer equall in weight with the Crowne, for the proportion might as well bee found in any other weight, yea although the masse of Gold were of one weight, and the masse of Siluer of another. As for example: If the crowne were of 8 poundes weight, as I did suppose, & I haue not so much other fine Gold, but onely one pound, and trying that by water, and finding that it doth expell but $\frac{3}{4}$ of an ounce of water, yet then by it may I inferre, that 8 poundes of Gold would expel 6 ounces of water. And likewise of the siluer: whereof if I had but two pounds, and find that it doth expell three ounces of water, then might I affirme that 8 pounds would expell 12 ounces that is, one poũd weight: & so is it as good as if ẏ 3 masses were al of one weight. And thus

for this time I will make an end of this other part of Arithmeticke.

Scholer. Although I can not sufficiently thanke you for this, yet your promise made me to looke for the Art of extraction of rootes, whereof hitherto I haue learned nothing.

Maister. I will not breake my promise, but intend (God willing) to performe it within these three or foure moneths, if I perceiue this my paines to be well taken in the meane season. And you shall not repent the tarrying for it: for it shall bee encreased by the tarrying. And in the meane time, you shall take this Addition, not for the second part of Arithmeticke which I promised, but for an augmentation of the first part, unto which I could haue annexed the extraction of rootes, square and cubike, namely for examples of the Statute of Assise of wood, but that in the second part I must write of diuerse other rootes, and thought it best to reserue those Rules also with their examples unto the same second part.

Scholer. Sir, although I cannot recompence your goodnesse, yet I shall alwayes doe mine endeuour to occasion you not to repent your benefite on me thus employed.

Maister. That recompence is sufficient for your part.

FINIS.

THE THIRD PART

or Addition to this Booke, entreateth of briefe Rules, called Rules of Practise, of rare, pleasant, and commodious effect, abridged into a briefer Method than hitherto hath bene published.

hath been published

VVith diuerse other necessarie Rules, Tables, and Questions, not onely profitable for Merchants, but also for Gentlemen, and all other occupiers whatsoeuer, as by the Contents of this Booke may appeare.

With

Set forth by Iohn Mellis Schoolemaister.

The first Chapter of this Addition entreateth of briefe Rules, called Rules of *Practise, with diuerse necessary Questions*, profitable, not onely for Merchants, but also for all other Occupiers whatsoeuer.

Rule.

The working of Multiplication in Practise, is no other thing than a certaine maner of multiplying of one kinde by another: whereupon is brought foorth the product of the proponed number, which is accomplished by meanes of diuision in taking the half, the third, the fourth, the fift, or such other parts of the summe which is to be multiplyed.

And for the better vnderstanding of such conuersions: you shall vnderstand that in the manner and vse of these rules of Practise, you ought first to know the euen or aliquot parts of a shilling, which in this Table following doth appeare.

$$\text{Item d}\left\{\begin{array}{c}6\\4\\3\\2\\1\end{array}\right\}\text{in the}\left\{\begin{array}{c}\frac{1}{2}\\\frac{1}{3}\\\frac{1}{4}\\\frac{1}{6}\\\frac{1}{12}\end{array}\right\}\text{of a s.}$$

Where-

Wherein as you see according to the order of these rules of Practise, at 6 pence the yard of any thing, you must take the ½ of your number which is to be multipled, and the product that commeth thereof shall be shillings, if any units doe remaine it is 6 pence.

For 4 pence take the ⅓ of the number that is to be multiplied, and the product also produceth shillings, if any unites doe remaine, each one shall be worth in value 4 pence. The like is to be understood of the other 3, &c.

Example 1	IIII
At 6 pence the yard: what 379 yards?	At 2 pence the yard: what 532 yards?
189 s. — 6 d.	88 s. — 8 d.
II	V
At 4 pence the yard: what 104 yards?	At 1 peny the yard: what 409 yards?
34 s. — 8 d.	34 s. — 1 d.
III	
At 3 pence the yard: what 5014 yards?	
1253 s. — 6 d.	

Here you may see in the first example, that 379 yards at 6 pence the yard, are worth 189 s. 6 pence, in taking the ½ of 379. And in the second example the 104 yards, at 4 pence the yard, are

worth 34 s 8 d in taking the ⅓ of 104. Likewise in the third example 5014 yards, at 3 d the yard bringeth forth 1253 s 6 d, in taking the ¼ of 5014. Also in the fourth example at 2 pence the yard, maketh 88 shillings 8 pence.

And lastly in the 5 example: 409 yards at 1 d the yard, amounteth to 34 s, and 1 d, in taking the $\frac{1}{12}$ of 409. And so is to be done of all other questions the like, when the number of the pence is any of the euen or aliquot parts of 12 d.

Item to bring the products of these shillings and all other the like into pounds is very easie in diuiding of it into your minde by 20, for it is to be understood, that as often as 20 is found in that product, so many poundes doth it containe, which with facility to performe, alwaies strike off the figure toward your right hand, with a right downe dash of your pen for the 0 that appertaineth to 20. And then begin at the left hand, in taking the ½ of the rest. And if at the last any unite doe remaine, the same shall bee ioyned with the figure that is cut off, which shal represent the odde shillings contayned in that worke.

As for example in your third question at 3 d the yard, which amounteth to 1253 s 6 pence, the product whereof maketh 62 l 13 s 6 d: as here you may see is easily performed by this example.

	1
	125|3
	62—13—6

Item also for the working of 1 penny the

yard

yard, it is something harsh and hard to take the $\frac{1}{3}$ of some products: Therefore to ease that hard worke, you shall first bring your deliuered sum into groats, by taking the $\frac{1}{2}$ part of the product. And if any vnites remaine of that $\frac{1}{4}$ part, as sometimes there may, they are pence: and must be signified with a line from the groates with their title of pence. And because that 60 groats maketh a pound or twenty shillings, strike off the first figure toward your right hand for the 0 that appertaineth to 60 (as you did euen now for the 0 that belongeth to 20:) then in taking the $\frac{1}{3}$ of that product, if there doe remaine any vnites, the same shall you ioine with the figure that you cut off, esteeming them as groates: which keepe in your minde, And by taking the $\frac{1}{3}$ part of them, you shall turne into shillings: and so haue you done: as for example by a question or two hereafter proponed shall more plainely by the worke appeare.

At 1 d the yard what 54368 yards?

1359 | 2 groates.

$\frac{1}{6}$ l — 226 — 10 s — — 8 d

Here in taking the $\frac{1}{6}$ part of 1359: in comming to the last worke the $\frac{1}{6}$ part 39 being taken, the remainer is 2, which ioyned with the 2 that was cutte off, maketh 32 groates: which conuerted into shillings by taking the $\frac{1}{3}$ part: maketh as appeareth tenne shillings

8 pence. Many other wayes there are, but none more apt for a yong learner to vnderſtand than this: wherefore this one way well empreſſed in memory, is better than 20 wayes doubtfully vnderſtood.

At 1 peny the yard, what 4533 yards?

113 | 3 groates — 1 d

li 18 — 17 — 9 d

At 1 peny the yard, what 64768 yardes?

1619 | 2 groates.

li — 269 — 17 — 4 d

2. Rule.

NOw followeth alſo to be vnderſtood that if the number of pence be not an aliquot part of 12, you muſt reduce them into ſome aliquot part of 12. And after the aforeſayde manner, you ſhal make of them 2 or 3 products as needs ſhall require, and adde them together into one ſumme. And here for thy furtherance appeareth a note of the order of their parts, as they are to be taken.

For pence	5. take	3 & 2	or	4 and 1
	7. take	4 & 3		6 and 1
	8. take	4 & 4		6 and 2
	9. take	6 & 3		4 4 & 1
	10. take	6 & 4		4 4 & 2
	11. take	6.4 & 1		4 4 & 3

Here in the firſt note of this Table at 5 pence, you ſhall firſt take for 3 d the $\frac{1}{4}$ of the number that

that is to be multiplied: & likewise for 2 pence the $\frac{1}{6}$ of the same number, adding together both the products. But if you will worke by 4 and 1, you must for 4 pence, first take the $\frac{1}{3}$ of the number that is to be multiplied And for 1 d take the $\frac{1}{12}$ of the whole summe, or rather which is more better, for 1 d, you may take the $\frac{1}{4}$ of the product which did come of the 4 pence: because that 1 d is the $\frac{1}{4}$ of the 4 pence: The totall summes of these two numbers shall bee the solution to the question. And in like maner is to be done of all others, as by these examples following shall appeare.

I

At 5 pence the yard what 758 yards?

3 d	189 — 6 d
2 d	126 — 4 d
	315 s — 10 d

Otherwise.

At 5 pence the yard what 758 yards?

4 d	252 — 8 d
1 d	63 — 2 d
	315 s — 10 d

II

At 7 pence the elle, what 563 elles?

4 d	187 — 8 d
3 d	140 — 9 d
	328 s — 5 d

III

At 8 pence ỳ pound, what 112 pounds?

4 d	37 — 4 d
4 d	37 — 4 d
	74 s — 8 d

Otherwise.		V	
At 8 pence ỹ pound what 112 pounds?		At 10 pence ỹ peece what 795 peeces?	
6 d	56 — 0	6 d	297 — 6
2 d	18 — 8	4 d	265 — 0
	74 s — 8 d		662 s — 6 d

IIII		VI	
At 9 pence the Elle what 356 Elles?		At 11 d the pound what 7579 pound?	
6 d	178 — 0	6 d	3789 — 6
3 d	89 — 0	4 d	2526 — 4
		1 d	631 — 7
	267 s — 0 d		694|7 s — 5 d
			li 347 — 7 s — 5 d

Here in this first example where it is demanded (at 5 d the yard) what shall 758 cost: First for 3 pence I take the $\frac{1}{4}$ of 758: and thereof commeth 189 s — 6 pence: Then for 2 pence I take the $\frac{1}{6}$ of the same product, which amounteth to 126 s 4 d, these two summes added together, doe make 315 s, 10 pence: And so much are the 758 yards worth at 5 pence the yard.

Item also for the same againe: First for 4 d I take the $\frac{1}{3}$ of 758: and thereof commeth 252 s 8 pence: then for 1 peny I take the $\frac{1}{4}$ of ỹ same product, that is to say, of 252 s — 8 pence, and it yeeldeth me 63 s, 2 pence: which both added together

gether maketh 315 s — 10 d, as before.

Item for 7 d, there is taken the $\frac{1}{3}$ and the $\frac{1}{4}$ of the whole summe, which is to be multiplied, and adde them together, that is to say, first, for 4 pence there is taken the $\frac{1}{3}$ of 563: which comes to 187 shillings — 8 pence, as appeareth by the worke: and for 3 d there is taken the $\frac{1}{4}$ of the whole summe, which amounteth to 140 s 9 d. Both which products added together maketh 328 s — 5 d: And so much comes 563 elles to at 7 pence the Elle.

Item, for the first 8 d there is taken for 4 d the $\frac{1}{3}$ of the whole summe: and another $\frac{1}{3}$ for the other 4 d, which added together as in the example doth evidently appeare amounteth to 74 shillings — 8 pence.

Againe, for the second worke of 112 l, there is taken first the $\frac{1}{2}$ of the whole summe for 6 d, which comes to 56 d: then for the 2 pence you have to take $\frac{1}{6}$ of the whole summe, or if you will the $\frac{1}{3}$ of the product that came of 6 d, either, which maketh 18 s 8 pence. These two summes being added together doe make 74 s 8 d, as in the third example appeareth.

Item, for 9 d there is taken for 6 pence the $\frac{1}{2}$ of the whole summe: and the $\frac{1}{4}$ of the whole sum for 3 d, or otherwise for the 3 d you may take the $\frac{1}{2}$ of the product that came of 6 pence, because 3 pence is the $\frac{1}{2}$ of 6 pence: which added together, as plainely appeareth in the fourth example, amounteth to 267 shillings — 0 pence.

Item, for 10 d, first there is taken for 6 pence the $\frac{1}{2}$ of the whole summe, which amounteth to 397 s 6 pence. Then for 4 pence there is found 265 s: both which added together maketh 662 s 6 pence, as appeareth in the 5 example. It may also bee wrought, as appeareth by the second note in the table by 4 pence twise taken, and the $\frac{1}{2}$ of the product of 4 pence: or else by the $\frac{1}{2}$ of the whole summe, &c.

Item, for 11 d, there is first taken the $\frac{1}{2}$ for 6 pence: then the $\frac{1}{3}$ of the whole summe for 4 d: lastly, the $\frac{1}{4}$ of ye last product for 1 d: All which 3 summes added together, maketh in s 6947, 5 d, and in pounds 347 — 7 shillings — 5 pence.

3. Rule. Item likewise by the same reason, when you will multiply (by shillings) any number that is under 20 shillings, you shall have in the product pounds, if you know the euen or aliquot parts of 20, which are here in this little Table set downe to sight.

$$\text{Item s}\left\{\begin{array}{c}10\\5\\4\\2\\1\end{array}\right\}\text{is the}\left\{\begin{array}{c}\frac{1}{2}\\\frac{1}{4}\\\frac{1}{5}\\\frac{1}{10}\\\frac{1}{20}\end{array}\right\}\text{of one l.}$$

So that for 10 s which is the $\frac{1}{2}$ of a pound, you may take the $\frac{1}{2}$ of the number which is to be multiplied: and you shall haue in your product

duct

vnit pounds: if a vnite doe remaine, it shall be worth 10 shillings.

Likewise for 5 shillings you must take the $\frac{1}{4}$ of the number which is to be multiplied: And if there doe remaine any vnites, they shall bee fourth parts of a pound, euery vnite being in value 5 shillings.

For 4 shillings take the $\frac{1}{5}$ of the number which is to be multiplied: and if there doe remaine any vnites, they shall be fift partes of a pound, each vnite being worth 4 shillings.

For 2 shillings you must take the $\frac{1}{10}$. of the number to be multiplied: wherefore to take the $\frac{1}{10}$. of any number, you must cut off the last figure of the same number (which is nearest your right hand) from all the other figures with a small right downe line or dash with a pen, & so haue you done: for all the other figures which do remaine toward your left hand frō the same figure that you doe separate shall be the sayd $\frac{1}{10}$ of a pound: And that figure so separated towards your right hand shall be so many peeces of 2 shillings the peece: the which figure you must double to make thereof the true number of shillings, as by the example shall appeare. 4. Rule.

Finally, for 1 shilling needeth small worke, for it is so many shillings as be proponed in the summe, which to bring into pounds hath beene already taught in the first rule.

Example.

At

At 10 s the peece
What are — 6543 — peeces worth?

$\frac{1}{2}$ li — 3276 — 10 s

At 5 s the Elle
What comes — 4373 — Elles to?

$\frac{1}{4}$ li — 1093 — 5 s

At 4 s the yard
What — 7839 yards?

$\frac{1}{5}$ li — 1567 — 16 s

At 2 s the pound weight
What are — 752|7 — pounds weight worth?

$\frac{1}{10}$ li — 752 — 14 s

At 1 s the peece
What comes — 757|3 peeces to?

$\frac{1}{20}$ li — 378 — 13 s

4. Rule. Nextly, now followeth in order to be understood, that if the number of shillings bee not some euen or aliquot part of twenty, you must then conuert the same number of shillings into the aliquot partes of twenty, and thereof make two or three products, as neede shall require: which done, adde them together, and bring them into poundes. And here for thy furtherance I haue sette downe a note of the order of their partes, as they are to be taken.

For

s.						s.		
3		2 & 1				13		10. 2 & 1
6		4 & 2		5 & 1		14		10. and 4
7		5 & 2				15		10. and 5
8	of	4 & 4	or	5. 2. 1.		16	of	10. 5. 1
9		5 & 4		4. 4. 1.		17		10. 5. 2
11		10 & 1				18		10. 4. 4
12		10 & 2				19		10. 5. 4

For 3 s according to the tenor that you see is expressed in the Table, you must first take for 2 shillings the $\frac{1}{10}$ of the number that is to be multiplied: Then for 1 shilling you must take the $\frac{1}{2}$ of the product which did come of the same $\frac{1}{10}$ part, which two summes added together produce the effect desired.

Item, for 6 shillings according to the note set forth in the Table, first for 4 shillings I take the $\frac{1}{5}$ of the number that is to be multiplied: Then for 2 shillings the $\frac{1}{2}$ of the product that came of 4 s, and adde them together.

Or els, as appeareth also in the Table, for 5 shillings you may take the $\frac{1}{4}$ and the $\frac{1}{5}$ part of the product that came of 5 shillings and adde them together.

Item, for 7 s first take for 5 s the $\frac{1}{4}$ of the product, that is to be multiplied, then for 2 shillings take the $\frac{1}{10}$. of the number that is to be multiplied, and adde them together, &c.

Item for 8 s, according to reason, and the intent of the Table, for the first 4 s take the

$\frac{1}{5}$ of the product, & the same number againe for the other 4 shillings, and adde them together.

Item for 9 shillings first for 5 shillings take the $\frac{1}{4}$: then for 4 shillings take the $\frac{1}{5}$: and adde them together.

Otherwise as you see by the intent of the table, worke twise for 4 shillings, as was taught euen now for 8: & then take the $\frac{1}{4}$ of the last product for the 1 shilling: But 5 & 4 is the shorter.

Item, for 11 shillings, first dispatch 10 shillings: for which you must take the $\frac{1}{2}$ of the product, then lastly for 1 shilling, take the $\frac{1}{10}$ part of the summe produced of the $\frac{1}{2}$ of the product, and adde them together.

Item, for 12 s, where I will end with the first part of my Table: first take the $\frac{1}{2}$ for 10 shillings, and then for 2 shillings take the $\frac{1}{5}$ of ÿ summe that came of 10 shillings and adde them together: or else, if you please, for 2 shillings you may take the $\frac{1}{10}$ of the whole giuen number.

To write more of the manner of taking the true parts, I omit. The desirous practitioner will (no doubt) conceiue it. Also the Table is some aide to helpe the unperfect, whereupon by and by I will set downe 3 or 4 of these notes in examples, and the rest I will leaue to thine owne industry and practise to labour upon.

This is the order most commonly vsed in Practise, whē the number of the shillings is not an aliquot part of a pound. But (louing Reader) after I haue touched the euen or aliquot parts

parts of a ℓ that falleth out in pence and shillings, I will deliuer two new rules that shall drowne this common order quite and cleane: wherein shall be comprehended in one line and working both euen and odde partes of shillings vnder 20: without regard whether it be an aliquot or not an aliquot part: which two rules, when they come in place, I cōmit to thy frendly iudgement in working.

Now follow the examples vppon the notes before sayd.

At 6 shillings the yard

What	——3215——	yardes
4 shillings	643——	
2 shillings	321——10	

ℓ——964——10 shillings.

Otherwise by Multiplication of 6.

3215——

6 shillings 1929|0

ℓ——964.——10 shillings

At 7 shillings the Elle

What	——4563	Elles
5 shillings	1140 .. 15	
2 shillings	456 .. 6	

ℓ——1597 .. 1 shilling.

Otherwise by multiplication of 7.

4563

7 s	3194|1
	1597 .. 1 s

At 8 s the peece what 7563 peeces?

4 s	1512 .. 12
4 s	1512 .. 12
	li—3025 .. 4 s

Otherwise by multiplication.

	7563
8 s	6050|4
	li—3025—4 s

At 13 s the peece what 401 peeces?

10 s	200 .. 10
2 s	40 .. 2
1 s	20 .. 1
	li—260 .. 13 s

Otherwise by multiplication.

	401
13 s	1203
	401
	521|3
	li. s 260—13 s

These & such like Questions of compound numbers which I haue here in this fourth rule for orders sake set downe for that it hath bin heretofore a common course of work, I account but superfluous. For in the 8 and 9 Chapters of this my simple Addition shall appeare that the given price of any euē or odde number of shillings either vnder or aboue 20, shall be wrought at one or two workings at the most, how difficult soeuer the Question be.

Item, there resteth yet a kind of practise how to bring pence into pounds at the first working: whereupon you must vnderstand that 240 pence maketh one pound, or 20 shillings. In consideration whereof I cut off the last figure or 0: and there remaineth but 24 (of which 24) 8 d is the $\frac{1}{3}$ part thereof: 6 d is the $\frac{1}{4}$ part, 4 d the $\frac{1}{6}$ part: and 2 pence is the $\frac{1}{12}$ part thereof. 5. Rule.

Whereupon if it were demaunded what 1486 yards or poundes of any thing commeth to, at 8 pence the yard, in pricking or cutting off the first figure towards your right hand, for the 0 that appertaineth to 240: There is remayning of the sayde summe 148, whereout I taking the $\frac{1}{3}$ part, and it commeth to 49 ℔, and there resteth 1: which 1 I put to the 6, that I picke or cut off, and it maketh 16 peeces of 8 pence, which I double to make into groates,

and they make 32, whereof the ⅓ part maketh 10 shillings, and there remaineth ⅔ s, which is 8 pence, whereby it followeth, that the 1486 yards at 8 pence the yard, maketh 49 l. 10 shillings, 8 pence, as by the example shall appeare.

Item for 6 pence, take the ¼ part of the number from the pricke figure: And if any vnites doe remaine, they are so many sixpences, whereof taking the ½ they are shillings, if there doe remaine yet one, it is in value sixe pence.

Item, for foure pence, take the ½ part of the number from the pricke figure: If any vnites remaine, they are so many groats, which to conuert into shillings take the ⅓ part: And if any thing yet remaine, they are thirds of shillings, each one in value being worth foure pence.

Item for three pence, take the ¼ part from the pricke figure, if any vnites remaine, they are so many peeces of 3 pence, whereof in taking the ¼ part, maketh shillings: If any thing yet remaine, they are fourth parts of shillings, each one being in value 3 pence.

Item, for 2 pence, as appeareth also by the table, take the $\frac{1}{12}$ part of the number from the pricke figure: If any thing remaine, they are so many peeces of 2 pence, which by taking the ⅙ part, you shall turne into shillings: and if any vnites remaine, they are so many sixt parts of shillings, or peeces of 2 d, whether you will.

If

If one cost 8 pence
What are 1486 worth?

maketh pounds. 49 — 1 — 0 pence.

If one cost 6 pence
What are 7865 worth?

maketh pounds 196 — 12 — 6 pence.

At 4 d the yard
What are 8736 yards worth?

maketh pounds 145 — 12 — 0 pence.

If one cost 3 pence
What are 6874 worth?

maketh pounds 123. — 8 — 6 d

At 2 d the Elle
What comes 7894 Elles to?

maketh pounds 65. — 15 — 8 d

BUt if your number of pence bee not an aliquot or euen part of 24: then must you bring them into the aliquot parts of 24, and make thereof diuerse products, which must bee added together, as by the questions hereafter following shall appeare.

Item for 5 d, first take for 3 d, then for 2 d: and adde them together, according to the instruction of the second rule: Or else first take

for 4 pence, then for 1 peny.

Item for 7 pence, first take for 4 pence: then for 3 pence and adde them together.

Item for 9, first take for 6 pence: then for 3 pence, and adde them together.

Item for 10 pence, first take for 6 pence: then for 4 pence, and adde them together.

Item for 11 pence, first take for 8 pence, then for 3 pence, and adde them together: as by these examples following doth appeare.

Examples.

If one yard cost 5 pence
what are 7596 worth?

	126.	12
4 pence		
1 peny	31.	13
maketh poundes	158.	5

Otherwise.

1	5		7596
	94	19	
3 pence			
2 pence	63	6	
maketh poundes	158	5	

If one cost 7 pence
what are 987 worth?

	16.	9	
4 pence			
3 pence	12	6	9
maketh pounds	28	15	9 pence.

Other-

Otherwise.

1 — 7 — 987

6 pence 24 — 13 — 0
1 peny 4 — 2 — 3

maketh pounds. 28 — 15 — 9

If one cost 9 pence
what are 987 worth?

6 pence 24 — 13 — 6
4 pence 12 — 6 — 9

maketh pounds 37 — 0 — 3

Otherwise.

1 — 9 — 987

3 pence 12 — 6 — 9

maketh pounds 37 — 0 — 3

If one cost 10 pence
what are 987 worth?

6 pence 24 — 13 — 6
4 pence 16 — 9 — 9

maketh pounds 41 — 2 — 6

If one cost 11 pence
what are 987 worth?

8 pence 32 — 18 —
3 pence 12 — 6 — 9

maketh pounds 45 — 4 — 9

But if you haue any shillings and pence to be multiplied together: Then are you to take for the shillings according to the instruction of the third rule: And for the pence according to the first rule before mentioned: vnlesse you can spy the aduantage thereof, and thereby helpe your selfe: as appeareth in the second example, where first I worke for 6 pence: which is to be rebated out of the giuen number, and I haue 719 li, 11 shillings my desire.

At 19 shillings 6 pence the yard what 738 yards?

	738	
10 s	369	
5 s	184	10
4 s	147	12
6 d	18	9
li	719	11 s

Otherwise by Rebating.

	738	
6 d	18	9
li	719	11 s

The like againe is done by rebating, as by these two examples appeareth.

At 18 s the Elle what 418 Elles?

2 s	41	16
li	376	4 s

At 16 s the Elle, what 517 Elles?

4 s	103	8
li	413	12 s

And

And now I will touch a litle the euen parts of a pound that falleth out in pence and shillings, whereof for those parts you shall take such like part of the giuen number that is to be multiplied, as the price of that giuen number beareth in proportion to a pound, which also for thy better ayde is here set downe.

1 s. 8 d.	is the	$\frac{1}{12}$	part of a pound.
2. 6		$\frac{1}{8}$	
3. 4		$\frac{1}{6}$	
6. 8		$\frac{1}{3}$	

Item first for 1 shilling 8 pence, take the $\frac{1}{12}$ part of the giuen number, and if any thing doe remaine, they are twelue parts of a pound, each one being in value 1 shilling 8 pence.

Item for 2 shillings 6 pence, take the $\frac{1}{8}$ part of the number that is to be multiplied. And if any thing doe remaine, they are eight parts of a pound, each one being in value two shillings 6 pence.

Item for 3 shillings 4 pence, as appeareth by the Table, you must take the $\frac{1}{6}$ part of the giuen number. And if any thing doe remaine, they are 6 parts of a pound, each one being in value 3 shillings 4 pence.

Item for 6 shillings 8 pence, take the $\frac{1}{3}$ part of the number that is to bee multiplied: And if any vnites doe remaine, they are thirds of

a pound euery one being worth 6 shilling eight pence.

Other infinite numbers there are, that may be reduced by abbreuiation into the proportionate parts of a pound: as 16 shillings 8 pence maketh $\frac{5}{6}$: which 16 shillings 8 pence is easily reduced into groats, by multiplying 16 by 3, and thereto adde 2, which maketh 50 groats: Then set 60 the groates of a pound vnder 50, cutting off the 2 ciphers as is here performed in the margent. And then haue you brought 16 shillings 8 pence into the knowen partes of a pound which maketh $\frac{5}{6}$.

16	8
3	
5|0	
6|0	

But yet gentle Reader, for thy further instruction I haue hereunto annexed in a Table, how pence and shillings beare proportion to a pound, which I commit to thy friendly beneuolence. it shall bee some ayde vnto the vngrounded practitioner: but I count him the best workeman that can presently reduse his giuen price vnto the knowne and proportionate parts of a pound.

s	d	l	s	d	l
0	2	1/120	8	4	5/12
0	3	1/80	8	9	7/16
0	4	1/60	9		9/20
0	6	1/40	10		1/2
0	8	1/30	11		11/20
1	0	1/20	11	3	9/16
1	3	1/16	11	8	7/12
1	8	1/12	12		3/5
2	0	1/10	13		13/20
2	6	1/8	13	4	2/3
3	0	3/20	13	9	11/16
3	4	1/6	14		7/10
3	9	3/16	15		3/4
4	0	1/5	16		4/5
5		1/4	16	8	5/6
6		3/10	17		17/20
6	3	5/16	17	6	7/8
6	8	1/3	18		9/10
7		7/20	18	4	11/12
7	6	3/8	18	9	15/16
8		2/5	19		19/20

Here follow foure Examples vpon the 4. notes deliuered.

At 1 s 8 d the yard
what are 3884 yards worth?

maketh pounds—323—13—4 pence.

At 2 s 6 pence the yard
what are 4563 yards worth?

maketh pounds—570—7—6 pence.

At 6 s 8 d the Elle
what comes 7562 Elles to?

maketh pounds—2520—13—4 pence.

Now by custome you are able to worke by all sortes of summes being delivered in shillings and pence, as 1 shilling one peny, 2 shillings two pence, 3 shillings three pence, and so of all other: wishing you to have some consideration of your questions, when they are set downe, for there are many subtill abbreviations, and great advantages to be gotten, and easily to be perceived.

At 3 shillings 8 pence of 2 shillings, & 1 s 8 d.
4 s 2 d, of 3 s 4 d, and 10 d, which 10 is $\frac{1}{4}$ of 3 shillings 4 pence.
5 s 8 d, of 4 shillings, and 1 shilling 8 pence.
5 s 10 d, of 5 s and 10 pence, which 10 pence is $\frac{1}{6}$ of 5 shillings.

And by this meane when you have taken one product, you may oftentimes upon the same

same take an other more briefly than upon the summe which is to be multiplied.

Now (gentle Reader) that you haue seene the vertue of the euen or aliquot parts of a pound in shillings alone, and also in the aliquot parts of shillings and pence: according to my promise, hereafter followeth a briefer and easier methode for any euen number of shillings eyther vnder or aboue twenty, than euer yet hath béene published: Notwithstanding Maister Humfrey Baker, whose trauell is worthy commendation, and whom for knowledge sake I reuerence, hath in some part touched this first part, though not in this methode. The worke of the rule is both pleasant, ready and briefe, as by the variety of the examples deliuered thereupon shall appeare. And first I will set forth a question, thereby the better to expresse or teach you the order thereof: which is this.

If one yard cost 6 s, what 8574?

1	6	8574

maketh pounds ——2572. ——4

To the vnderstanding of this example, after you haue set downe your giuen number in forme of the rule of 3, with a line drawen vnder it, you shall presently set a pricke vnder your first figure 4, towards your right hand.

drawing from the pricke as heretofore hath
beene practised, a little short line, thereto set
downe the shillings anon, which done, multi-
plie the first figure 4 by 6 the value of your
price (which here you see standeth in sight a-
boue the line) it maketh 24s which is 1 pound
4 shillings. The 1 pound keepe to carie to the
next place, and the foure shillings set downe at
the end of the prescribed line towardes your
right hand. Thus haue you done now with
fire aboue the line, and also with 4 in the first
place (for the pricke vnder foure doth represent
that 4 hath done his office.) Then secondarily
for a general rule take but the ½ of the giuen
price which here is 3, which 3 is the number that
shall now continue the rest of the multiplicatiō
and end the worke, whereupon I multiplie
three into seuen standing in the second place, it
maketh 21, and with the one l' I kept in minde
22, set downe 2 and keepe 2 in mind working
according to the rule of Multiplication, deliue-
ring the tens in mind in their due place, which
done, the product from the pricke to your left
hand representeth the pounds, and the other at
the end of the line the shillings, as appeareth
by the examples.

If one yard cost 2 s what 7536 ?

		7536	
1	2		
maketh pounds —		753. —	12 s

If one yard cost 4 s what 8792?

1	4	8792
maketh pounds ——		1758.—8 s

If one péece cost 6 s what 9537?

1	6	9537
maketh pounds ——		2861.—2 s

If one cost 8 s what 7509?

1	8	7509
maketh pounds ——		3003.—12 s

If one cost 12 s what 5794?

1	12	5794
maketh pounds ——		3476.—8 s

If one cost 14 s what 3705?

1	14	3705
maketh pounds ——		2593.—10 s

If one cost 18 s what 5703?

1	18	5703
maketh pounds ——		5132.—14 s

If one cost 22 s what 953

1	22	953
maketh pounds ——		1048.—6 s

Let these suffice (gentle Reader) for an entrance into euen numbers: And nowe I will shew the like rule for any od or vneuen partes of a pound.

The 9 Rule.

TO helpe you to the vnderstanding of these other questions that do hereafter followe: wherein my first example the giuen number is 6487, at thrée shillings the yard. I multiplie 3 aboue the line into 7, it maketh 21. The one shilling I set downe, and the 1 l' I kéepe. Now am I to take the $\frac{1}{2}$ of thrée, which because it is an odde number I cannot. Therefore I shall kéepe and continue my multiplication by thrée still, and worke by the $\frac{1}{2}$ of the rest of the giuen figures or numbers, to wit 648. And first the $\frac{1}{2}$ of 8 which is 4 multiplied into thrée maketh 12, thereto ioyne the 1 l' in minde, it maketh 13: set downe 3, kéepe one. Then againe multiplie by two the $\frac{1}{2}$ of foure, it maketh sixe, and with one in minde it maketh 7. Then lastly take the $\frac{1}{2}$ of sixe which is thrée, saying: 3 times thrée is nine, which 9 set downe, and so is the question answered, as appeareth by the practise and the examples following.

At 3 s the yard, what 6487 l
1 — 3 6487 l

maketh pounds — 973 — 1 s

If one yard cost 5 s, what 4269?

1	5	4269

maketh pounds — 1067.—5 s

At 7 s the Elle what 6489?

1	7	6489

maketh pounds — 2271.—3 s

If one Elle cost 9 s what 2807?

1	9	2807

maketh pounds — 1263.—3 s

At 11 s the Pistolet what 8263?

1	11	8263

maketh pounds — 4544.—13 s

If one peece cost 13 s what 4629?

1	13	4629

maketh pounds — 3008.—17 s

But now note (gentle Reader) when the giuen price falleth vppon any odde number. As 3.5.7.9.11.13.&c. Then it is to be presupposed, that the giuen summe to be multiplied must be a summe made of euen numbers, as 2.4.6.8.10. &c. else cannot that question be wrought at one line or working.

Prouiding alwaies that it may beare an odde figure in the first place towardes your right hand: as appeareth in these 6 examples which last were wrought, and such like, &c.

which may beare an odde number for the price, and be done at one line or working very well.

But if the given price bee an odde number, and the summe to be multiplied odde numbers also. Then can it not be done at one working, but requireth the aid of two workings, for odde with odde will not agree, which notwithstanding to bring to passe, take this for a generall rule: First worke for the euen number, contained in that question, or giuen price, according as you haue learned, and then afterwards for the one odde shilling, take the $\frac{1}{2}$ of the summe giuen to bee multiplied, omitting the first pricke place, as was taught for the working of one shilling in my first rule of Practise, and adde those two together, and you shall haue your desire.

A generall Rule.

Example.

At 3 s the yard what are 7539 yards worth?

2 s	753—18
1 s	376—19
maketh pounds	—1130—17 s

At 7 s the Elle what 7539?

1	7	7539
6 s		2261—14
1 s		376—19
maketh pounds—		2638—13

At

At 13 s̄ the yard, what 7534 ?

1	13	7534	
13 s̄		4520 — 8	
1 s̄		376 — 14	
maketh pounds		4897 — 2	

And thus haue I abridged into theſe two rules how to bring any number of ſhillings: whatſoeuer they be into pounds, with a briefer method than euer yet hath beene publiſhed, which I commend vnto thy friendly cenſure and iudgement in the vſe of practiſe thereof.

If one coſt 6 s̄ 5 d, what 1231 ?

1	6	5	1231
6 s̄			309 — 6
4 d			20 — 10 — 4
1 d			5 — 2 — 7
maketh pounds			394 — 18 — 11

At 14 s̄ 2 d, what 2825 ?

1	14	2	2825
14 s̄			1977 — 10 —
2 d			23 — 10 — 10
maketh pounds			2001 — 00 — 10 d

At 16 s̄ 4 d, what 2531 ?

1	16	4	2531
16 s̄			2024 — 16
4 d			42 — 3 — 8
maketh pounds			4066 — 19 — 8

At 3 s̄ the Pistolet, what 8325?

1	3	8325
maketh pounds		1248·—15 s

At 7 s the crowne, what 6529?

1	7	6529
maketh pounds		2285·—3 s

At 9 s̄ the peece, what 6767?

1	9	6767
maketh pounds		2955·—3 s

These three last questions may seeme something hard, yet are they easie inough if you marke them well, if I should explane them, then are they too easie: therefore I leaue them to whet the mindes of the desirous.

10 Rule. ITem, when any one of the summes which is to be multiplied, is composed of many denominations, and the giuen number but of one figure alone: then shall you multiply all the denominations of the other summe by the same one figure, beginning first with that summe which is least in value towards your right hād, and bring the product of those d into shillinges, and the product of the shillings into pounds, as by this example doth appeare.

At 3 li 7 s 4 d what are 9 worth?

maketh pounds	30	6	0 d

But

But if in any of the sums that are to be multiplied there be a broken number: First worke for the whole according to the instructions that you haue learned: and then take such part of the giuen price, as that broken number beareth in proportion to the price, as in the example. After you haue wrought for thrée shillings & for 6 d, then are you to take the ½ of 3 s 6 d for the ½ yard, and adde that to the summe: so adding all thrée products together, which maketh 43 li. 2 s. 9 d the iust price of 246 ½ Elles: and thus must you do of all other.

At 3 s. 6 d the Elle, what 246 ½?

1	3	6	246 ½
	3 s		36 — 18
	6 d		6 — 3
	½		1 — 9
maketh			43 — 2 — 9

At 16 s 4 d the péece, what 14 ¼?

1	16	4	14 ¼
	16 s		11 — 4
	4 d		0 — 4 — 8
	¼		0 — 12 — 3
maketh pounds			12 — 00 — 11 d

If one péece cost 4 li. 3 s 6 ½ d what are 12 péeces worth?

4 li.	48.
3 s	1 — 16
6 d	0 — 6
½	0 — 0 — 6
maketh pounds	50 — 2 — 6 d

The Proofe.

If 12 peeces cost 50 li, 2 s, 6 d what one peece?

50 2 6

maketh pounds 4—3—6 $\frac{1}{2}$

Item touching the maner how to understand the order of this question, and others the like: first seeke how many times 12 is contained in 50, which is 4 times, and so resteth 2 li, which 2 li converted into shillings, and ioyned with the other 2 shillings, maketh 42 shillings, wherin is found 12 three times: resteth 6 shillings, which turned into pence, putting thereto the 6 d in the first place, it maketh 78: wherein 12 is found 6 times, resteth 6 d, which containeth 12 but $\frac{1}{2}$ a time, put that $\frac{1}{2}$ to the 6 d: And then the solution is 4 li, 3 s 6 $\frac{1}{2}$, as appeareth by the practise thereof.

Item, the like is to be done of any thing that is bought or [illegible] after 5 score to the hundreth, or the Quintall: As for example.

If

If 100 cost 27 l 13 s 4 d
What one pound?

27——13——4			27——13——4		
20			20		
s	5	53 12	s	5	53 12
	1 5	10 3	d	6	40
d	6	4 | 0 10 | 0 $\frac{2}{5}$			100 $\frac{2}{5}$
maketh		5 s —— 6 $\frac{2}{5}$ d	makes 5 s 6 $\frac{2}{5}$ d		

I haue wrought this at length for the ayde of the young learner, because he shoulde understand how all the multiplication is set downe.

But to worke it more neatelp, it is by a litle understanding ended thus.

Item to the understanding of this and such like questions, the right downe line is all the guide, which is pulled downe close by 20, as you see in the example, where 27 pound 13 shillings is reduced all into shillings, and maketh 553 shillings.

The 5 towardes your left hand being sepa-

rated with the hanging or right downe line is the iust number of shillings that answereth to the question: Nextly, fiue shillings is multiplied by 12 to reduce them to pence, putting to the 4 d, it yéeldeth for the multiplication of the first figure two 110: the one beyond the line towards the left hand, is 1 peny towards the rest of the price: then 53 also multiplied by 1 yéeldeth 53: but the fiue behind the line towards the left hand is also 5 d more, towardes the price, which 1 and 5 I adde together vnder the line; it maketh 6 pence. So is there found now as appeareth by the Titles of shillings and pence 5 shillings 6 pence.

Finally, I come now on this side the line towardes the right hand, and vnder 12 I finde first 10, and then 3, which added together maketh 40, vnder which 40 you must put the 100, and it maketh $\frac{40}{100}$, which abbreuiated commeth to $\frac{2}{5}$. So the iust price of one pound after fiue score to the hundreth, maketh 5 s 6 $\frac{2}{5}$ d.

One example more, and so I will leaue this rule.

I

If 100 cost $10\frac{3}{4}$ d

What 9874

100 · · $10\frac{3}{4}$ · · 9874

6 d		246.	17	0
4 d		164.	11	4
$\frac{3}{4}$ d		20.	11	5
$\frac{1}{4}$		10	5	$8\frac{1}{2}$

maketh	l	4	42	5.	$5\frac{1}{2}$
			20		
	s	8	45		
			12		
	d	5	$\frac{45\frac{1}{2}}{100}$	$\frac{91}{200}$ parts of a d.	

Also the like may bee done of our usuall weights here in England (which is 112 l for every hundreth weight) in case you know the aliquot parts of a hundreth weight, which are these, 56 l, 28 l, 14 l, and 7 l: For 56 l is the $\frac{1}{2}$ of 112 pound, 28 l is the $\frac{1}{4}$ of 112 l, 14 l is the $\frac{1}{8}$, and 7 l is the $\frac{1}{16}$ part.

Therefore for 56 l take the $\frac{1}{2}$ of the summe of money that 112 l weight is worth.

For 28 l take the $\frac{1}{4}$ of the summe of money that 112 l weight is worth.

For 14 l, take ye $\frac{1}{8}$ of the sum that 112 l is worth.

And for 7 l, the $\frac{1}{16}$ of the summe of money that 112 is worth.

As for example: 17 l 19 s the hundreth

pounds weight, that is to say 112 ℔, what shall 3 quarterns and 7 ℔ cost?

1 · ·	17 ·	19 · ·	3—7
2 quarterns	8 ·	19·	6·
1 quarterne	4 ·	9·	·9
7 pounds	1	2·	·5 1/4
maketh pounds.	14 ·	11·	·8 1/4

The second Chapter entreateth of the Reduction of diuerse measures to others value by Rules of Practise.

14 Rule. Now will I shew a few examples of Practise in reducing of measures: as Elles, Yards, Braces, Pawnes of Jeanes, &c. Much more would I haue touched, but that I feare the booke will rise to too great a volume.

In 864 Elles of Antwerpe, how many yardes of London?

864	864
432	216
216	648

maketh 648 —— yardes of London.

Item, in these and such like questions of Flemmish measure to be brought into yardes English: first take the ½ of the giuen number, as

as appeareth in the first example towards your left hand. Then take the $\frac{1}{3}$ of that product, or the $\frac{1}{2}$ of the given number, and adde those 2 products together, they shall be yards English, as by the example you may perceiue.

The second example towardes your right hand is yet briefer then the first, whose worke is this: take the $\frac{1}{4}$ of the deliuered number, and that product subtract out of the giuen number, and the rest sheweth your desire. Of these two waies vse which you thinke best.

The Proofe.

In 648 yards of London,
how many Elles of Antwerpe?

	648
	216
maketh	864 Elles of Antwerpe.

Item for the understanding of this worke: first take the $\frac{1}{3}$ part of the yardes of London, which found, adde that $\frac{1}{3}$ part and the yardes together, as appeareth by the practise: and the product sheweth the Elles of Antwerpe. 15 Rule.

In 320 yards of London, how many Elles of Antwerpe maketh 426 $\frac{2}{3}$ Elles.

320 yards	Proofe.
$\frac{1}{3}$ 106 $\frac{2}{3}$	426 $\frac{2}{3}$ Elles
426 $\frac{2}{3}$ Elles	$\frac{1}{4}$ 106 $\frac{2}{3}$
	320 yards.

Other Reductions.

16 Rule. Item you shall understand, that for as much as sixe Braces of Millane make 5 Elles of Antwerpe, whereuppon, according to the rules of Practise, you may reduce the one into the other by the like reasons aforesayde, in taking the $\frac{1}{6}$ part, and then subtract the same to make Elles of Antwerpe: And againe by the contrary in taking the $\frac{1}{5}$ part with adding the given number, to turne the Elles to Braces, as for example.

In 876 Braces how many Elles of Antwerpe?

876 Braces	The contrary.
146	730 Elles Flem:
Elles 730 Antwerps	146
	876 Braces.

Elles 730 Antwerpe.

182 $\frac{1}{2}$

Yards 547 $\frac{1}{2}$ English.

Thus

Thus appeareth, that 876 Braces by practise, make 730 Elles Flemmish: which Elles Flemmish reduced into English yardes by the rules aforesayd, make 547 ½ yardes.

So againe upon the same first question of Braces, I would know how many yardes English they make.

After the rate that 100 Braces are worth 62 ½ yards.

876 Braces

438
109 ½

I answer, 547 ½ Yards

Item, to the understanding of this worke and such like, first take the ½ of the given Braces, and after take the ¼ of that halfe, or the ⅛ of the given number, and adde them together, and the product are also yards English.

Item three Elles of Rochell make 5 Elles at Lisbone: So likewise three Elles at Lions make 5 Elles at Antwerpe.

To worke these and such like, double the Elles of Lyons, and the Elles of Rochell; and from their products subtract the ⅓: And the rest shalbe Elles of Antwerpe, or Elles of Lisbone.

Example.

In 63 Elles of Lions how many Elles of Antwerpe?

63
63

126

$\frac{1}{6}$) 21

Ans. 105 Elles Ant.

In 100 Elles of Rochell, howe many Elles of Lisbone.

100

200

$\frac{1}{6}$) $33\frac{1}{3}$

Ans. $166\frac{2}{3}$ els of Lisb.

Touching the proofe or returne of these and such like questions, for a generall rule, you shall first take the $\frac{1}{5}$ of the giuen number: and adde that $\frac{1}{5}$ and the giuen number together, and the $\frac{1}{2}$ of that product shall be your desire.

Example.

In 105 Elles of Antwerpe, how many Elles of Lions.

105

$\frac{1}{5}$) 21

$\frac{1}{2}$) 126

Ans. 63 els of Lions.

In $166\frac{2}{3}$ Elles of Lisbone, howe many Elles of Rochell?

$\frac{1}{5}$ $166\frac{2}{3}$

$33\frac{1}{3}$

$\frac{1}{2}$ 200

Ans. 100 els of Roch.

The

The third Chapter entreateth of the order and work of the Rule of three in broken Numbers, after the trade of Merchants, digressing something from M. Recordes which is comprehended in 3 Rules.

Now that I haue somewhat intreated of the Rules of Practise; I will giue a few instructions, after my simple order, for the working of the Rule of thrée in broken Numbrs. wherein I shall néed to say the lesse, because I hope the studious Learner, that hath travelled any thing in the Ground of Artes, is not unfurnished of knowledge capable to understand mee. But before I deliuer any instructions for broken numbers, I will propone a question, which shall be wrought thrée sundry wayes, whereby to shew as it were thrée degrees of comparison how farre the Rule of thrée in broken, for more speede of worke, differeth from the whole, which I rather set downe for a view, that the studious herein may be more desirous to attain broken: leauing any more to discourse in Dialogue forme, but onely to giue instructions where neede is: and in the rest to put foorth the questions with their answers.

My first question is this.

If one yard cost 6 s — 8 d
What are —— 789 worth at that rate?

1 — 6 — 8 —— 789

80

12 ——— 80 ——— 63120 d

Here the product of the summes are pence, according to the nature of the middle number.

587
63120 | 5260 | 263
222 | 2220

I answere 263 l.

1 —— $6\frac{1}{3}$ —— 789

20 20

15780

Here the product of the summe is s, according to the nature of the middle number.

25750 | 5260 | 263 pounds.
333 | 2220

1 —— $\frac{1}{3}$ —— 789

3 1

1
789

Here

Here the product is pounds according to the title of the second number.

2
789 (263
333

I answere 263 l.

Now that you haue séene the 3 former vertues of the Rule of thrée, whose products haue first brought forth d, next s, and lastly pounds: I will deliuer thrée notes in order following, and with them a dozen questions, that shall shew the work of the Rule of 3 in broken numbers or Fractions.

1 The first foure shall be sundry questions of a Fraction comming in the second place.

2 The second foure shall be of two Fractions comming in the second or third place.

3 The third foure of Fractions in all thrée places.

My first question is this.

If one yard cost me 3 s — 4 d, what are 756 worth at that price?

In setting downe the question to performe the worke I turne 4 d into the parts of a shilling, which is $\frac{1}{3}$, and then the question standeth thus. 1. Rule.

1 ———— $3\frac{1}{3}$ ———— 756

TO the ready working of this question, and all such other like, my first note is this: which take for a generall rule, that when any one Fraction shall come either in the second or third place, that the Denominator of that Fraction or Fractions, must alwayes be brought unto the number or Numerator of the first place: and thereby multiplied the one into the other.

And this benefite is alwayes gotten by the vertue of bringing the denominator of the second numbers Fraction unto the first place. For the Fraction in the middle number is now released: & the product that commeth of the multiplication, is of the nature and like Denomination of the whole number in the second place which here are shillings.

Whereupon now to worke the question I bring three the denominator of the Fraction in the second place, unto my first number one with a line set under $\frac{1}{}$ thus: and the three under it $\frac{1}{3}$ thus: saying, once 3 is 3 my Divisor: That done, reduce three $\frac{1}{3}$ saying three times 3 is nine, and the 1 over 3 make 10: my second number in the rule of three; by which 10 I do multiply my last number 756 as appeareth by the worke thereof, and it yeeldeth 7560 shillings my dividend.

Then dividing 7560 by 3 my Divisor, it yeeldeth in quotient 2520 shillings, which maketh 126 pounds, as appeareth here most plainely,

both

both by the example and the worke.

At 3 s 4 d the yard what 756 yardes?

$\frac{1}{3}$ —— 3 $\quad$ $\frac{1}{3}$ .. 756

3 $\quad$ 3 $\quad$ 10

10 $\quad$ 7560 s

1	1	
7560	2520	126 I answere 126 l.
3333	2220	

Yet otherwise upon the same question, altering the price now into the proportion it beareth to a pound, for the 3 s 4 d is $\frac{1}{6}$ part of a pound: which example first standeth thus, as appeareth on the left hand, and afterwards wrought as appeareth on the right hand.

$\frac{1}{6}$ —— $\frac{1}{6}$ —— 756

1 —— $\frac{1}{6}$ —— 756 $\quad$ 1 $\quad$ 1

756 pounds

As soone as I have caried 6 the denominator of my middle number unto my first place, as afore hath beene taught, I pull downe one, the numerator of 6, with a line under 5 thus: $\frac{1}{1}$: And that one of custome I pull downe in sight being the figure that I shall multiply my third or last number by, according to the tenor of the rule of three: And because one can

neither multiply nor yet diuide (though here it is set downe in forme of Multiplication the rather for your vnderstanding) the product of the multiplication according to the declaration of this my first rule or note is conuerted into the title of my second number, which here are pounds: Now followeth the diuision performed by my diuisor 6, to make an end of the question.

x3
756 (126 : which maketh 126 l, as before.
666

And thus much for the variety in working that question.

And now followeth another.

If one yard of cotten cost 8 $\frac{1}{4}$, what 859 ?

1 ——— 8 $\frac{1}{4}$ ——— 859

4   33   33

2577

28347

2 (3		11	l s d
28347	708(6	590	29 —— 10 —— 6 —— $\frac{3}{4}$.
4444	12	320	

This

This question was also wrought like the first, and bringeth forth 29 l.——10 s.——6 $\frac{2}{3}$ d. the price of 859 yards.

If 7 l. of any thing cost 3 l.——10 s. what comes 987 pounds to?

7——3 $\frac{1}{2}$——987

2 7 7

14 6909

(7
6909 | 493 $\frac{7}{14}$ | $\frac{1}{2}$ I answere 493 l. 10 s.
14

Notes vpon my second Rule, for two Fractions comming in the second and third place.

My first question is this.

If one Elle cost 13 s. 4 d.: what halfe a quarter or $\frac{1}{8}$ of an Elle?

Answere. First bring 13 s. 4 d. into the parts of a l. which is $\frac{2}{3}$, and then will the question stand thus.

1——$\frac{2}{3}$——$\frac{1}{8}$

Item for the performance of this worke, doe as before was taught in the first Rule, first

bring 3 the denominator of the second Fraction unto your first number 1: setting a line under 1 thus: Saying once 3 is 3: that done, bring 8 the denominator of the third fraction setting it under 3, and multiply them together, saying 3 times 8 makes 24: which 24 is your Divisor: (Now have you done with the denominator 3, and also with the denominator 8.) Therefore you shall put a line under 3 thus. And the like line also under 8, setting or pulling downe under them their owne numerators, that is 2 under 3, and also 1 under 8, as appeareth in the example, which numerators for a general rule are evermore to be pulled downe of custome in sight, to multiply the one by the other, according to the tenor or Rule of three. Then I multiply the one by the other, saying: once 2 is 2, which signifieth 2 pounds, being of the nature and like denomination of the middle number, which 2 poundes is to be reduced into shillings, otherwise it cannot be divided by my first number 24. Then dividing 40 by 24, the quotient bringeth forth 1 $\frac{2}{3}$ shillings. So much is $\frac{1}{8}$ of an elle worth after that rate. Otherwise although 2 poundes could not be divided by 24: yet it might have beene abbreviated to $\frac{1}{12}$, of a pound: which is worth 1 shilling 8 pence, as before.

1 ——— $\frac{1}{3}$ ——— $\frac{1}{8}$

3	2	1	16
8	20		40 ($1\frac{1}{3}$
			24
24	40		

Second question.

If one pound of any weight cost 13 shillings 4 pence, what are $\frac{7}{8}$ of the pound worth after that rate: Answere. Reduce the 13 shillings 4 pence into the parts of a pound: which is $\frac{2}{3}$: and then will the question stand thus.

£

1 ——— $\frac{2}{3}$ ——— $\frac{7}{8}$

Item for the understanding of this, if you marke well the last example, this and the rest lieth open, and neede small instruction. For as you did last, so now againe bring the denominator of the second and third fraction unto the first figure 1, multiplying the one into the other, which maketh also 24 your Diuisor.

Then making a line under 3 thus, and a line under 8 thus: and pulling downe their Numerators under each figure, that is 2 under 3, and 7 under 8, which as I sayd before for a generall rule I pull downe of custome in sight: to be the two numbers that of duety

ought to be multiplied together, which done I bring 2 being the lesser figure under 7: multiplying them together, it maketh 14: which are of the nature of the middle number. That is to wit, pounds: which 14 cannot aptly be diuided among 24, therefore are reduced into shillings, as is plainely to be seene in the example. Then 280 s̄ parted among 24, yeeldeth for his quotient 11 s̄ 8 d̄ your desire, and the iust price of $\frac{7}{8}$ of an Elle. Otherwise, 14, though it could not be diuided by 24, might by mediation or diuision in broken numbers haue bene diuided or abbreuiated to $\frac{7}{12}$, which in effect being reduced to his knowne parts, maketh 11 s̄ 8 d̄ as before. But my good will and meaning is to aide yong beginners. Therefore haue I reduced the 14 li into s̄, which is the easier way.

Now followeth the example.

$\frac{1}{3}$	$\frac{2}{3}$	$\frac{7}{8}$	(1
3	3	8	4
8	2	7	46
24		2	280 | 11 s̄
		14	244
		20	2
		280 s̄	

If one yard cost me 2 shillings 6 pence, what 345 $\frac{1}{4}$ yards?

Answere. First put 6 d into the parts of a s: and then the question standeth thus.

1 ——— 2 $\frac{1}{2}$ ——— 345 $\frac{1}{4}$:

Item, to the ready understanding of this and all such like, according as before hath bene declared, bring the denominators of the second and third Fractions unto the first place, multiplying them the one into the other, all which make 8 your Divisor common. Then next reduce your second number, saying 2 times 2 is 4: and 1, is 5: as was taught in the example aforesayd. Lastly reduce your third number 345 $\frac{1}{4}$ all into fourths, and they make 1381: which 1381 is to be multiplied by 5, according to the tenor of the rule of three: which done, maketh 6905 shillings: and divided by 8 your divisor, yeeldeth in quotient 863 $\frac{1}{8}$ s, which maketh in poundes 43 li ——— 3 s ——— 1 d $\frac{1}{2}$: And so much are the 345 and $\frac{1}{4}$ yards worth at that price.

The same question wrought againe by 2 s 6 d, is now converted into the parts of a li, and standeth thus:

1 ——— $\frac{1}{8}$ ——— 345 $\frac{1}{4}$

Item, after I have brought here my second and third fraction unto my first place, and found 32 to be my divisor: having thus furni-

thed my firſt place with all things vnto him belonging (which is meant of bringing and multiplying the Denominators of the ſecond and third Fractions into him) I then go in hand to ſee what is to doe in my ſecond place, where preſently of cuſtome I pull downe my Numerator 1 vnder 8: being the figure in ſight that ſhall multiply my third number.

Then laſtly I reduce 345 $\frac{1}{4}$ all into fourths, as afore was practiſed, which maketh 1381, the which 1381 I am to multiply by 1 my ſecond number: they are nothing increaſed, but by the Metamorph. of my worke, they are now 1381 poundes: being of the nature of the middle number, as I haue often ſhewed you, which diuided by 32 my Diuiſor, yeeldeth 43 poundes and $\frac{5}{32}$: which $\frac{5}{32}$ of a pound reduced into knowne numbers, make 3 ſhillings 1 d $\frac{1}{2}$ as before.

Example.

			1		
			10 (5		
1 ——	$\frac{1}{4}$ ——	345 $\frac{1}{4}$	1381	5	
32	1	1381	322	(43 ——	
			3	32	

3. Rule. Now follow 4 other queſtions, which are in all three places, broken numbers, or whole and broken together.

Item

Item first for the finding out of your Divisor: you shall take this for a most certaine and generall rule: That you must multiply the numerator of the first number in the question by the denominator of the second: And also all that again by the denominator of the third: and the totall thereof shall be your divisor.

Secondarily, for a generall rule to finde out your dividend, multiply the denominator of the first number by the numerator of the secōd, and the whole thereof by the denominator of the third. And the totall thereof shall evermore be your dividend.

Now for an example I propone this question thereby to make my meaning the more plaine, and to shew you as I have done in the rest, the manner and order of the worke.

If $\frac{2}{3}$ of any weight or measure cost $\frac{5}{6}$ of a £ or 20 s, what are $\frac{3}{8}$ of the like weight or measure worth after that rate?

Example.

$\frac{2}{3}$ —————— $\frac{5}{6}$ —————— $\frac{3}{8}$

Item for the more playner understanding hereof, and all other the like, in broken numbers: First you shall pull downe 2 the Numerator of the first number or Fraction, with a line under $\frac{2}{3}$ thus: that done, according as you

haue learned before, bring 6 the denominator of the second fraction, and set it vnder 2 multiplying the one into the other, which maketh 12. Then lastly bring 8 the denominator of the third fraction, and set it vnder 12, multiplying that 12 by 8, which amounteth to 96: (or else for more briefe multiply 6 by 8: saying 6 times 8 makes 48, which 48 set vnder two, and multiply the one into the other, maketh 96 as before.) And this 96 is the first number in the rule of three. That shall alwaies for a most generall rule be your diuisor.

Secondly to worke for your diuidend, you shall, as hath beene sufficiently declared afore, pull downe 5 the numerator of your second fraction. And set it vnder 6 with a line vnder 6 thus. That done, as you know, you are to pull downe 3 the numerator of the third fraction and set it vnder 8 with a line vnder 8 thus, multiplying the one into the other, according to the tenour of the rule of 3, which maketh 15. Then according to my note, forget not to bring ẏ denominator of the first Fraction which is 3 vnder 15, and multiply them together, which maketh 45. which 45 is your diuidend. Which 45 are of the nature or denomination of the middle number, as I haue oft taught you before. And therefore are 45 ł, which aptly cannot bee diuided by 96. Therefore you shall reduce that 45 ł into s̄, as you see is performed in the example, which

which amounteth to 900 s, which divided by 96 your divisor, it yéeldeth 9 s, and $\frac{36}{96}$ of a s, which in lesser termes is $\frac{3}{8}$, which $\frac{3}{8}$ in money maketh $4\frac{1}{2}$ d: And so much will the aforesayd $\frac{3}{5}$ cost, as by the worke following shall appeare.

Example.

$\frac{3}{4}$	$\frac{3}{5}$	$\frac{5}{8}$	
2	5	3	
48		15	36
96		3	$900\vert\frac{36}{96}\vert\frac{3}{8}$
		45	96
		20	
		900	

Otherwise though 45 could not bee divided by 96: yet by Division in broken numbers it might haue bene abbreviated to $\frac{15}{32}$ of a l, which reduced into knowen partes will make 9 s 4 $\frac{1}{2}$ d as before.

Now my second example shall be the proofe of this question.

If $\frac{3}{8}$ yards cost $\frac{15}{32}$ of a pound or 20 shillings, what shall $\frac{3}{5}$ cost?

Answere. Worke as was taught you before, and you shall haue your desire.

1	11	1
3	15	2
3	2	
32		
96	30	
3	8	
288	240	

Here, appeareth by the worke, the multiplications being ended, 240 is to be diuided by 288, which to some perchance may seeme hard, yet notwithstanding is the worke good: Therefore abbreuiate 240 by 288, as you see here is practised: and the end of your abbreuiation shall come to $\frac{5}{6}$ your desire $\frac{240}{288}\big|\frac{10}{36}\big|\frac{5}{6}$

Otherwise. $\frac{240}{288}\big|\frac{120}{144}\big|\frac{60}{72}\big|\frac{30}{36}\big|\frac{5}{6}$

Otherwise. $\frac{240}{288}\big|\frac{40}{48}\big|\frac{5}{6}$

The third question.

If $\frac{3}{4}$ elles cost 13 s 4 d, what 156 $\frac{1}{3}$ elles?

Answere. To worke this question the shortest way: reduce 13 s 4 d into the parts of a l, which is $\frac{2}{3}$.

Then as you did afore, after you haue set downe the question, the numerator of the first Fraction 3, is pulled downe vnder 4, and the

De-

Denominators of the other 2 fractions multiplied into him which maketh 18 your diuisor.

Then the numerator of the second fraction two is pulled downe vnder three of custome now in sight ready to multiply my third number by: which is performed as soone as the last number 156 $\frac{1}{2}$ is reduced into halfes.

Then lastly I multiply that product by 4, the denominator of my first fraction: it yeeldeth 2504: which I diuide by 18, and my quotient is 139 li. and $\frac{1}{9}$ of a li. remayning, which is worth 2 s 2 $\frac{2}{3}$ d: And so much will 156 $\frac{1}{2}$ Elles cost, as by the worke following doth appeare.

$\frac{3}{4}$ · · $\frac{8}{3}$ · 156 $\frac{1}{2}$ 1

3   2   313   47
6   2   76 (2

18   626   2504 | 139 $\frac{1}{9}$.
4   1888

2504   11

The fourth example.

If two $\frac{1}{2}$ Elles cost 1 $\frac{1}{3}$ li, what commeth 29 $\frac{1}{2}$ Elles to?

Item to the workemanship of this question: first reduce your first number to one direct Numerator: in saying two times 2 is 4, and 1 is 5: Then bring the multiplication of the Denominators of the second and third Fracti-

ons, which maketh 12: and multiply that 12 by fiue your first numerator, it maketh 60, which is your diuisor.

Then the reduction of the second number, which is 5 multiplied with 117 the product of ÿ last nūbers reduction, make 585, which 585 yet resteth to be multiplied by two, the denominator of the Fraction in the first place yeeldeth 1170: which diuided by your diuisor 60: yeeldeth 19 pounds—10 shillings, as appeareth by the worke thereof.

Thus hauing now touched the 12 questions whereof I first pretended, which with diligence and oft practise I trust are sufficient to ayd the desirous vnto the working of any broken numbers, I will now entreate of diuerse necessary Rules incident vnto trafficke, as hereafter followeth.

The fourth Chapter entreateth of Losse and Gaine, in the trade of Merchandise.

If one yard cost 6 shillings—8 pence: and the same is sold againe for 8 s—6 pēce: the question is, what is gained in 100 pounds laying out on such commodity.

Answere. The rule of three direct, applieth two manner of wayes to doe the same, the one is, to say: If 6 $\frac{1}{3}$ giue 8 $\frac{1}{2}$, what giueth 100?

Mul-

Multiply and diuide, and looke what your quotient bringeth forth aboue your laying out, is the neate gaines, and the solution to your question: If you follow the worke, your quotient will bring forth 127 l. —10 shillings, which is 27 l. —10 s more then your principall, and so much is gained in the 100 pounds laying out.

Item, to worke it the other way, which I take the nearest, seeke the difference betwixt the iust price, and the ouer price, which is one shilling 10 pence? Then say by the rule of 3: If 6 $\frac{2}{3}$ shillings gaine 1 $\frac{5}{6}$ shillings, what shall 100 pounds gaine? Multiply and diuide, & you shall finde 27 poundes —10 shillings, and so much is gained in 100 poundes laying out.

Use which of these two wayes you thinke good.

The proofe.

If a yard of cloth be deliuered for 8 s —8 d, whereuppon was gained after the rate of 27 l. 10 s in 100 pounds laying out. The question is, what the yard cost at the first hand?

Answere. Put your gaines to 100 l. all maketh 127 poundes —10 shillings: then say, if 127 l. —10 s giue but 100 l. what giueth 8 $\frac{1}{2}$ shillings? Worke, and you shall finde 8 s —8 d, the true solution to your question.

Yet another Branch or proofe vpon the same first question.

If one yard cost 6 s — 8 d, the question is what price the same is to be sold againe for, to gaine 27 l — 10 s in 100 l laying out.

Answere. Say by the rule of three, if 100 l giue 127 l — 10 s, what giueth $6\frac{2}{3}$ s? Multiply and diuide, and you shall finde 8 s sixe pence, your true solution.

If one Elle cost 7 s — 8 pence, and sold againe for 8 s — 6 d. Question, What is gayned in 20 l, laying out in such commodities.

Answere. Seeke the difference betwixt the iust price and the ouerprice, which is 10 pence, and then apply the Rule of three, as before is taught, saying, If $7\frac{2}{3}$ s giue $\frac{5}{6}$ shillings, what giueth 20 l? Multiply and diuide, and you shall finde 2 l $3\frac{11}{23}$ s, and so much is gained in 20 pound laying out.

The proofe also by an example of losse.

A Merchant hath bought Holland cloth at 8 s — 6 d the Elle, which prooueth not to his expectation, whereuppon he is content to loose 2 l — 3 s — $\frac{11}{23}$ s in 20 pounds laying out. The question is what price ought to bee made of the Elle abating his losse.

Answere. Doe as before in gaines hath bene taught, putting 2 l — $3\frac{11}{23}$ s to your 20 l, all together maketh 22 — $3\frac{11}{23}$ s. Then say by the rule of three: if 22 — $3\frac{11}{23}$ s giue but 20,

whi

what shall come of 8 $\frac{1}{2}$ s? worke and you shall finde 9 s — 8 pence the iust price that the Elle ought to be sold for after the rate of this losse.

Thus it appeareth euidently as in company the rule is appliable as well to gains as losse.

If 20 $\frac{1}{2}$ yards cost 26 l. — 10 s, how shall I sell the same againe to gaine $\frac{1}{3}$ of the principall, or to make of 3, 4: which is all one.

Answere. By the rule of 3, if 3 doe giue 4, what will 36 $\frac{1}{2}$ giue? Multiply and diuide, and you shal find 48 pounds — $\frac{2}{3}$. Then say againe, if 20 $\frac{1}{2}$ yards do giue 48 $\frac{2}{3}$ pounds, as well principall as gaine, what will one yard be worth at that price? Multiply and diuide, and you shall finde 2 pounds — 8 $\frac{5}{241}$ shillings.

If one Elle of cloth cost me 8 s — 8 d, and afterwards I sell 10 $\frac{1}{2}$ Elles thereof for 5 pounds 13 s — 4 d, I would know whether I win or lose: and how much upon the 100 l. of money.

Answere. See first at 8 s — 8 d the Elle, what 10 $\frac{1}{2}$ elles comes to, and you shall find 4 pounds 11 s, and I sold the same for 5 l. — 13 s — 4 d, so that I did gaine vppon the 10 $\frac{1}{2}$ yardes 1 — 2 — 4 d. Then if you would know how much is gayned in the 100 l., say by the rule of three, if 4 l. — 11 s did gaine 22 $\frac{1}{3}$ s, what will 100 l.

gaine: Multiply & divide, & you shal find—24
pounds—10 s—1 d. And so much is gained
in the 100 pounds of money.

If 12 ½ yards cost me 11 pounds—5 s, and
I sell the yard againe for 16 s, the question is,
whether I doe winne or loose, and how much
in or upon the pound of money.

Answere. Looke what the 12 ½ yardes come
to at 16 s the yard, and you shall find 10 pound.
But they cost 11 poundes—5 s. So there is
lost upon the whole, 1 pound—5 s. Then to
know how much is lost in the pound, say by the
rule of 3, if 11 ¼ pounds do loose 1 ¼ pound, what
will 1 pound loose. Multiply and divide, & you
shall finde 2 s—10 d, and so much is lost in the
pound of money.

If I sell the C. weight of any commodity for
4 l, whereupon I doe loose after 10 pounds in
the 100 pounds, I demaund how much I shall
loose or gaine in the 100 poundes, if in case I
had sold the same for 4 pounds 10 s.

Answere. Say if 90 pounds yeeld 100, how
much will 4 give: Multiply and divide, and you
shall finde 4 ⅘. Then say againe, if 4 ⅘ give me
4 ½, what will 100 come to: Multiply and di-
vide and you shall finde 101 pounds ¼, which is
more than 100 pounds by 1 pound—5 s. And
so much is gained in the 100 pounds.

A Merchant hath sold Currans for the
summe of 430 pounds, & he hath gained therein
after

after 10 pounds in the 100 pounds. The questi-on is to know how much he gained in all.

Answere. Say by the rule of three, If 100 pounds doe gaine 10 poundes, what will 430 pounds gaine? Multiply and divide, and you shall find 43, and so much hath he gained in all.

If one yard be worth 28 $\frac{1}{2}$ shillings, for how much shall 10 yardes bee sold, to gaine after 8 l.—6 s.—8 pence in the 100 pounds.

Answere. First adde 8 l.—6 s.—8 pence to 100: Then say if 100 l. do give 108 $\frac{1}{3}$ for princi-pall and gaine, what wil 28 $\frac{1}{2}$ s principall yeeld? Multiply and divide, and you shall finde 30 $\frac{7}{8}$ s. Then say againe by the rule of three, If 1 yard doe give 30 $\frac{7}{8}$ shillings (which is as well the principall as the gaine), what shall 10 yardes give? Multiply and divide, and you shall finde 15 l.—8 s.—9 d. And for the same price shal the 10 yards be sold, for to gaine after the rate of 8 l.—6 s.—8 pence upon the 100 pounds.

A Branch or Proofe out of this Question.

A Merchant hath sold Clothes for 15 l.—8 s.—9 pence, and he hath gained in the whole, the summe of 1 l.—3 s.—9 pence. The questi-on is to know how much he hath gained in the 100 pounds.

Answere. To know this, first rebate the gaines from the price, and there will remaine

14 l—5 s—0 d. Then say by the rule of 3 direct, if 14 l $\frac{1}{4}$ giue me 1 l—3 s—$\frac{1}{4}$ d, what will 100 l giue? Multiply and diuide, and you shall finde 8 l—6 s—8 d the effect desired: the proofe is apparant to the question before.

Yet an other Branch or Proofe of the first question.

If 10 yards bee deliuered for 15 l 8—9, whereuppon was gayned after the rate of 8 l 6 s—8 d vpon the 100 pound, the question is, what the yard did cost at the first hand.

Answere. First say by the rule of three, if 10 with principall and gaine yeeld 15 l—8 $\frac{3}{4}$, what shall 1 yeeld? Multiply and diuide, and you shall finde 30 $\frac{7}{8}$ shillings. Then say againe by the rule of three, if 108 $\frac{1}{3}$ principall & gaine giue but 100, what shall 30 $\frac{7}{8}$ shillings of principall and gaine yeeld? worke and you shall find 28 $\frac{1}{2}$ shillings. And so much did the yard cost at the first peny.

If one yard cost 36 shillings, how much shall 12 yards be sold for to gaine after the rate of 10 pounds in the 100 pounds.

Answere. First say, if 100 giue 110 pounds principall & gaine, what will 36 shillings giue? Multiply and diuide, and you shall finde 36 $\frac{3}{5}$ shillings. Then say againe by the rule of three: If 1 yard of principall and gaine yeelde 39 $\frac{3}{5}$ s, what

what shall 12 yardes gaine? Multiply and divide, and you shall finde 23 l—15 $\frac{1}{7}$ s, which $\frac{1}{7}$ shillings in knowne number is $2\frac{2}{7}$ d. And for the same price shall the 12 yardes be sold, to gaine after the rate of 10 in the 100.

The proofe.

If 12 yards be sold for 23 l—15 s—$2\frac{2}{7}$ d, whereupon is gained after 10 poundes in the 100 poundes. The question is, what the yard cost at the first peny.

Answere. First say, if 12 giue 23 l—15 $\frac{1}{7}$ shillings, what 1 yard? Multiply and diuide, and you shall finde 39 $\frac{4}{7}$ shillings. Then say againe by the rule of three, if 110 poundes giue but 100, what shall 39 $\frac{4}{7}$ giue? worke and you shall finde 36 shillings, the iust price of the yard at the first hand.

Item, when one Merchant selleth wares to another, and he giueth to the buyer 1 l—6 s 8 pence vpon the score, or 20 pounds. The question is, how much shall the buyer gaine vppon the 100 pounds after that rate.

Answere. First adde 1 l—6 s—8 pence vnto 20 pounds, and they are 21 $\frac{1}{3}$. Then say, if 20 l giue 21 $\frac{1}{3}$, what shall 100 giue? Multiply and diuide, and you shall finde 106 $\frac{2}{3}$. So the buyer

getteth after the rate of $6\frac{2}{3}$ l. vpon the 100 l.

Gentle Reader, other necessary questions appertaining to Losse and Gain, you shal haue in the eight Chapter of this Treatise.

The fifth Chapter entreateth of Losse and Gaine vpon Time, wrought by *the double Rule of three: or by the Rule of three* composed, which is contained in foure speciall selected branches or questions of diuers formes, each one of them springing from the first question, and each one of them also being a proofe to other, &c.

If one yard cost me 2 shillings 8 pence ready money, and after I sell the same againe for 2 s 10 d, to be payd for it at the end of the 3 moneths. The question is, what I gaine vpon the 100 pounds in 12 moneths.

Answere. First say, if $2\frac{2}{3}$ gaine $\frac{1}{6}$, what shall 100 pounds gaine? Multiply & diuide and you shall finde $6\frac{1}{4}$ pounds. Then say againe by the rule of three, if three moneths gaine $6\frac{1}{4}$ l, what shall 12 moneths gaine? Worke and you shall finde 25 pounds, and so much shall I gaine in 12 moneths after that rate.

Item, you may also worke it at one working by the first part of the rule of 3 composed, saying

saying, if 2½ shillings in three moneths doe gaine ⅙ of a shilling (which is 2 pence) what will 100 l. gaine in 12 moneths: which for thy further encouragement, the worke of this one example I haue here put downe to verifie that I affirmed in the first part of this Ground of Arts, that this rule, and so all others more reioyceth in Broken then in Whole.

s	moneths	s	l.	moneths
$2\frac{1}{2}$	3	$\frac{1}{6}$	100	12
8				100
3		~~2~~		1200
24		~~7~~		3
60				
~~4~~		262		3600
144		3600 | 25.		
		1444		
		14		

Where the Multiplication and the Diuision being ended maketh 25 pounds your desire.

If a yard be deliuered for 2 shillings 10 pence, to be paid at 3 moneths, whereupon was gained after the rate of 25 l. in the 100 for 12 moneths, the question is now, what the yard cost at the first hand.

Answere. First say, if 12 moneths gaine 25 pounds, what shall 3 moneths gaine? Worke and you shall finde $6\frac{1}{4}$ l. Then say againe the

second time, if 106 $\frac{1}{4}$ pounds giue but 100, what shall 2 $\frac{2}{3}$ shillings giue? Worke and you shall finde 2 shillings 8 pence, which is the iust price that the yard cost at the first hand.

If one yard of Cloth cost mee 2 s — 8 pence ready money, for what terme shall I sell the same againe for 2 shillings — 10 pence, so that I might gaine after the rate of 25 pounds vpon the 100 pounds in 12 moneths.

Answere. First say, if 2 $\frac{2}{3}$ gaine $\frac{1}{6}$, what shall 100 pounds gaine? Multiply and diuide, and you shall finde 6 $\frac{1}{4}$ pounds. Then say againe for the second worke, if 25 pounds become of 12 moneths, what shall come of 6 $\frac{1}{4}$? Worke, and you shall finde 3 monethes, the iust terme of time that the Cloth ought to bee deliuered at 2 shillings 10 pence, to gaine 25 poundes vppon the 100 pounds and 12 moneths.

If one yard cost me 2 s 8 pence in ready money, for what price shall I sell the same againe to be paide at the end of three monethes, so that I may gaine after the rate of 25 pounds in the 100 pounds for 12 moneths.

Answere. First say, if 12 gaine 25 l. what shal 3 moneths gaine? Multiply and diuide, and you shall finde 6 $\frac{1}{4}$ l. Then say for the second worke, if 100 pounds giue 106 $\frac{1}{4}$, what giueth 2 $\frac{2}{3}$ s? Worke and you shal find 2 s — 10 pence, and for that price must the yard be sold to gaine after 25 pounds in ye 100 pounds for 12 moneths.

Many

Many other of these questions I might here haue deliuered, but for feare the booke would rise to too thicke a volume, and so to make the price so much the dearer, whereby it might not be so partable to my Countreymen as I wish it. But these foure, I haue of purpose framed in this order, hauing relation one to another, assuring you, that what questions soeuer may bee proponed within the compasse of this rule, you shall finde by one of these foure to make a solution. And moreouer, diuerse other are yet to be deliuered: where the Creditor giueth diuerse dayes of payment, which can neuer bee well wrought, nor yet vnderstood, vnlesse you can first find by Art, the iust time that al those payments, how different soeuer they be, ought to be payde at once: whereuppon first I thinke good here to giue some instructions into such a Rule, for it is the onely ayde for the finishing of such questions as hereafter shall follow.

The sixt Chapter entreateth of rules of Payment, which is a right necessary Rule, and one of the chiefest handmaides that attendeth vpon buying and selling, &c.

An Example.

A Merchant doeth owe a summe of money, whereof the $\frac{1}{3}$ is to bee payd at 6 monethes, the $\frac{1}{2}$ at 8 monethes, and the rest at a yeare. If he would pay at one payment, the question is, what time ought to be giuen him?

Answere. I haue omitted the quantity of the summe, for you shall vnderstand, the rule is appliable, and yeeldeth a true solution to what summe soeuer shall be proponed. But now for orders sake in teaching, I doe imagine the sum to be 60 pounds, whereupon the maner of this worke is to multiply the proportionate part of the money by the time, as in company. Then 20 being the first payment and the $\frac{1}{3}$ of 60, which $\frac{1}{3}$ multiplied in broken numbers by 6, his time of payment maketh $\frac{6}{3}$, which in whole numbers as appeareth by the example in the margent, maketh two months, next 30, which is the $\frac{1}{2}$ multiplied by his terme 8, yeeldeth 4 monethes, then the rest

$\frac{1}{3}$ by $\frac{6}{1}$ $\frac{6}{3}$	2 monethes
$\frac{1}{2}$ by $\frac{8}{1}$ $\frac{8}{2}$	4 monethes
$\frac{1}{6}$ by $\frac{12}{1}$ $\frac{12}{6}$	2 monethes
	8 monethes

which is 10 ℓ, must needes be abreuiated into the proportiō it beareth to 60, which is $\frac{1}{6}$, which $\frac{1}{6}$ multiplied by his time 12 moneths, produceth $\frac{12}{6}$, maketh 2 moneths. All which added together, as appeareth in the margent, maketh 8 moneths, which is the iust time that all those payments ought to be paide at once.

A Merchant hath 800 ℓ to pay, the $\frac{1}{6}$ thereof ready money, the $\frac{1}{4}$ at two monethes, the $\frac{1}{2}$ at 4 moneths, and the rest at a yeare. The question is, if he would pay all at one payment, what time ought to be giuen him.

Answere. The ready money is neuer multiplied: then $\frac{1}{4}$ multiplied by 2 moneths, as you did before, maketh $\frac{1}{2}$, then $\frac{1}{2}$ by 4 produceth 2 moneths, as appeareth here in the margent. But now for the rest of the money you can not multiply it vntill you haue sought what proportion it beareth to 800 pounds. Therefore you must subtract the ready money, the $\frac{1}{4}$ and the $\frac{1}{2}$ out of the principall. The rest will be $66\frac{2}{3}$ ℓ, which you must looke what part it beareth to the principall, which you shall find to be $\frac{1}{12}$, the same you must also multiply by his time 12 moneths, and it yeeldeth 1 moneth, so all make three $\frac{1}{2}$ moneths, as appeareth in the margent.

$\frac{1}{4}$	$\frac{2}{1}$	
$\frac{2}{4}$		$\frac{1}{2}$
$\frac{1}{2}$	$\frac{4}{1}$	
$\frac{4}{2}$		2
$\frac{1}{12}$	$\frac{12}{1}$	
$\frac{12}{12}$		1
		$3\frac{1}{2}$

A Merchant is to pay 1200 l, in thrée termes, That is to wit 400 l at two wéekes, and 600 l at foure moneths, lastly 200 l at 5 monethes. The question is in what time, they ought to be payd at once.

Answere. Proportionate the parts, and you shall finde that 400 is $\frac{1}{3}$ part, and for 600 you shall finde $\frac{1}{2}$, and likewise 200 is the $\frac{1}{6}$ part, which multiply by their times as before, and you shall haue $\frac{2}{3}$ wéekes, more 8 weekes, and lastly thrée $\frac{1}{3}$ weekes, which together maketh 12 weekes, or thrée moneths your desire.

A Merchant is to pay 600 l in thrée termes, whereof 100 l is payd present, more 300 l at 20 dayes, and the rest at 5 moneths, accounting 30 dayes to a moneth. The question is, what time ought these payments to bee payd at once.

Answere. Worke and you shall finde two moneths.

The

The seuenth Chapter entreateth of *buying and selling in the trade of Merchandise*, wherein is taken part ready money, and diuerse daies of payment giuen for the rest, and what is wonne or lost in the 100 li forbearance for 12 monethes more or lesse according to the quantity of money, or proportion of time, &c.

A Merchant hath bought Sattens which cost 8 shillings the yard ready money. And he selleth the same againe to an other man for 10 shillings the yard, but he giueth two daies for the payment, that is to say, three moneths for the one halfe, and fiue moneths for the other halfe. The question is to know how much the seller doth gaine vpon the 100 li in 12 monethes after that rate.

Answere. Seeke first by the rules of payment at what time those two payments ought to be payd at once, and you shall find foure moneths, at which time the second Merchant ought to haue payd the whole entire payment. And therefore say by the first part of the Rule of three composed. If 8 s in 4 moneths doe gaine 2 s, what will 100 li gain in 12 monethes

8	— 4	— 2	— 100	— 12	
4			12		
32			1200	2400	75
			2	232	
			2400		

Multiply and diuide, and you shall finde 75 poundes, as appeareth in the example, and so much doth the first Merchant gaine vppon 100 pounds in 12 moneths.

A Merchant hath sold fifty clothes at $9\frac{1}{2}$ ℓ the price, to be paide the one $\frac{1}{3}$ at foure moneths, the $\frac{1}{3}$ at fiue moneths, and the $\frac{1}{3}$ at 7 moneths, and the sellers minde is to take no more but after 8 pounds in the 100 for 12 moneths. The question is now what the first Merchant gaineth in the sale of these clothes after that rate.

Answere. First looke what the 50 clothes come to at that price, and you shall finde 475 pounds. Then secondly, according to your direction in the Rules of payment, seeke at what time all the paymentes are to be performed at once. And you shall finde $4\frac{1}{8}$ monethes. Then thirdly say by the first part of the rule of three composed. If 100 ℓ in 12 monethes gaine 8 ℓ, what will 475 ℓ gaine in $4\frac{1}{8}$ moneths? Worke, and you shall find 15 pounds, and $\frac{4}{9}$ of a pound, which is the neate gaines that the first Merchāt hath after the rate aforesayd.

A Merchant hath bought Holland at 7 shillings 3 d the Elle ready money, and he selleth the same againe for 8 s 4 pence the Elle, to bee payd $\frac{1}{4}$ part in ready money more $\frac{1}{3}$ part at two monethes, and the rest at 4 moneths. The question

stion is now to know how much the first Merchant doth gaine vppon the 100 pounds in 12 monethes after the same rate.

Answere. According to the direction deliuered you in the rule of payment, the ready money is not to be multiplied. Then working for the other two payments, to finde out the true proportion at what time they ought to be payd at once, you shall finde for $\frac{1}{3}$ at 2 monethes $\frac{2}{3}$ of a moneth. And the rest of the money which is $\frac{5}{12}$ multiplied by his terme 4 monethes, yeeldeth 1 $\frac{2}{3}$ monethes, both which added together make 2 and $\frac{1}{3}$ monethes, the iust time that both the payments ought to be performed at once. And therefore say by the first part of the rule of 3 composed, if 7 $\frac{1}{2}$ in 2 $\frac{1}{3}$ monethes doe gaine $\frac{13}{24}$. of a li, what shall 100 pounds gaine in 12 monethes after that rate? worke and you shall finde 76 pounds, 16 shillings, 11 pence $\frac{71}{303}$ poundes. And so much doth he gaine vpon 100 pounds in 12 monethes.

A Merchant hath bought 30 clothes at sixe pounds the peece for ready money. Afterward he selleth 10 of them for 7 pounds the peece, for three monethes terme. And the other 20 he selleth for 8 poundes the peece for foure monethes terme. The question is now, what he gaineth vpon 100 pounds in 12 monethes.

Answere. First finde the value of the 30 clothes, which amount to 180 pounds. Secondarily, seeke what the 10 peeces come to at 7 poūds, and what the 20 peeces come to at 8 poundes

the one comes to 70, and the other to 160: both which together make 230, which is 50 l more than they cost. Thirdly, as I haue taught you in the rule of payment, proportionate the first and second prices, vnto the proportion they beare vnto 230 the product of their two prices, and you shall finde $\frac{7}{23}$ for the first, and $\frac{16}{23}$ for the latter. Then fourthly, multiply those parts by their times, and you shall haue $\frac{21}{23}$ and [illegible]: both which together maketh 3 whole moneths, and $\frac{16}{23}$ of a moneth, which is the iust time that both these payments are to be paide at once.

Then say by the first part of the Rule of 3 composed: If 180 l in $3\frac{16}{23}$ moneths doe gaine 50 l, what shall 100 gaine in 12 moneths? Multiply & diuide, and you shall finde $90\frac{10}{51}$ pounds. And so much doth he gaine vpon 100 pounds in 12 moneths.

A Merchant hath bought Synamon which cost him 9 s the l ready money. The question is now, at what price hee ought to sell the 100 weight. To wit 112 pounds, to be paide the $\frac{1}{2}$ at two moneths and the residue at the end of three moneths, so that he may gaine after the rate of 10 pounds vpon a 100 l, for a 12 moneths.

Answere. Seeke first by the Rules of payment what terme both the paymentes ought to be payd at once, where the $\frac{1}{2}$ multiplied by his terme two moneths, maketh $\frac{1}{2}$ monethes.

Like-

Likewise the next payment which is $\frac{3}{4}$ multiplied by his terme 3 moneths, maketh $2\frac{1}{4}$ moneths, both which added together, maketh $2\frac{1}{4}$ moneths, which is the time, that both the payments ought to be paide at once. Then say by the Rule of 3, if 12 moneths doe giue me tenne pound, what will $2\frac{1}{4}$ moneths giue? Multiply and diuide, and you shall finde $2\frac{1}{4}$ pound. Then say againe by the Rule of 3: If one pound cost me 9 $\frac{1}{}$, what will 112 pound cost? Multiply and diuide, and you shall finde 50 pound—8 s. Then say once againe, If 100 pound doe giue $102\frac{7}{24}$, what will $50\frac{2}{5}$ pound giue? Multiply & diuide and you shal find 51 pound—11 s—$1\frac{5}{7}$ d. And for that price ought I to sell 112 pound of Synamon to be payd at two seuerall paiments aforesayd, to gaine thereby after the rate of 10 pound vpon the 100 pound in 12 moneths.

Briefe Rules for our hundreth *waight here at London, which is* after 112 pound for the C.

Item who that multiplieth the pence that one pound weight is worth by 7, and diuideth the product by 15, shall finde how many pounds in money the 112 pound weight is worth.

And contrariwise, he that multiplieth the pounds that 112 pound weight is worth by 15, and diuideth the product by 7, shall finde how

many pence in money the 112 pound weight is worth.

Example.

At 10 d the pound weight, what is 112 pound weight worth?

Answere. Multiply 10 by 7, and thereof commeth 70, the which divide by 15, and you shall finde 4 $\frac{2}{3}$. And thus the 112 poundes is worth 4 l.—13 s.—4 d., after the rate of 10 d the pound aforesaide.

At 6 pound the 112 pound weight, what is one pound worth?

Answere. Multiply 6 l. by 15, and thereof commeth 90: the which divide by 7, and you shall finde 12 d $\frac{6}{7}$. So much is one pound worth when the 112 l. did cost 6 pounds.

The eight Chapter entreateth of Tares and allowances of merchandise *sold by weight, and of losses and* gaines therein, &c.

At 16 l. the 100 suttle, what shall 895 l. suttle be worth in giving 4 l. weight uppon every 100 for treate?

Answere. Adde 4 unto 100, and you shall have 104. Then say by the rule of three, If 104

104 be worth 16 ℓ, what are 895 ℓ worth? Multiply and diuide, and you shall finde 237 pounds 13 —10 $\frac{2}{13}$ d. And so much shall the 895 pounds weight be worth.

Item at 3 s 4 d the pound weight, what shal 754 $\frac{1}{2}$ be worth, in giuing 4 ℓ weight vpon euery 100 for treate.

Answere. See first by the rule of three, what the 100 pound is worth, saying: If 1 cost 3 $\frac{1}{3}$ s, what 100? Multiply and diuide, and you shall finde 15 ℓ $\frac{2}{3}$. Then adde 4 vnto 100, and they are 104. Then say againe by the rule of three, if 104 be sold for 16 $\frac{2}{3}$, for how much shall 754 $\frac{1}{2}$ be sold for? Multiply and diuide, and you shall finde 120 ℓ—18 s 3 $\frac{3}{13}$ d. And for so much shall the 754 $\frac{1}{2}$ be sold for at 3 s—4 d the pound, in giuing 4 vppon the 100.

Other necessary briefe rules there are for the finding of treats, or casting vp of chests of sugar, &c. which for that it is a mystery, I omit. If any lacke instruction that way, they shall finde me ready to pleasure them.

Item if 100 ℓ be worth 36 s 8 d, what shall 800 ℓ be worth in rebating 4 pound vpon euery 100 for tare and closse.

Answere. Multiply 860 by 4, and thereof commeth 3440, the which diuide by 100, and you shall haue 34 $\frac{2}{5}$ ℓ, abate 34 $\frac{2}{5}$ from 860, and

there will remaine 825 ⅓. Then say by the rule
of three. If 100 l cost 36 ⅓ s, what will 825 ⅓ cost
after the rate? Multiply and divide, and you
shall finde 15—2—8 $\frac{16}{31}$. And so much shall
the 860 cost in rebating 4 pound upon every
100: for tare and closse.

Item whether doth he loose more that giveth
4 l upon the 100: or he that rebateth 4 l upon
the 100.

Answere. First note that hee that giveth
4 l upon 100 giveth 104 for 100. And he which
rebateth 4 pound upon the 100 giveth the 100
for 96. Therefore say by the rule of three, if 104
be delivered for 100, for how much shall the
100 bee delivered? Multiply and divide and
you shall finde 96 $\frac{2}{13}$, and hee which rebateth 4
in the 100 maketh but 96 of 100, so that hee
looseth 4 in the 100, and the other which gi-
veth 4 upon the 100, looseth but 3 $\frac{11}{13}$ upon the
100. Thus may you see that he which rebateth
4 in the 100, looseth more by $\frac{11}{13}$ in the 100 l,
than the other which gave 4 upon the 100, for
tare and closse.

If 100 l of any thing cost me 23 s 4 d, the
question is how I shall sell the l to gayne after
the rate of 10 l upon the 100 l.

Answere. Say by the rule of three, if 100 l
give 110 l, what shall 23 ⅓ s give? Multiply
and divide, and you shall finde 1 l $\frac{17}{30}$. Then say
againe

againe, if 100 pound be worth 1 $\frac{1}{9}$ pound, what is one pound worth? Multiply and diuide, and you shall finde 3 d $\frac{1}{17}$. And so much is the pound worth in gaining 10 pound vpon the 100.

Item a Grocer hath bought a C weight of a commodity for 6 pound 10 s. The question is now to know how many pounds thereof he shal sell for 32 s, 4 d, to gaine 20 s in the C weight.

Answere. Adde 20 s vnto 6 pound 10 s, and they make 7 pound — 10 s. Then say if 7 $\frac{1}{2}$ yeeld me 112 pound, what shal 1 $\frac{1}{5}$ pound yeelde? Multiply and diuide, & you shall find 24 pound $\frac{1}{5}$. And so many pound ought hee to sell to gaine 20 shillings in his C weight.

If one pound weight cost 3 s — 4 d, and I sell the same againe for 4 s, what is gained in 100 pound of money layd out in that commodity.

Answere. You may say if 3 $\frac{1}{3}$ s giue 4 s, what will 100 pound giue? But then when you haue found, you must subtract 100 pound out of the product, the rest is your neat againe, or else to produce the neat gaine in your worke at the first. Then subtract the iust price out of the o-ner price, as I taught before in the first begin-ning of losse and gaine, and your conclusion shall bee all one. Multiply and diuide by which of the two waies you thinke good, and you shall finde that he gayneth twenty pound

in the 100 pound.

Item if the pound weight which cost 4 s, bee sold againe for 3 s 4 d, I demaund what is lost in the 100 l of money?

Answere. Say, if 4 s loose $\frac{2}{3}$ s, what shall 100 l loose? Multiply and diuide. and you shall finde 16 l—13 s—4 d, and so much is lost vpon 100 l of money.

Item if a C weight of any commodity cost 45 l, and the buyer repenting would loose 5 l in the 100 l of money, I demand how the pound may be sold: his losse to be neither more nor lesse than after the rate aforesayd of 5 by the hundreth?

Answere. By the rule of 3, if 100 l loose 5 l, what shall 45 l loose? Worke and you shal find 2 $\frac{1}{4}$ pound, which rebated from the principall 45, resteth 42 l—15 s. Lastly say if 112 l yeeldeth but 42—15 s, what 1 pound? Multiply and diuide, and you shall finde 7 s 7 d $\frac{17}{28}$. And so much is the pound worth after that losse.

A Grocer hath bought 3 peeces of raysons, weying 175 $\frac{1}{2}$ pound, 182 $\frac{1}{4}$ l, 191 l, tare for each fraile 2 $\frac{1}{4}$ l, at 25 $\frac{1}{2}$ s the C. weight. The question is what they amount to in money.

Answere 6 l 3 s—4 d $\frac{11}{28}$.

A

A Grocer hath bought 3 sacks of Almonds, weyghing $267\frac{1}{2}$ l' tare 2 pound: $257\frac{1}{2}$ pound tare $2\frac{1}{2}$ pound, 252 pound tare 3 pound at 2 s 10 $\frac{1}{2}$ the pound, what amount they to in money?

I answere 110 pound—12—$3\frac{3}{4}$ d.

The ninth Chapter entreateth of Lengths and Breadths of Arras, and other clothes, with other questions incident vnto Length and Breadth.

If a peece of Arras be 7 elles and $\frac{1}{4}$ long, and 5 elles, and $\frac{2}{3}$ broad: how many elles square doth the same peece containe?

Answere. Multiply the length by the breadth, that is to say $7\frac{1}{4}$ by $5\frac{2}{3}$. And thereof will come $41\frac{11}{12}$ Elles: so many Elles square doth the same peece containe.

Item more, a peece of Arras doth containe 22 elles square, and if the same were in length $3\frac{1}{4}$ elles, I demand how many elles in breadth the same peece doth containe?

Answere. Diuide 22 elles by $3\frac{1}{4}$, and thereof commeth $6\frac{10}{13}$. So many elles doth the same containe in breadth.

Item more, a Merchant hath $3\frac{1}{4}$ Elles of

Arras, at 1 $\frac{2}{3}$ elles broad, which he will change with an other man for a peece of Arras, that is $\frac{7}{8}$ elles square. The question is how many elles of that squarenesse ought the first Merchaunt to haue.

Answere. Multiply the first Merchants peece his length by the breadth, and you shall find it containeth 5 $\frac{5}{12}$ elles, which $\frac{5}{12}$ Elles you shall diuide by $\frac{7}{8}$, and you shall finde 6 $\frac{4}{21}$ elles, and so many elles of that squarenesse ought the latter Merchant to giue the first.

Item a student hath bought 3 $\frac{1}{2}$ yardes of broad cloth, at seuen quarters broad, to make a Gowne, and should line the same throughout with Lambe, at a foote square each skin: the question is now how many skinnes he ought to haue?

Answere. Seeke first the number of yardes square that his cloth containeth, which to doe, multiply 3 $\frac{1}{2}$ his length, by 1 $\frac{3}{4}$ his breadth, and you shall finde 6 $\frac{1}{8}$ yardes square, then say by the rule of three, if 1 yard square giue 9 foote, what shall 6 $\frac{1}{8}$. Worke and you shall finde 55 $\frac{1}{8}$ skinnes.

Item more, a Lawyer hath a rich peece of seeling come home, which is 24 foote and three Inches long, and 7 foote and 2 $\frac{1}{2}$ Inches high, the Ioyner is to bee paide by the yard square: The question is, how many yardes this peece con-

containeth?

Answere. Multiply his length by his breadth, that is to wit $24\frac{1}{4}$ foote by $7\frac{1}{3}$, foote, & you shall finde $174\frac{77}{98}$ foote square, which 174 you shal diuide by 9 (for so many foot make a yard square, and you shall finde 19 yards, 3 foote, and $\frac{42}{77}$ of a foote, and so many yards doth this peece hold.

Item, bought a peece of Holland cloth conteyning 36 Elles $\frac{1}{5}$ Flemmish. The question is, how many Elles English it makes?

Answere. You must note, that fiue Elles Flemmish doe make but three Elles English: Therefore say by the Rule of three, if 5 Elles Flemmish make but three Elles English, how many Elles English will $36\frac{1}{5}$ Elles Flemmish make? Multiply and diuide and you shall find $21\frac{4}{5}$, and so many Elles English doth $36\frac{1}{5}$ Els Flemmish containe. The like is to be done of all others.

Item more, I haue bought 342 Elles Flemmish of Arras worke at two Elles broad, Flemmish, and I would line the same with Elle broad Canuas of English measure. The question is, how many Elles English will serue my turne?

Answere. For as much as three Elles English are worth fiue Elles Flemmish therefore put 3 Elles English into his square, and multi-

plying 3 by himselfe, which maketh 9 Likewise multiply the English Elle which is fiue quarters euery way into him self squarely, and you shall finde 25. Then multiply 342 which is the length of the peece, by two which is the breadth, and thereof commeth 684. then say by the rule of three, as before: if 25 Elles square of Flemmish measure be worth 9 Elles square of English measure, what are 684 of Flemmish measure? Multiply and diuide, and you shall find 246 $\frac{6}{25}$ Elles English.

The same is also wrought by the backer rule of three, in seeking the squares contained in the Flemmish Elle of two Elles broad (which are 18): and also in seeking the squares conteined in the English Elle (which are 25) then say by the rule of three backeward, if 18 quarters require 342 Elles, what shall 25 quarters giue? Multiply & diuide by the rule of three. Reuerse, and you shall finde as before 246 $\frac{6}{25}$ Elles English.

Item more, at two shillings foure pence the Flemmish elle, what is the English Elle worth after the rate?

Answere. Say if three quarters giue 3 $\frac{1}{3}$ s, what giueth fiue quarters? Multiply and diuide, and you shall finde 5 shillings —6 $\frac{4}{5}$ d.

Item more, at 8 s 4 d the Flemmish Elle square, what is the English Elle worth after

ter that rate?

Answere. According to the reason of the last question, consider that a Flemmish Elle square is equall to 9 quarters of a yard English, and an English elle square is equall to 25 quarters of a yard. Therefore say by the rule of 3, if 9 quarters giue $8\frac{1}{5}$ s, what 25 quarters? Worke and finde 23 s — $1\frac{7}{9}$ pence. And so much is the English Elle worth.

Item more, at 6 s — 8 d the Elle square, what shall a peece of cloth cost that is $7\frac{1}{2}$ Elles long, and three $\frac{1}{4}$ Elles broad?

Answere. Multiply the breadth by the length and you shall finde $24\frac{3}{8}$ Elles square. Then say by the rule of three, if 1 Elle square cost $6\frac{2}{3}$ s, what $24\frac{3}{8}$? Multiply and diuide, and you shall find 8 pounds — two s — 6 pence, and so much the same peece of cloth shall cost.

Item more, a Mercer sold 3 peeces of silke. To wit $24\frac{1}{4}$, $23\frac{1}{3}$, and 25 yardes at $9\frac{1}{4}$ shillings the yard, & was glad to receiue in part of payment againe, a cloth contayning $34\frac{1}{2}$ yards at $7\frac{2}{3}$ shillings the yard. The question is now, what the Debitor is in the Creditors debt. Worke, and you shall finde he oweth the Mercer two and twenty pounds — 3 s — two $\frac{1}{2}$ d.

The tenth Chapter entreateth of *the reducing of the Pawnes of Geanes* into English yardes.

Note, that 100 Pawnes do make 26 yards, whereupon 3 Pawnes $\frac{11}{13}$ doe make one yard, and one Pawne after the rate and proportion is $\frac{13}{50}$ of a yard.

In 4563 Pawnes of Geanes, how many yards English?

Answere. Say by the rule of three, if 100 pawnes doe make 26 yardes, what will 4563 pawnes make? Multiply and divide, and you shall finde 1186 yards $\frac{19}{50}$. So many yards doe 4563 pawnes make.

Otherwise, take some other number at your pleasure, as 10 pawnes, which is the $\frac{1}{10}$ part of 100, then to finde his proportion, take the $\frac{1}{10}$ part of 26, which is 2 $\frac{3}{5}$, and then say also by the rule of three, if 10 pawnes giue 2 $\frac{3}{5}$ yardes, what will 4563 pawnes giue? Worke, and you shall finde 1186 $\frac{19}{50}$ yards, as before.

More, at two shillings—6 d the pawne of Geanes, what will the English yard be worth after the rate?

Answere. Say by the rule of three, if $\frac{13}{50}$ of a yard cost two $\frac{1}{2}$ s, what one yard? Multiply, and

and diuide, and you shall finde 9 s — 7 $\frac{1}{13}$ d.

More, if 346 $\frac{1}{3}$ Pawnes cost 30 pounds —— 13 shillings 4 pence sterling, what is that the English yard after the rate?

Answere. Say by the rule of three, if 346 $\frac{1}{3}$ Pawnes cost 30 $\frac{2}{3}$ poundes, what are three $\frac{11}{13}$ Pawnes worth (for so many Pawnes make a yard:) Multiply and diuide, and you shall find $\frac{2300}{27027}$ parts of a pound, which in knowen numbers is worth 6 shillings —— 9 pence $\frac{6271}{9009}$.

The eleuenth Chapter entreateth of Rules of Loane and Interest, with certaine necessary Questions and Proofes incident thereunto, &c.

Item lent my friend 326 l. for 5 $\frac{1}{2}$ monethes simply without any Interest, vppon condition, to haue the like courtesie againe when I neede. But when I came to borrow, hee could spare me but 149 poundes — 8 shillings — 4 d, the question is now how long time I ought to haue the vse thereof, to counteruaile my frendship before time shewed him.

Answere. Say by the backer rule of three if 326 giue 5 $\frac{1}{2}$ moneths, what time will 149 poundes $\frac{5}{12}$, giue? Multiply and diuide, and you shall find 12 moneths, and so long time ought I

to vse his money.

The Proofe.

Item, lent my friend 149 pounds — 8 s. — 4 d, for 12 monethes. The question is now, how much money hee ought to lend me againe for fiue $\frac{1}{2}$ monethes to recompence my friendship shewed him.

Answere, Say by the backer or reuerse rule of 3: if 12 moneths giue 149 $\frac{5}{12}$, what shall 5 $\frac{1}{2}$ monethes giue? Worke, and you shall finde 329 pounds, and so much ought he to lend me to requite my gentlenesse or good turne.

Two other Branches yet more for proofe out of the same questions.

Item, lent my friend 149 poundes — 8 s. — 4 d for 12 monethes, to haue the like friendship againe when I neede. And comming to borrow of him, he very courteously tooke me 326 l. (for that he could well then spare the same.) The question is now, how long I ought to occupie it, not vsurping friendship, but in his due time to restore it againe.

Answere. Say by the Rule of three Reuerse, if 149 $\frac{5}{12}$ giue 12 monethes, what shall 326 poundes giue? Multiply and diuide, and you shall finde, that at 5 $\frac{1}{2}$ monethes terme, I ought to restore it againe.

Proofe

Proofe.

Item lent my friend 326 pound for $5\frac{1}{2}$ monethes. The question is now, how many li. he ought to lend me for 12 monethes to recompence the pleasure againe.

Answer. Worke by the rule of three reuerse, as you haue done before, and you shall finde 149 pounds — 8 s — 4 pence.

Againe, foure other selected questions of *Loane and Interest*, all out of one braunch, and each one also a necessarie Question, *and a particular proofe to other.*

Item, lent my friend 430 pounds at interest for three moneths to receiue after the rate of 8 pounds in the 100 far 12 monethes. The question is, what the Interest commeth to?

You may if you please worke it at two workings by the rule of three direct, in saying: If 12 monethes giue 8 pounds what giueth three moneths? Multiplie and diuide, and it giueth two pound.

Then to the second worke say, if 100 pounds yeeld 2 pounds, what yeeldeth 430 li? Multiplie and diuide, and you shall find 8 li — 12 s, and so much comes the loane of 430 li to for 3 mo-

neths, after the rate of 8 poundes in the 100 poundes of 12 monthes.

Otherwise wrought thus by the rule of thrée at twise also.

If 100 li give 8 li, what giveth 430 li? Multiplie & divide, and you shall find 34 pounds $\frac{2}{5}$.

Then againe for the second worke say. If 12 monethes give 34 pounds $\frac{2}{5}$, what giveth 3 monethes? Worke and finde eight pounds —— 12 shillings as before.

Otherwise yet at one working. By the first part of the rule of 5 numbers forward, in saying, if 100 pounds in 12 monethes gaine 8 li, what shall 430 pounds gaine in thrée monethes? Multiplie the first by the second for your divisor, and the other thrée the one into the other for the dividend, & you shall finde 8 pounds —— 12 shillings, as aforesaied.

Proofe.

Item a friend of mine received of me eight poundes 12 shillings for the interest and use of 430 pounds for thrée monethes terme: the question is now, what he tooke in the 100 poundes for the twelve monethes after that rate.

Answer. For most briefe, say by ẏ first part or rule of 5 numbers forward. If 430 poundes in thrée moneths did pay 8 li —— 12 s. what doth 100 pounds in 12 moneths take after the rate? Worke, & you shall find 8 pounds, and so much he tooke upon the 100 poundes for 12 monethes.

A third

A third queſtion and proofe alſo wrought by the backer rule of 5 Numbers.

Item lent my friend 430 pound to receiue for the intereſt thereof, after the rate of 8 pound in the 100 li. for 12 monethes. The queſtion is now, howe long time my friende ought to giue the vſe thereof, that it may be returned with 8 li.——12 s. gaines.

You may worke it if you pleaſe, by the rule of 3 direct at twice, in ſaying: if 100 poūd yeeld 8 pound, what yeeldeth 430 li.? Multiplie and diuide, and find 34 li. and $\frac{2}{5}$.

Then againe for the ſecond worke ſay. If 34 $\frac{2}{5}$ giue 12 moneths, what giueth 8 $\frac{3}{5}$ pound? Multiplie and diuide, and you ſhall find 3 moneths, and ſo long time ought my friend to vſe it to returne with 8 li.——12 s. gaine.

Otherwiſe at one working by the backer rule of 5 numbers, in ſaying: If 100 li. in 12 monethes do gaine 8 li., howe long time ſhall 430 li. be a gaining of 8 pound——12 s.? Multiply the firſt and the ſecond into the laſt for your diuidend, and the third and fourth multiplie together for your diuiſor, and then diuide, & you ſhall finde thre moneths the iuſt time that my friend ought to vſe it, to returne it with eight pound——12 s. gaine.

A fourth deriued question out of *this Braunch which is a Proofe of* this last, and also of the other two going before.

Item how much money ought a Merchant to deliuer after 8 l. in the 100 for 12 monethes that in 3 moneths he may gaine 8 l. — 12 s.

Answer. You may also if you please worke it by the Golden Rule of three at twise, first saying, if three moneths gaine 8 pound, what will 12 moneths gaine? you shall finde 34 l. Then say againe, if 8 l. — be come of 100 l. what shal come of 34 l. — 8 $\frac{3}{5}$? Worke and you shall find the answer to the questiō, which is 430 pound, and so much ought the Merchant to deliuer.

But most briefly it is answered by the Backer Rule of fiue numbers, where I argue thus, saying: If 100 l. be twelue months a gaining of 8 l. then but for 3 moneths terme onely to take 8 pound — 12 shillings must needes be a good round summe: to worke it, set your number thus, 100 — 12 — 8 — 3 — 8 $\frac{3}{5}$: multiplying the first into the second, and also by 43 the product of the fifth, for your diuidend, and the third and fourth together with 5 the Denominator of your fraction, for your Diuisor: then diuide, and you shall finde as before 430 l. The true solution to your question.

The twelfth Chapter treateth of *the making of Factors which is* taken in two ſorts.

The firſt is, when the eſtimation of the Factor, is taken vpon the ſending of the Merchant, as if the eſtimation of his perſon be $\frac{1}{4}$ it is vnderſtood, that he ſhall haue $\frac{1}{4}$ of the gaine, and the Merchant the other $\frac{3}{4}$.

The other ſort is when the eſtimation of his making is out of the ſending of the Merchant, as if the order and agréement betwéene them were ſuch, that the Merchant ſhall put in 800 ℓ, and the Factor for his making ſhall haue $\frac{1}{4}$, neuertheleſſe he ſhall haue but $\frac{1}{5}$ of the gaine or profite, for the $\frac{1}{4}$ of 800 is 200 (for the eſtimation of his making) which with the 800 pound make 1000 ℓ, whereof the 200 ℓ is $\frac{1}{5}$.

A Merchant doth put in 800 ℓ into the hands of his Factor: vnder ſuch condition, that the ſayd Factor ſhall haue $\frac{1}{4}$. And after certaine time, they finde in profite 124 ℓ — 6 s 8 d, I demaund how much the merchant ſhal haue hereof, and how much ought the Factor to haue?

Anſwere. When the eſtimation of the Factor is out of the ſending of the Merchant, it maketh.

li	s	d	
99	9	4	for the Merchant.
24	17	4	for the Factor.

But if that his estimation be at the sending of the Merchant, then it maketh but

li	s	d	
93	5	0	for the Merchant.
31	1	8	for the Factor.

For the Merchant is then to haue $\frac{3}{4}$, and the Factor $\frac{1}{4}$.

A Merchant doth put into the handes of his Factor 800 li, and the Factor 400 li to haue the $\frac{2}{7}$ of the profite, I demaund now for how much his person is esteemed, when the same is counted vppon the sending of the Merchant.

Answere. According to the tenor and order before prescribed in the first rule, that is, if his estimate be $\frac{1}{4}$, hee shall haue the $\frac{1}{4}$ of the gaine. Therefore say by the rule of 3 direct, If $\frac{1}{4}$ taken put in 400 li, what is the estimate, or putting in of $\frac{1}{7}$ taken? Multiply and diuide, and you shall find 320 li, and so much is the person of the Factor estimated.

Otherwise.

To finde the estimation of the person of the

the Factor, you ſhall conſider, that ſeeing it was agreed betweene them, that the Factor ſhould take the $\frac{4}{9}$, then the Merchant ſhall haue the reſidue, which are $\frac{5}{9}$: wherefore the gaine of the Merchant vnto that of the Factor is in ſuch proportion as 5 vnto 4. Then if you will know the eſtimation of the perſon of the Factor, ſay if 5 giue 4, what will 400 giue? Multipy and diuide, and you ſhall finde 320 l. And ſo much is the perſon of the Factor eſteemed to bee worth.

Other conditions than theſe aforeſaide, may alſo be betweene Merchants and Factors without reſpect, either of ſending or not ſending of the Merchant, where moſt commonly the eſtimation of the body of the Factor is in ſuch proportion of the ſtocke which the Merchant layeth in, as the gaine of the ſaid Factor is vnto the gaine of the Merchant. As thus, If a Merchant doe deliuer into the handes of his Factor 400 pound, and he to haue halfe the profite. The perſon of the ſaide factor ſhall bee eſtéemed to bee worth 400 pound, and if the Factor doe take but $\frac{1}{3}$ of the gaine, he ſhould haue but $\frac{1}{2}$ ſo much of the gaine as the merchant taketh which muſt haue $\frac{2}{3}$, wherefore the perſon of the Factor is eſtéemed but the $\frac{1}{2}$ of that which the Merchaunt layeth in. That is to ſay 200 pound.

And if the Factor did take the $\frac{1}{4}$ of the gaine

then the Merchant shall take the residue which are $\frac{3}{5}$, wherefore the gaine of the Merchant unto the Factor is then in such proportion as 3 unto 2: whereuppon if you will then know the estimation of the person of the Factor; Say if 3 giue 2, what shall 400 giue? Worke and you shall finde $266\frac{2}{3}$ poundes. And so much is the person of the Factor esteemed to bee worth.

And if the Merchant should deliuer unto his Factor 400 pound, and the Factor would lay in 80, and his person, to the end he might haue the $\frac{2}{5}$ of the gaine, I demand at how much shall his person be esteemed.

Answere. Abate 80 from 400, and there will remaine 320, And at so much shall his person be esteemed.

A Merchaunt hath deliuered unto his Factor 900 li to gouerne in the trade of Merchandise, uppon condition that he shall haue the $\frac{1}{3}$ of the gaine, if any thing be gained, and also to beare the $\frac{1}{3}$ of the losse, if any thing bee lost. Now I demaund how much his person was esteemed at.

Answere. Seing that the Factor taketh the $\frac{1}{3}$ of the gaine, his person ought to be esteemed as much as $\frac{1}{2}$ of the stocke, which the Merchant layeth in. That is to say the $\frac{1}{2}$ of 900 pound which is 450. The reason is, because $\frac{1}{3}$ of the gaine that the Factor taketh, is the $\frac{1}{2}$ of the $\frac{2}{3}$ of the gaine that the Merchant taketh, and so the

the Factor his perſon is eſtéemed to be worth 450 pound.

A Merchant hath deliuered vnto his Factor 600 pound. And the Factor layeth in 250 pound and his perſon. Now becauſe he layeth in 250 pound and his perſon, it is agréed betwéene them, that he ſhall take the $\frac{2}{5}$ of the gaine, I demaund for howe much his perſon was eſtéemed?

Anſwer. For as much as the Factor taketh $\frac{2}{5}$ of the gaine, he taketh $\frac{2}{3}$ of that which the Merchant taketh: for $\frac{2}{5}$ are the $\frac{2}{3}$ of $\frac{3}{5}$. And therfore the Factors laying in, ought to be foure hundreth pound, which is $\frac{2}{3}$ of 600 pound that the Merchant laid in. Then ſubtract 250, which the Factor did lay in, from 400 pound which ſhould haue bene his whole ſtocke, and there remaineth a hundreth fiftie pound for the eſtimation of his perſon.

More. A Merchant hath deliuered vnto his Factor 800 pound, vpon condition that the factor ſhall haue the gaines of 160 li, as though he layed in ſo much readie money, I demaund what portion of the gaine the ſayed factor ſhall take?

Anſwer. Sée what part the 160 li (which the Factor layd in) is of nine hundreth ſixtie which is the whole ſtocke of their companie, and you ſhall finde $\frac{1}{6}$. And ſuch part of the gaine ſhall the Factor take.

But in caſe, that in making their coue-

nants, it were so agreed betwéene them that the factor should haue the gaine of 160 pounds of the whole stocke which the Merchant layeth in. That is to say of the 800 pounde, then should the Factor take $\frac{1}{5}$ of the gaine: for 160 is $\frac{1}{5}$ of 800 pounds.

The thirteenth Chapter entreateth *of Rules of Barter, and exchanging Mer-* chandise, which is distinct into 7 Rules, with diuerse other necessarie Questions incident thereunto.

The first Rule

Two Merchants willing to chaunge their merchandise the one with the other. The one hath 24 broad clothes at 10 pound——10 s the péece. The other hath Mace at 12 shillings the pound. The question is how many pound of Mace he ought to giue him for his clothes, to saue himselfe harmelesse and be no loser.

Answere. Séeke first by the Rule of thrée, what the 24 clothes cost at 10 pound———10 s the péece, and you shall find 252 l. Then to find the quantitie of Mace, say againe by the Rule of 3, if 12 s buy one pounde, what shall 252 pound buy me? Worke, and you shall find 420 pound of Mace. And so many pound ought he to

to giue for his clothes.

The Proofe.

Two barter, the one hath 420 pound of Mace at 12 s the pound, to barter or change for broad clothes at 10 pounds ——— 10 s the peece. The question is, how many broad clothes he ought to giue for all his Mace.

Answer. First saye if one cost 12 shillings, what 420? You shall find 5040 shillings. Then say againe, if 10 $\frac{1}{2}$ poundes giue 1 cloth, what shall 5040 shillings giue? Worke, and you shall find 24 clothes your desire.

The second rule.

Two change merchandise for merchandise. The one hath Pepper at 2 s ——— 4 pence the pound, to sell for readie money. But in barter he will haue no lesse than three shillings the pound. And the other hath Holland at 5 shillings ——— 6 pence the Elle readie money. The question is nowe at what price he ought to deliuer the Elle in barter to saue himselfe harmlesse.

Answer. Say by the rule of three direct: if two $\frac{1}{3}$ shillings readie money giue three shillings in barter, what shall 5 $\frac{1}{2}$ shillings giue in barter? you shall find 7 $\frac{1}{14}$ shillings, and at that price ought the second Merchant to sell his Holland in barter.

The Proofe.

Two barter: The one hath Holland at fiue shillings — 6 pence the Elle to sell for readie money: and in barter he will haue 7 $\frac{1}{4}$ s. The other hath pepper at 2 s — 4 pence the pound to sell for readie money. The question is now how he ought to sell in barter.

Answer. Say by the Rule of thrée direct: If 5 $\frac{1}{2}$ readie money giue 7 $\frac{1}{4}$ s in barter, what ought two $\frac{1}{3}$ s to take in barter? Multiplie and diuide, and you shall find thrée s your desire.

The third Rule.

Two barter. The one hath cloth of Arras at 30 s the Elle readie money, but in Barter he will haue 35 $\frac{1}{2}$ shillings. And the other hath white wines, which he deliuered in barter for 16 l for a Tunne. The question is now, what his wines cost the Tunne in readie money.

Answer Say by the Rule of thrée direct. If 35 $\frac{1}{2}$ s in barter giue but 30 readie mony, what did sixtéene pounds in barter cost? Worke and you shall finde 13 pounds — 10 s $\frac{30}{71}$. And so much cost his wines for a Tun readie money.

The Proofe.

Two barter merchandise for merchandise. The one hath white wines at 13 l — 10 s $\frac{30}{71}$

the Tunne to sell for ready money. But in barter he delivered it for 16 pounds. The other to make his match good & saue him selfe harmelesse, delivereth Arras at $32\frac{1}{2}$ s the elle. The question is now, what an Elle of his Arras cost in ready money.

Aunswere. Saye by the rule of three direct. If 16 l in barter giue but 13—10 $\frac{2}{71}$, s in ready money, what shall $35\frac{1}{2}$ shillings yeeld in barter? Worke and you shall finde 30 s your desire.

The fourth Rule.

Two barter. The one hath Kerseis at 14 pounds the peece ready money. But in barter he will haue 18 poundes. And yet hee will haue the $\frac{1}{3}$ part of his ouerprice in ready money. And the other hath ginger at eight groats the pound to sell for ready money. The question is how he ought to deliuer the Ginger by the pound in barter to saue himselfe harmelesse, and make the barter equall.

Answere. Item for the working of this question, and such other the like, you must vnderstand, if the party ouerselling his wares, require to haue also some portion in ready money, as $\frac{1}{2}$: $\frac{1}{3}$: $\frac{1}{4}$ &c. Then shall you first rebate the same demaunded part whatsoeuer it be from the ouer price, and also from the iust price. And those two numbers that shall remaine after the subtraction is made, shall bee the

the two first numbers in the rule of thrée. And the iust price of the same merchandise shall bee the third number, which by the operation of the rule of thrée direct shall yéeld you a true solution how and at what price you shal ouersell that your merchaundise, to saue your selfe harmelesse, and make the barter equall.

Example.

Take the $\frac{1}{3}$ (of eightéene) which is the ouer price of his cloth, which $\frac{1}{3}$ of eightéene is sixe, which as appeareth here in the margent, you must subtract from 18, there resteth 12. And also abate it from 14, which is the iust price of the cloth, and there remaineth 8, which eight and 12 are the two first numbers in the rule of thrée. Then take eight groats or two $\frac{2}{3}$ shillings for the third number. Then say by the rule of 3 direct, If 8 poundes giue 12 pounds, what shall $2\frac{2}{3}$ s giue? Multiply and diuide, and you shall finde 4 s. And for so much shall the second Merchant sell his Ginger, or his commodity in barter, to ballance the same equall.

14 ··	18
6	6
8 ··	12

The proofe.

Two barter. The one hath fine Kerseys at 14 pounds the péece ready money. But in barter

ter he will haue 18 poundes, and yet he will haue the $\frac{1}{3}$ part of his ouerprice in ready money. And the other hath Ginger, which hee hauing cuning inough to make the barter equall, deliuered in barter for 4 shillings the pound. The question is now what his Ginger cost him in ready money.

Anſwere. After you haue made the ſubtraction, abating 6 the $\frac{1}{3}$ part of 18, both from 18 and 14, as before was taught you, then will there remaine 8 and 12 for your two firſt numbers in the rule of three. Then ſay, if 12 giue but 8, what ſhall come of 4 the ouerprice of the pound of Ginger? Multiply and diuide, and you ſhal find two ſhillings —— 8 pence your deſire.

Two Merchants barter merchandiſe for merchandiſe. The one hath Denſhire whites at 7 poundes —— 13 ſ —— 4 d the peece ready money, but in barter he doth them away for 8 l —— 3 ſ —— 4 d. And yet he will haue the $\frac{1}{4}$ part of his ouerprice in ready money. And the other hath Cottens at 3 l the peece ready money. The queſtion is now at what price hee ought to ſell or exchange his Cottens in barter to ſaue himſelfe harmeleſſe, and make the barter equall.

7 —— 13 —— 4	8 —— 3 —— 4
2 —— 14 —— 5 $\frac{1}{3}$	2 —— 14 —— 5 $\frac{1}{3}$
4 —— 18 —— 10 $\frac{2}{3}$	5 —— 8 —— 10 $\frac{2}{3}$

Answere. First seeke $\frac{1}{3}$ part of 8 l —— 3 s —— 4 d, which is 2 l —— 14 —— 5 $\frac{1}{3}$ d, which rebated from 8 —— 3 —— 4 d, there resteth as appeareth by the example aboue said 5 —— 8 —— 10 $\frac{2}{3}$ pence, which $\frac{1}{3}$ of 8 —— 3 —— 4 d also rebated frō 7 —— 13 —— 4 d, there resteth 4 —— 18 —— 10 $\frac{1}{3}$ the two first numbers in the rule of 3. And the 3 pounds which is the steate price of the peece of Cotten is the third number. Then say by the rule of three direct, as was taught before: If 4 —— 18 10 $\frac{1}{3}$ d did giue 5 l —— 8 s —— $\frac{2}{3}$ d, what shall 3 l giue? Multiply and diuide, and you shall find 3 —— 6 —— 8 $\frac{1}{3}$ pence the iust price that he ought to deliuer his Cottens in barter.

The fifth Rule.

Two Merchants will change merchandise for merchandise. The one hath Kerseys at 46 s the peece to sell for ready money. And in barter he will sell them for 56 shillings —— 8 d, and he will gaine after 10 l vpon the 100 l. And yet he will haue the $\frac{1}{3}$ of his ouerprice in ready money. The other hath Flaxe at 3 d the pound ready money. The question is now how he shall sell the pound of his Flaxe in barter.

Answere. See first at 10 l vppon the 100 l what the 56 $\frac{2}{3}$ s commeth to, in saying by the rule of three direct. If 100 l giue 110 l, what 56 $\frac{2}{3}$ s? Multiply and diuide, and you shall finde 3 l —— 2 s —— 4 d of which the $\frac{1}{3}$ that he demandeth

doth in ready money, is 1 li.—11 s.—2 pence, the same 31 s.—2 pence abated from 40 s. and also from 56 s.—8 d. there will remaine 8 shillings—10 pence, and 25 s.—6 d. for the two first numbers in the rule of three, and $\frac{3}{4}$ pence the price of the pound of flaxe for the third number. Then multiply & divide, and you shall find $8\frac{35}{53}$ pence. And for so much shall hee sell the pound of flaxe in barter.

The sixth Rule.

Two are willing to exchange Merchandise, The one hath Norwich Grograines at 25 s. the peece ready money. And in barter he will have 30 s. and he will have the $\frac{1}{4}$ part of his overprice in ready money. The other hath Norwich stockings at 40 s. the dozen to sell for ready money. But in as much as the first Merchants Grograines are no better, hee would deliver them so to ballance the barter, that he may gaine after 10 pounds in the 100 pounds. The question is now how hee shall sell his hose the dozen in barter, according to his request.

Answere. Say, if one 100 give 110, what shall 40 s. give, which is the just price of the dozen of stockings? Multiply and divide, and you shall finde 44 s. Then take the $\frac{1}{4}$ of 30 s. which is 7 s.—6 d. And subtract it from 25 s. and also from 30 shillings, & there will remaine

17 s̄——6 d——and 22 s̄——6 pence, for the two first numbers in the rule of three, and 44 shillings which is the iust price (with his gaine in the dozen of stockings) for the third number. Then multiply and diuide, and you shall finde 56 shillings 6 $\frac{6}{7}$: and for so much he is to sell his dozen of stockings in barter.

The seuenth Rule.

Two Merchants will change their merchandise one with the other. The one hath 720 Elles of Cambricke at 5 shillings the Elle to sell for ready money, but in barter he requireth 6 shillings——8 d. And yet notwithstanding he looseth by it after 10 pounds vpon the 100 poūds, whereupon he requireth the $\frac{1}{3}$ of his ouerprice in ready money, and the other Merchant hauing skill inough to make the barter equall, deliuereth English Saffrons at 30 s the pound. The question is now what his saffron cost the pound in ready money.

Answere. You must first seeke what is lost vpon the 100 pounds, which to do you may say if you please, If 100 l loose 10, what shall 6 $\frac{2}{3}$ loose? Worke & you shall find $\frac{2}{3}$ s̄ (or 8 d) which must be rebated from 6 s̄——8 d. So resteth 6 s still: or you may say if 100 pounds giue me but 90 pounds, what shall 6 s——8 d giue? Worke this way eyther, and you shall finde also as before, directly in your quotient 6 s your desire.

fire. Then are you next to cast vp what the 720 Elles of Cambricke commeth to at 6 s 8 d the Elle, and you shall finde 240 poundes; the $\frac{1}{2}$ whereof the Cambricke Merchant will haue in ready money (which is 120 pounds:) nextly you must cast what the Cambricke commeth to after his losse in the 100 pou̅d, which as you found is but 6 s an Elle, and you shall find 216 pound. Now must you subtract his ready money (which is 120 l.) out of 240 l. and also out of 216 l. and there will remaine 120 pound, and 96 pound for your two first numbers in the rule of three, and 30 shillings is the ouerprice of your Saffron for the third number. Then multiply and diuide, and you shall finde 24 shillings. And so much did his Saffron cost in ready money.

Two Merchants barter. The one hath 50 Clothes to put away for ready money at 11 pound the Cloth, and in barter putteth them away for 12 pound, taking Holland Cloth at 20 d the Flemmish elle, which was worth no more but 18 d. The question is now, what Holland payeth for the Cloth, and what he winneth or looseth by the bargaine?

Answere. 50 Clothes at 11 l. the Cloth commeth to 550 l. and put away at 12 l. the peece, maketh 600 l.: Then to finde what Holland payeth for the Cloth, say by the rule of three direct, If 20 d buy 1 Elle, what 600 l.? Worke,

and you shall finde 7200 Elles. Now to finde the estate of his gaine or losse, you must seeke what his 7200 Elles commeth to at 18 d the Elle. Worke by the rule of Proportion direct and you shall finde 540 pounds, which is not so much as his Clothes were worth in ready money by 10 pounds, and so much lost the first Merchant by his exchange.

A Venetian hath in London 100 peeces of silke, to put away for ready money at 3 pounds the peece. But in barter he delivereth them for 4 pounds the peece, taking wools of a Felmonger at 7 pounds — 10 s the C. weight, which was worth no more but 6 l the C. in ready money. The question is now, what wools payeth for the silks, and which of them winneth or looseth by the barter.

Answere. 100 peeces of silke at 3 pounde, is 300 l, and at 4 l is 400 l. Then to finde what woolles payeth for the silke, say by the rule of three direct. If $7\frac{1}{2}$ l buy me 1 C. weight, what 400 pounds? Worke, and finde $53\frac{1}{3}$ C weight of wooll. Now to finde the estate of their gaine or losse, cast up his wooll at 6 pounds the C (for so much they were woorth ready money) and you shall finde 320 pounds, which is 20 pounds more than the silkes were to be sold for in ready money, whereupon the Venetian gained 20 pounds by the barter.

A

A Merchant hath $53\frac{1}{3}$ C. weight of wooll at 6 pounds the C. to sell for ready money, but in barter he will haue 7 pounds — 10 shillings: and an other doth barter with him for Silkes which are worth 3 pounds a peece ready money. The question is now, how hee ought to deliuer his silkes the peece in barter? and how many payeth for the woolles.

Answere. Say by the rule of Proportion, or the rule of three direct, If 6 li for a C. weight ready money yeeld me 7 li — 10 s, what will 3 li yeeld, which is the iust price of a peece of silke in barter, to make the Trucke equall: Worke, and finde 3 pounds — 15 shillings, the price of a peece of silke in barter: then say, if 3 li — 15 s require 1 peece of silke, how many peeces of silke are bought with 400 li, which is the value of the $53\frac{1}{3}$ C. weight of wooll at 7 li — 10 shillings. Worke by the rule of three direct, & you shall finde 106 peeces of silke, and $\frac{2}{3}$ of a peece, and so many peeces of silke paieth for the wols, and neither party hath aduantage of other.

Two men will change merchandise the one with the other. The one of them hath Beere at 6 s — 3 d the barrell to sell for ready money. But in barter hee will sell the barrell for 8 s, and yet he will gaine moreouer after 10 li vpon the 100 li. And the other hath white Spanish wooll at 20 s the Roue, to sell for ready money. The question is now, how hee shall

deliuer the stone of wooll in barter to saue himselfe harmelesse.

Answere. Say if $6\frac{2}{7}$ s which is the iust price of the barrell of Beere, be sold in barter for 8 shillings, for how much shall 20 s (which is the iust price of the stone of wooll) be sold in barter? Worke by the Rule of three direct, and you shall find 24 s. Then for because the first Merchant will gaine after 10 l. vpon the 100 l., hee maketh of his 100 l.—110 l. And therefore say by the rule of three, if the second Merchant of 110 pound doe make but 100 pound, how much shall he make of 24 s? Multiply and diuide, and you shall find 21 s—9 d $\frac{9}{11}$ of a penny. And for so much shall he sell the stone of wooll to be deliuered in barter, to the end the first Merchant may gaine 10 in the 100.

Two Merchants will change their commodities the one with the other. The one of them hath white paper at 4 s the reame, to sell for ready money. And in barter he will do it alway for 5 shillings, and yet he wil gaine moreouer after the rate of 10 l. vpon the 100 l.: and the other hath Mace at 14 s—6 d the pound weight, to sell in barter. Now I demaund what the pound did cost in ready money?

Answere. Say, if 5 s which is the ouer price of the paper in barter be come of 4 s the iust price, of how much shall come 14 s $\frac{1}{2}$ which is the surprice of the pound of Mace in bar-

barter? Multiply and diuide, and you shall find 11 s̃ $\frac{1}{5}$. Then for because the first Merchant of Paper will gaine after 10 vpon the 100. Say, if 100 doe giue 110, what shall 11 $\frac{1}{5}$ shillings giue? Worke, and you shall finde 12 s̃ — 9 d̃ $\frac{11}{25}$, and so much did the pound of Mace cost in ready money.

The fourteenth Chapter entreateth of exchanging of money from one place to another.

EXchange is no other thing than to take or receiue money in one City, to render or pay the value thereof in an other City, or else to giue money in one place, and receiue the value thereof in an other, at terme of certaine dayes, moneths, or fayres, according to the diuersity of the place.

But this practise chiefely consisteth in the knowledge of the Money or Coynes in diuerse places, of which, for thy benefite, (after a few examples giuen to the Introduction of this worke) I will set downe certaine notes of the diuersity of the common and vsuall Coynes in most places of Christendome for trafficke.

And first I will begin at Antwerpe, where they vse to make their accounts by Deniers de

gros: that is to say by pence Flemmish, whereof 12 doe make 1 shilling Flemmish, and 20 shillings doe make 1 pound de gros.

Item a Merchant delivered at Antwerpe 400 pounds Flemmish, to receive in London 20 s sterling for every 23 s — 4 d Flemmish. The question is now, how much sterling money is to be received at London for the sayde 400 pounds Flemmish?

Answere. Say by the rule of three, if 23 $\frac{1}{3}$ Flemmish give 20 shillings sterling, what 400 pounds Flemmish? Worke, and you shall find 342 pounds — 17 shillings — 1 $\frac{1}{7}$ d, & so much sterling shall I receive in London for the 400 pounds Flemmish.

Otherwise also wrought by Rules of Practise in taking the $\frac{1}{7}$ of the Flemmish money delivered, and abating the same from the principall, the rest is English money sterling, as before.

400	0	0	
57	2	10	$\frac{2}{7}$
342	17	1	$\frac{5}{7}$ sterling.

A Merchant at London delivereth 200 pound sterling for Antwerpe at 23 shillings 5 d Flemmish the pound sterling. The question is, how much he must receive at Antwerpe.

Answere.

Answere. Say by the Rule of three, if 1 l sterling giue 23 s—5 d Flemmish, what 200 pounds sterling? Worke, and you shall finde 234 l—3 s—4 d. So many pounds Flemmish shall he receiue at Antwerpe for the sayed 200 pounds sterling.

Otherwise also by Practise.

1 ·· 23 ·· 5 ·· 200

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3 s 4 d   33 ·· 6 ·· 8

1 d   ·· 16 ·· 8

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maketh 234 l ·· 3 ·· 4 sterling.

In London 200 l sterling is deliuered by Exchange, for Antwerpe at 23 s—9 d Flemmish the l sterling. The question is, what rate the Flemmish money ought to bee returned to gaine 4 l vpon the 100 l sterling at London.

Answere. First say by the Rule of three direct. If 1 pound sterling giue 23 ¾ Flemmish what 200 pounds sterling? Multiply and diuide and you shall finde 237 poundes—10 s. The which to returne to gaine 8 pounds sterling in London, say by the backer Rule, If 200 pounds sterling requirs the exchange 23 s—9 d Flemmish, what the exchāge to make 208 l sterling? Worke by the Rule, and finde 22 s—10 d $\frac{1}{26}$ d Flemmish, the effect in the question required.

If I take vp money at Antwerpe after 19 s̄—4 d̄ Flemmish, to pay for the same at London 20 shillings sterling, and when the day of payment is come, I am forced to returne the same money againe in London to pay my Bill of exchange, so that for 20 shillings which I take vp here at London, I must pay 19 s̄—6 d̄ at Antwerpe, I demaund whether I do win or loose, and how much in or vppon the 100 pound of money.

Answere. Say by the rule of three: If 19 $\frac{1}{3}$ giue 19 $\frac{1}{2}$, what will 100 l̄ giue? Multiply and diuide, and you shall finde 99 l̄—2 s̄ $\frac{106}{117}$ which being abated from 100 l̄, there will remaine 17 shillings $\frac{11}{117}$, and so much doe I loose vppon the 100 l̄ of money.

If I take vp at London 20 shillings sterling to pay at Antwerpe 22 s̄—4 d̄, and when the day of paiment is come, my Factor is constrained to take vp money againe at Antwerpe, wherewith to pay the aforesayd summe, and there he doth receiue 23 s̄—4 d̄ Flemmish, for the which I must pay 20 shillings at London, the question is now whether I doe win or loose, and how much vppon the 100 pound of money after that rate.

Answere. Say by the Rule of Proportion. If 22 $\frac{1}{3}$ s̄, giue 23 $\frac{1}{3}$ s̄, what will 100 pound giue? Mul-

Multiply & diuide, & you shall find 104 pounds 9 $\frac{1}{3}\frac{1}{7}$, from the which abate 100 pounds, & there will remaine 4 pounds 9 $\frac{1}{3}\frac{1}{7}$ s, and so much is there gained vpon the 100 pounds of money.

In Antwerpe is deliuered 200 pounds Flemish by exchange for London, at 20 s sterling for euery 23 s — 4 d Flemmish. The question is at what rate the same is to bee returned to gaine 10 pounds vpon the 100 poundes Flemmish in Antwerpe.

Answere. First say by the rule of three, if 23 $\frac{1}{3}$ Flemmish giue 20 s, what shall 200 poundes giue? Worke, and you shall finde 171 poundes — 8 s 6 $\frac{6}{7}$ d. Then say againe by the rule of 3 direct, if 171 l 8 s 6 $\frac{6}{7}$ sterling, giue me 210 l Flémish, what shalll 20 s sterling giue? worke, and you shall finde 24 s — 6 d Flémish. And at the same rate ought ẏ same to be returned at Antwerpe to gaine 10 l vpon the 100 Flemmish.

A Merchant of Antwerpe deliuereth 234 l — 3 s — 4 d Flemmish, to receiue at London 200 pounds sterling. The question is now how the exchange goeth after this rate.

Answere. Say by the rule of three direct, If 200 giue 20, what giueth 234 $\frac{1}{6}$? Multiply and diuide, and you shall finde 23 s — 5 d? And for so much goeth the Exchange.

Item the Exchange from London into

France, is not like as it is in Flaunders, but is deliuered by the French Crowne, which is worth 50 ſoulx Tournois the peece.

Whereuppon alſo you muſt note, that in France they make their accounts by Franckes, Soulx and Deniers Tournois, whereof 12 Deniers maketh one ſoulx Tournois, and 20 ſoulx maketh 1 li Tournois, which they call a Liure or Francke. But the Merchants to make their accounts doe vſe French crownes, which is curant among them for 51 Soulx Tournois. But by exchange it is otherwiſe, for it is deliuered but for 50 Soulx Tournois the Crowne, or as the taker vp of the money can agree with the deliuerer. And note that this ▽ Caracter repreſenteth the Crowne by exchange, and is euer 50 Soulx Tournois or French money.

A Merchant deliuereth in London 240 poũds ſterling after 5 ſhillings — 8 d the crowne, to receiue at Paris 50 Soulx Tournois for euery Crowne, I demaund how much Tournois or French money payeth the bils for the ſayd 240 pounds ſterling?

Anſwere, Say by the rule of three, if $\frac{1}{2}$ s ſterling giue mee 50 s Tournois, what ſhall 240 li ſterling giue? Reduce the poundes into ſhillings, then multiply and diuide, and you ſhall find 2181 Liures — 16 Soulx, — 4 Deniers

niers, and $\frac{2}{1}$ Tournois, and so much paieth the Billes at Paris for the 240 pounds sterling.

A Merchant deliuereth in Roan, or else where in France 1430 pounds, or Franckes, the which Francke or pound is 20 soulx, or a pound Tournois, to receiue in London 6 s—4 pence sterling for euery △ of 50 soulx Tournois. The question is, how much sterling money I ought to receiue at London for my 1430 poundes Tournois.

Answere. Say, if two $\frac{1}{2}$ poundes giue me 6$\frac{1}{3}$ shillings, what will 1430 giue mee? Worke, and you shall finde 3622 s $\frac{2}{3}$ sterling, which maketh 181 l 2 shillings 8 pence, & so much money is to be receiued at London for the sayd 1430 Liures Tournois, after 6 shillings—4 pence euery Crowne of 50 Soulx.

In London is deliuered 200 l sterling by exchange for Paris, at 5 s—9 pence the ▽ of 50 soulx Tournois. The question is at what price the sayd crowne is to bee returned to gaine 6 l vpon the 100 pounds sterling at London.

Answere. First say by the rule of three direct, if 5$\frac{3}{4}$ s sterling giue 50 soulx Tournois, what shall 200 pounds sterling giue? Worke, & you shall finde 1739 Franckes or Liuers, two soulx $\frac{14}{23}$. Then, the which to returne and gaine 6 pounds vpon the 100 l in London, say by the rule of 3 direct, if 1739 Francks, two Soulx $\frac{14}{23}$

yeeld 212 pounds, what the ∧ of 50 Soulx? worke, and findes 6 s — 1 d $\frac{7}{7}$. the effect required in the question.

A Merchant delivered in London 160 pound sterling, to receive in Biskay for every 5 s 6 d, 1 Duckate of 374 Marveides. The question is, how many Marveides I ought to receive at Biskay.

Answere. Say, if $5\frac{1}{2}$ s sterling give 374 Marveides, what shall 160 poundes sterling give? Multiply and divide, and you shall finde 217600 Marveides, and so many I ought to receive at Biskay for my 160 pounds sterling.

A Merchant delivereth in Bayon 20000 Marveides to receive in London 5 s — 6 pence sterling, for every Duckate of 374 Marveides. The question is now, how much sterling money paieth the Bils of Exchange for the said 20000 Marveides.

Answere. Say, if 374 Marveides make 1 Duckate, what 20000 Marveides? Multiply and divide, and finde 106 Duckates $\frac{178}{187}$.

Then say againe, if 1 Duckate give $5\frac{1}{2}$ s, what giveth 106 $\frac{178}{187}$ Duckates? Worke, and find 30 pounds — 6 s and $\frac{14}{561}$. s which is worth $\frac{408}{561}$ parts of a peny.

Otherwise it is wrought more briefer at one working, as in the last question before, in con-

considering that 5 s̄—8 d, containeth one Duckate, or 374 Marueides. Therefore say by the rule of three, if 374 Marueides giue 5 $\frac{2}{3}$ s̄, what 40000 Marueides? Worke, and you shall also finde in your quotients 30 poundes —— 6 s̄ $\frac{14}{56}$. And so many pounds sterling is to bee receiued for the 40000 Duckates.

In London 200 l' deliuered by Exchange for Vigo 374 Marueides the Duckate, of 5 s̄ 10 d sterling, maketh 256457 $\frac{1}{7}$ Marueides, ẏ which to returne and gaine 10 poundes vpon the 100 pounds in London, say by the rule of three direct, if 220 pounds require 256457 $\frac{1}{7}$ Marueides, what 5 s̄—— 10 d? Worke, and finde 340 Marueides, prices of euery Duckate in returne: which is the effect in the question required.

These may seeme sufficient for instructions.

NOtwithstanding for thy further ayde and benefite hereafter, follow 6 speciall & most briefe rules of Practise for English, French and Flemmish money.

1	teacheth	how to turn Flemish to Eng. sterling.
2		how to turn Eng. sterling to Flemish.
3		how to turne Flemmish to French.
4		how to turne French into Flemmish.
5		how to turne sterling into French.
6		how to turne French into sterling.

The fifteenth Chapter entreateth of the sayd sixe Rules of Breuity, and of valuation of English, Flemmish, and French money, and how each of them may easily be brought to others value.

How briefly to reduce l, s, and d Flemmish, into l, s, and d English Sterling.

It is to be noted, that 7 l Flemmish maketh but 6 pounds sterling, 7 s Flemmish maketh 6 s sterling and 7 d Flemish 6 d sterling, so that 7 yeeldeth but 6. Wherein is euident, that there is lost $\frac{1}{7}$ (if it may be so called) when it is reduced into English money. Wherefore to know how much 233 poundes, 13 s, 4 d, Flemmish maketh English, you must subtract from it $\frac{1}{7}$, beginning with the pounds, &c. and that which resteth after this subtraction is the summe required: so that 233 poundes, 13 s, 4 d Flemmish maketh 200 pounds, 5 s, 8 $\frac{4}{7}$ pence sterling.

Example.				Another Example.		
l	s	d		l	s	d
233	13	4		311	0	0
$\frac{1}{7}$ 33	7	7 $\frac{1}{7}$		$\frac{1}{7}$ 44	8	6 $\frac{6}{7}$
200	5	8 $\frac{4}{7}$ ster.		264	11	5 $\frac{1}{7}$

To

To reduce l, s, and d Sterling, into l, s, and d, Flemmish.

Note that a pound sterling, maketh 1 pound. 3 s 4 d Flemmish, that is 1 pound $\frac{1}{6}$: 1 s sterling maketh 1 s $\frac{1}{6}$ Flem. & 1 sterl. maketh 1 $\frac{1}{6}$ Flem. So that there is gayned (if it may be so called) $\frac{1}{6}$ of the summe being thus reduced to Flem. For of $\frac{6}{6}$ is made $\frac{7}{6}$, which is one whole, and $\frac{1}{6}$. Then to know how much 237 pounds, 7 s 6 d sterl maketh flem. subtract from your sterl. the $\frac{1}{6}$ of the whole summe, and adde it to the same summe, & it maketh 276 pounds, 18 s 4 pence, which is the summe required.

Example.			Another Example.		
l	s	d	l	s	d
237	7	6	337		
$\frac{1}{6}$ 39	11	3	$\frac{1}{6}$ 56	3	4
276	18	9 sterl.	393	3	4

Yee shall note, that the equality of Flemmish and French money is this, that is to say, the pound Flemmish maketh 7 pound $\frac{1}{2}$ Frēch or Turnois, 1 shilling Flemmish maketh 7 s $\frac{1}{2}$ French, and a groat Flemmish, maketh 7 d $\frac{1}{2}$ French.

Wherefore to know how much 143 poundes 4 s, 9 d Flemmish maketh French, yee must multiply the whole number twise by 6, begin-

ning at 8 d, and so forward, and the product of your second multiplication diuided by 5, so that worke is finished. Or multiply the sayd summe by 7, and take out of it 1/5, adding it to the product of your multiplication by 7, and that is your number required. So that as well by the one as by the other, 243 l, 4 s, 9 d Flemmish, maketh 1031 l, 6 s, 2 pence 2/5 French or Turnois.

Example.

l	s	d
143	4	9 flem.
		6
859	8	6
		6
5156—	11—	0
1/5 1031—	6—	2 2/5 fren.

The same otherwise.

l	s	d
143	4	9
		7
1002	13	3
1/5 28—	12—	11 2/5
1031—	6—	2 2/5 fre.

Another example.

143 l Flem.
6
858
6
5148
1/5 1029 l, or 12 s Frẽch.

Or thus.

143
7
1001
1/5 28—12
1029 l, 12 s French.

A

A briefe Reduction of l̄, s̄, and d̄ French, into l̄, s̄, and d̄, Flemmish.

Multiply 233 l̄, 8 s̄, 4 d̄ French by 5, and diuide the product twice by 6, that is the sayde number by 6, and the product againe by 6, and the quotient of this second Diuision is the thing required. So that 233 l̄, 8 s̄, 4 d̄, French, maketh 32 l̄, 8 s̄, 4 d̄, $\frac{1}{9}$ Flemmish.

Example. Another.

	l̄	s̄	d̄		l̄	s̄	d̄
	233	—8—	4 fre.		753		fren.
			5		5		
	1167	—1—	8		2756		
$\frac{1}{6}$	194	—10—	$3\frac{2}{6}$	$\frac{1}{6}$	—627	10	
$\frac{1}{6}$	32	—8—	$\frac{1}{9}$ Fle.	$\frac{1}{6}$	—104	11	8 flem.

To reduce l̄, s̄, and d̄ Sterling, into l̄, s̄, and d̄ French or Turnois.

The l̄ sterling maketh 8 l̄, 8 s̄ French, that is to say 8 l̄ $\frac{2}{5}$: the s̄ maketh 8 s̄ $\frac{2}{5}$, and the peny 8 d̄ $\frac{2}{5}$ French. Wherefore to know what 231 l̄, 13 s̄, 4 d̄ sterling maketh French, yee must multiply your whole summe by 42, that is by 7, and the product of it by 6, and diuide this

second product by 5, and that is the summe required.

Otherwise multiply the summe sterling by 8, and adde twice to the product $\frac{1}{7}$, and it shall produce the sum required. So that both waies 231 l. 13 s. 4 d sterling, maketh 1946 l French, as here under followeth.

Example. | The same otherwise.

l	s	d	l	s	d
231	13	4 ster.	231	13	4 ster.
		6			8
1390	0	0	1853	6	8
		7	48	6	8
			46	6	8
$\frac{1}{7}$ 9730	0	0			
$\frac{1}{7}$ 1946	0	0 fren.	1946	0	0 fre.

Another Example. | The same.

753 ster.	753 ster.
6	8
4518	6024
7	$\frac{1}{7}$ 150 — 12
	$\frac{1}{7}$ 150 — 12
31626	
$\frac{1}{7}$ 6325 — 4 fren.	6525 — 4 fren.

To

To reduce l, s, and d Fren, into l, s, d Ster.

To know how much 1256 l 12 s 6 d Fren. maketh in sterling money, multiply the summa by 5, and divide the product by 7 and 6 at twice, and the last quotient shalbe the thing required, that is to say, 1256 l, 12 s, 6 d, maketh 149 l, 11 s, 11, d $\frac{1}{7}$ sterling.

Example.			Another Example.		
l	s	d	l	s	d
1256	12	6 fren.	2531		fren.
		5	5		
6283	2	6	12755		
$\frac{1}{6}$ 1047	3	9	$\frac{1}{6}$ 2109	3	4
$\frac{1}{7}$ 149	11	11 $\frac{1}{7}$ ste.	$\frac{1}{7}$ 301	6	2 $\frac{2}{7}$ ster.

Note that when any money is giuen by exchange at London for Roan at 71 d $\frac{1}{7}$, or rather 71 $\frac{1}{7}$ for the crowne of 50 s French, there is neyther gaine nor losse, for it is one money for another, accounting 8 l, 8 s French, for 1 l sterling. So the giuer loseth the time of payment, which is about 15 dayes, and hee that taketh it, hath gaine of the same.

They of Roan that put forth or take money by exchange for London, ought to haue like consideration.

Item when any man giueth at London 64 pence $\frac{1}{3}$, or rather 64 pence $\frac{2}{7}$, to haue at one of the Faires of Lions a crowne de Marc, he that ſo giueth the money looſeth the time, & hee that taketh it, gaineth the ſame: for 62 pence $\frac{2}{7}$ is equall in value to 45 s French. He that putteth or taketh money at Lions for London, ought to conſider the ſame.

Item, when any deliuer in Antwerpe 75 pence, to receiue at Lions a crowne of Marke, he that putteth it foorth, looſeth the time, and he that taketh it, gayneth the ſame. For 75 groats Flemmiſh is equall in value to 45 ſhillings French.

Thus for this time I make an end of the practiſe of Exchange and the inſtructions thereunto belonging, and according to my promiſe, yet further to gratifie ſuch as are deſirous to know the common Coynes vſed for trafficke among Merchants in theſe Cities following, here followeth a briefe declaration of their moneis and the reckonings and accounts of them.

The

The sixteenth Chapter contayneth a declaration of the valuation and diuersity of Coines of most places of Christendome for trafficke. And the manner of exchange in those places from one City or Towne to another, which knowne is right necessary for Merchants, by meanes whereof they do find the gaine or losse vpon the Exchange.

ITem for as much as the greatest diuersity of money of exchange is at Lions. Therefore I will begin duely of the money of that place.

At Lions they vse Francks, Soulx and Deniers Tournois: A Francke maketh twenty Soulx, and one Soulx 12 Deniers. But the Merchants to keepe their bookes of accounts do vse French Crownes of the marke at 45 Soulx the peece, and doe diuide it into 20 shillings, 1 shilling, and 12 pence.

Item a Marke of Gold maketh 65 △̃ of the Marke, which serueth for exchange, and diuide it into 8 ounces. The ounce into 24 pence or Deniers, the denier into 24 graines. And so the summe or whole by imagination or gesse.

Also at Lions there are 4 faires in a yeare, at the which they doe commonly exchange, which are from thrée moneths to thrée moneths.

At Geanes they vse the Soulx: one Ducket maketh l 3.

At Naples they vse Duckets, Tary, and Graines. The Ducket maketh 5 Taries, and one Tary 20 Graines: but they take 6 Duckets (which maketh thirty Taries) for the ounce.

A Ducket maketh 10 Carlins, and a Carlin 10 graines, so that 2 Carlins make a Tary, and 100 graines make a Ducket.

At Rome they vse the Duckets of the chamber: one ducket is worth 12 Guylis, & a Guylis 10 Soulx.

At Venice they vse Duckets currant at 124 Soulx a peece or 24 deniers, and 1 denier maketh 32 picolis.

At Palerme and Messine they write, after ounce, tary, and graines, and 1 ounce is worth 6 duckets or 30 taries, and 1 tary is 20 graines, and 1 graine 6 picolis, 1 Ducket is also worth 24 Carlins.

At Millan they vse l, s, d, of duckets Imperials, and △ of exchange is worth 4 l.

At Lucques, Florence and Ancone, they vse the △ of gold: in golde the French Crowne is worth l 7, but at Buloigne l 3, 10 s.

At Barselone they vse the Soulx, the Ducket of exchange is worth 22 Soulx.

At Valence and Saragosse they vse the Liuer, Soulx and Denier, the French crowne of exchange is worth 20 Soulx, and 1 Soulx is

12 Deniers.

At the Fayres of Castil they vse the Maruedes, the Ducket is worth 375 Marueides.

At Lisbone they vse the Raies, one Ducket of exchange is worth 400 rayes.

At Noremburge, Franckford, and August in Germany, they vse the Krentzers, whereof 60 make a Florin.

At Antwerpe they vse ℔, and pence de Gros, and they exchange into the denier de Gros, to wit, our English peny.

At London they vse the 1 ℔ sterling, & 1 peny sterling, and they exchange in 1 peny sterling.

The exchange of Lyons at sundry places.

Item at Lyons there is exchange in three sorts, at the cities and townes following.

First they deliuer at Lyons one Marke to haue or receiue at Naples almost 41 $\frac{1}{3}$ duckets, at Venice 70 duckets currant, at Rome 53 duckets of the Chamber, Luques and Florence 65 △ of gold, at Millan 82 △.

And contrariwise at the sayd Cities aforesayd they doe giue so much of money to haue a Marke at Lyons.

Secondly, they giue at Lisbone one △ of Marke of 45 Soulx Turnois a peece, to haue at Gennes almost 68 Soulx. At Palerme and Messine almost 24 Carlins. At Barselone 22 Soulx, at Valence or Saragosse 20 Soulx. At the faire at Castil 350 Marueides, at Lis-

bone 360 Rais, in Antwerpe 57 deniers de Gros, and at London 70 pence sterling.

And contrariwise they doe giue in the sayde Cities almost as much of their money to haue a French crowne, of the marke at Lions.

Thirdly they doe giue at Lions a △ of the sunne to haue almost 93 Krentzers at Franckford, Ausburg, Noremberge, or other Cities in Almaine.

Also at Lions onely they doe pay the change the $\frac{1}{3}$ in gold, and $\frac{1}{3}$ in money, or else all in money, in giuing 1 $\frac{1}{3}$ for the hundreth.

Changes at Naples and other Townes.

Item at Naples they giue or deliuer almost 112 Duckets to receiue at Rome 100 Duckets of the chamber at the old valne.

Through Luques and Florence they deliuer 100 Duckets Carlins, to receiue there almost 86 △ of gold.

Through Palerme and Messine one Ducket of 5 Cary to receiue there almost 164 graines.

Through Millan one ducket, to receiue there almost 90 Soulx.

Through Geanes one Ducket, to receiue there almost 65 Soulx. The whole summe to be payde within 10 dayes after the sight of the bill of Exchange.

Also at Naples they deliuer one Ducket, to receiue in Antwerpe almost 67 pence or deniers de Gros, within 2 moneths. At London almost

most 60 pence sterling in thrée moneths. At Barselone almost 20 Soulx within two monethes. At Valence almost 18 Soulx within two moneths. At Lisbone 333 Raies, within thrée moneths, & at the faire at Castill, almost 340 Marueides at the same faire.

Change of Venice to other places.

At Venice they deliuer 100 duckets currant, to receiue in Almaine almost 140 Florenes, at 60 Krentzers the peece.

At Lucques and Florence almost 180 △ of gold in 10 daies.

Likewise at Venice they deliuer a Ducket currant, to receiue at Palerme and Messine almost 21 Carlins, at Millan almost 93 Soulx. At Geanes almost 62 Soulx, the whole at 10 daies end.

Of the Pair or Pari.

As touching the Exchange, it is necessary to vnderstand or know the Pair, which the Italians call Pari, which is no other thing than to make the money of the change of one City or Towne to agrée with the money of another, by meanes whereof they doe finde the gaines or losse vpon the exchange.

Example.

Item hauing receiued letters of credite of one of Antwerpe that the △ of the Sunne is there worth 7 Soulx. The question is what the same is worth at London when the Pair

or exchange goeth for 23 shillings.

Answere. Say if 23 giue but 20, what giueth 7? Worke and find 6 shillings 1 $\frac{1}{3}$ d. and so much is the △ of the Sunne worth at London.

The seuenteenth Chapter contayneth also a declaration of the diuersity of the weights and measures of most places of Christendome for trafficke. At the end of which discourse are two Tables, the one for weight, and the other for measure, proportionate and reduced to an equality of our English measure and weight, by the aide whereof the ingenious may easilly by the Rule of three, conuert the one into the other at their pleasure, &c.

At London and so all England through are vsed two kindes of weights and measures, as the Troy weight & the Haberdepoise: from the Troy weight is deriued the proportion and quantity of all kinde of dry & liquide measures, as Pecks, Bushels, Quarters, &c. wherewith is bought and sold all kinde of grayne and other commodities mette by the Bushell. And in Liquid, Ale, Beere, Wine, Oyle, Butter, Hony, &c. vpon these groundes and statutes is bread made, and sold by the Troy weight. And so is Golde, Siluer, Pearle, Precious stones and Jewels. The least quantity of this Troy weight

weight is a graine: foure and twenty of these graines make a peny weight, twenty peny weights an ounce, and 12 ounces a pound: 2 pounds or 2 pintes of this weight maketh a quart. And so ascending into bigger quantities is produced the measures whereby are sold our other naturall sustenance, viz. Ale, or Beere, with also other necessary commodities, as Butter, Hony, Herrings, Eeles, Sope, &c. All which last before rehearsed, though their measures (wherein they are contayned) bee framed and deriued from the Troy weight, yet are they in trafficke, with diuerse commodities, as Lead, Tinne, Flaxe, Waxe, with all other commodities both of this Realme, & of other forraine Countries whatsoeuer, bought and sold by the Haberdepoise weight, after 16 ounces to the pound, and 112 pounds to the C. weight. And vnto euery C. is allowed 12 l. weight at the cōmon beam. From hence is also deriued ẏ weigh of Suffolke Chéese, which containeth 32 Cloues, 8 pounds to a Cloue, and weygheth in all 256 pounds. And also the barrell of Suffolke butter is or should be of like weight with the weigh of Chéese, viz. 256 poundes. More 14 of these pounds make a stone, and 26 stone contayneth a sacke of English wooll. Forraine wools, to wit, French, Spanish, and Estrich, is also sold by the pound or C. weight, but most commonly by the Roue, 25 pounds to a Roue: other commodities of Sale are bought and solde by the C.

fiue scores to the C. Except headed ware, to wit, cattell, nailes, and fish, which are sold after sixe score to the C. There is also two other sortes of measures, to wit, the Elle, and the Yard. By the Elle is vsually mette linnen Cloth, as Canuas, &c. And by the yard, Silkes, woollen clothes, &c.

Antwerpe.

At Antwerpe are also 2 sortes of waightes, their gold and siluer weight, and their common weight. Gold and siluer is weighed by the Marke, the Marke is 8 ounces, the ounce 20 Esterlings, & the Esterling 32, as our graines. The Goldsmiths diuide that into smaller, but not the Merchants: the proofe of gold is made by Caracts, whereof 24 maketh a Marke of fine gold: the Caract is 24 graines, the proofe of the money is made by Deniers, 12 Deniers is 1 shilling fine, that is a Marke of fine siluer, the Denier also is diuided into 14 graines, and the graine into foure quarters.

Item, 100 Markes in Antwerpe, Troy weight, maketh at Lions 103 Markes, two $\frac{1}{2}$ ounces, and 20 graines 23 p̃. At Noremburg 103 Markes, 2 $\frac{1}{2}$ ounces, 2 Quints, 3 Deniers, at Franckford 105 Markes, at Ausburge 104 Markes, three ounces, 1 quint. At Venice 103 Markes, 1 ounce, 7 Deniers, 18 graines. At London 66 pounds.

The Marke of Gold or Siluer at Antwerpe

Troy

Troy weight, which is 8 ounces, maketh 7 $\frac{1}{2}$ ounces common weight, with which all other merchandise is weyed. So that ÿ Troy weight is greater than the common weight by sixe $\frac{1}{4}$ in the C. By this weight of Troy they also weigh Muske, Amber, Perle, &c.

All silkes are bought at Antwerpe, by the Bruges Elle, which is greater than the cōmon measure, by which they retaile, by two in the C. Their common Elle is $\frac{3}{4}$ of our yard, and $\frac{1}{7}$ of our Elle.

Lyons.

At Lyons is used 3 sorts of weights, whereof the first is the common Towne weight, with which they weigh all kinde of Spicery, and diverse other merchandise. The second is called Geneua weight, which is eight in the 100 greater than the common weight, with which they weigh silkes, &c. The third is French weight, called commonly the Marke weight, and 100 poundes thereof maketh 106 $\frac{1}{4}$ poundes Geneua, and 114 $\frac{1}{4}$ of their common weight: with which French weight is weyed all things that payeth custome or toll.

At Lyons is also used two sortes of Elles or Aulnes. The one wherewith they measure grosse Clothes, as Canuas, and such like. The other is called the French Elle or Aulne, with which they measure all other kinde of Merchandise, whereof seuen common Towne Elles

maketh 11 ordinary French Elles.

Roan.

At Roan 6 $\frac{1}{2}$ Muydes of salt being the mea-
sure of the place, make an hundred at Armui-
den in Zeland, and the C. of Bronage measure
of Armuiden maketh at Roan 11 Muides. 30
Mines make a Last of Corne, and 16 a Last of
Oates, 100 pounds weight there, maketh at
London 114 $\frac{1}{4}$, and 109 $\frac{2}{3}$ at Antwerpe. And 100
Elles make at London 115 $\frac{2}{5}$.

Noremburge.

A 100 pounds weight of Noremburge, ma-
keth at London 111 $\frac{1}{3}$: at Antwerpe 107 $\frac{1}{2}$. And
100 Elles of Noremburge, make at London 75
$\frac{3}{5}$, at Antwerpe, 95 $\frac{1}{5}$, &c.

Lisbone.

The 100 weight of Lisbone maketh foure
Roues, euery Roue 32 pounds, so that their C.
weight is 128 pounds, and their pound contay-
neth 14 ounces, & 100 pounds of their weight,
maketh at London 113 $\frac{1}{2}$.

Their Silke, cloth of Gold, and woolen, is
measured with a measure which they call a
Cubite, contayning about $\frac{3}{4}$ of a Varre of Ca-
stile. Howbeit, their common measure is cal-
led a Varre which maketh 5 Palmes, and con-
taineth 1 $\frac{1}{4}$ of a Varre of Castile, our Elle of
London is equall with the Varre at Lisbone.

All

All kinde of merchandise brought from Flanders, Roan, or Brittaine, payeth at Lisbone, as a duety or custome to the King, 20 in the 100, which they call the tenth in merchandise, and the other tenth in money.

Note also that all kind of merchandise comming to Lisbone by land, payeth lesse in custome than that that commeth by water.

Ciuill.

The Roue of Ciuill is 30 poundes, 4 Roues make their C. weight, which is 120 poundes. The 100 poundes of Ciuill maketh at London 120 pounds. Their other common measure is a Uarre, whereof 100 maketh at London 74 Els, and at Rome 40 Canes, &c.

Venice.

At Venice be two sortes of weights, the one called La Grosse, the other La Suttle, with the grosse is weighed all kinde of great wares, and with the small, all kind of spicery, and such like: 96 pounds of grosse weight there, maketh at London 100 pounds, and 100 pounds of spicery there without any tare or allowāce, make at London 64, and with tare 56.

Their other common measure are Braces, whereof 100 make at London 55 $\frac{1}{2}$ Elles, at Antwerpe 92 $\frac{1}{2}$, &c.

Florence.

At Florence the 100 ℓ weight, maketh at

Aquila, for Saffron 110, and 145 poundes of Florence make at Roan 100 pounds, the waight of Florence, and that of Luke is all one.

Their other measures are braces, whereof 100 maketh at Antwerpe Burges measure 81 $\frac{2}{3}$ Elles, 100 Braces there make at London 49 Elles, &c.

Lucque.

The Lucque Sattens are commonly sold at Lions by weight, & 133 $\frac{1}{3}$ pounds, maketh at Lions 100 li, so that 1 li $\frac{1}{3}$ maketh at Lions but 1 li.

Their other measures are Braces, whereof 100 of them make at London 50 Elles, of Antwerpe 83 $\frac{1}{3}$ Elles, &c.

Aquila.

At Aquila their 100 pounds maketh at London 71$\frac{1}{2}$, their 136 $\frac{2}{3}$ pounds of Saffron maketh at Geneua but 100, and 11 pounds of Geneua maketh 15 pounds at Aquila.

Valentia.

At Valentia be 2 sortes of weights, a great and a small. The C. weight of great weight containeth 4 Roues, the Roue 36 li, so the 100 great weight is 144 li, and the C. weight small containeth but 120 poundes, and is also parted into 4 Roues, which is 30 pounds to a Roue. By the small is sold the Scarlet graine, with all other kind of spicery, & by the great is sold wooll with all such like grosse wares. The 1 $\frac{1}{3}$ poũds of

of silke at Valentia maketh at Lions 1 ℔ Geneua weight. The charge of great merchandise at Valentia containeth 432 ℔, & in small wares 360 ℔. The weight here and at Barsellone is all one. Their 100 poundes waight maketh at London 78 ℔, and at Antwerpe 75.

Dansicke.

At Dansicke or Spruce land the rule is, that whosoeuer buyeth any merchandise there, buyeth it by the Ship-pound, which is 320 ℔, 20 Lispounds make a Shippound, and the Lispound containeth 16 ℔, which Shippound of Dansicke maketh at Antwerpe 266 $\frac{1}{3}$ ℔. Their 100 pounds waight maketh at London 86 $\frac{5}{8}$, &c.

Their other common measures are Elles, whereof 100 make at London 72 $\frac{1}{4}$, and at Antwerpe 120 $\frac{1}{2}$ Elles.

Toulouse.

At Toulouse 6 Cabes of Woad, maketh a Charge, 2 Cesternes of Corne, and all kind of graine maketh a Charge, the Cesterne weigheth 169 ℔ weight of that place. Their 100 ℔ in weight maketh at London but 91 $\frac{1}{4}$ ℔.

Geanes.

At Geneua or Geanes, a 100 ℔ of their weight maketh at London 71 $\frac{1}{4}$, and at Antwerpe 68 $\frac{1}{3}$, 100 ℔ weight at Geneua maketh at Venice, to wit, Suttle 106 ℔.

Their other common measures are Palmes, whereof 100 make at London 20 1/4 Elles, and at Antwerpe 34 1/3.

The eighteenth Chapter entreateth of *Sportes and Pastimes*, done by Number.

If you would know the number that any man doth thinke or imagine in his minde, as though you could diuine, bid him triple it, or put twise so much more to it, as it is, which done, aske him whether it be euen or odde: if he say odde, bid him take 1 to it, to make it euen, and for that 1 keepe 1 in your minde. Now after he hath taken 1 to it to make it euen, bid him giue away halfe, and keepe the other halfe for him selfe, which when he hath done, bid him triple that halfe, and againe after hee hath tripled it, aske him whether it be euen or odde: if he say odde: then bid him take to 1 make it euen again, and for that last 1 keepe 2 in your mind, now after he hath made his number euen, bid him cast away the one halfe, and keepe the other still, from which halfe that hee keepeth, cause him subtilly to put away or giue you 9 out of his number, and for each 9 that he giueth you, keepe 4 in your mind, and thereunto ioine the 3 which I bad you keepe, and you shall haue your desire.

Example.

Example.

Imagine he thought 7, the triple whereof is 21, and because it is odde, he is to take 1 to make it euen, which first 1 giuen is, for you to keepe in minde. Then the halfe of his 22 being cast away, he reserueth still 11, which after you haue bid him triple, it maketh 33, then in giuing of him 1 againe to make it euen: vpon that last 1 reserue 2 in your minde, then his halfe of 34 maketh 17, from whence he can giue you 9 but once. Therefore that yeelding to you 4, and the 3 that you keepe, make 7 your desire.

An other kinde of Diuination, to tell your friend how many pence or single peeces, reckoning them one with another, hee hath in his purse, or should thinke in his minde.

Which to doe, first bid him double the peeces he hath in his purse, or the number he thinketh, if he participate his number or secrecy vnto some one friend that sitteth by him, that can but multiply, and adde neuer so little: if their number be great, then shall they worke as you bid them so much the surer.

Now after hee hath doubled his number, bid him adde thereunto fiue more, which done, bid him multiply that his number by fiue also: which done, bid him tell you the iust summe

of his last Multiplication, which summe the giuer thinking it nothing auaileable, because it is so great aboue his pretended imagination: yet thereby shall you presently with the helpe of Subtraction tell his proposed number.

The Rule is this.

Imagine he thought 17: double 17, and it maketh 34, whereunto if you adde 5, it maketh 39: which multiplyed by 5, as here is practised in the margent, it yeeldeth 195, which 195 is the summe deliuered you in the worke: then for a generall rule, you shall euermore cut off the last figure towards your right hand, with a dash of your pen, as here is performed, as a figure nothing auaileable vnto your worke,	17 2 34 5 39 5 19 | 5 5 | 17

and then rebate 2 from your first figure, after 5 is cut off, and the rest shall euermore bee your desire, as by this example doth appeare.

If in any company you are disposed to make them merie by manner of Diuining, in deliuering a Ring vnto any one of them, which after you haue deliuered it vnto them, that you will absent your selfe from them, and they to deuise after you are gone which of them shal haue the keeping thereof, and that you at your returne will tell them what person hath it, vpon what hand, vpon what finger, and what ioynt.

Which

Which to do, cause the persons to sit downe all on a row, and to keepe likewise an order of their fingers: now, after ye are gone out from them to some other place, say unto one of the lookers on, that he double the number of him that hath the King, and unto the double bid him adde 5, and then cause him to multiply that Addition by 5, and unto the product bid him adde the number of the finger, of the person that hath the King. And lastly, to end the worke, beyond that number towardes his right hand, let him set downe a figure signifying uppon which of the ioynts he hath the ring, as if it be uppon the second ioynt, let him put downe 2. Then demaund of him what number he keepeth, from the which you shall abate 250. And you shall haue three figures remaining at the least. The first towardes your left hand shall signifie the number of the person which hath the King, the second or middle number shal declare the number of the finger, and the last figure towardes your right hand, shall betoken the number of the ioynt.

Example.

Imagine the seventh person is determined to keepe the King upon the fifth finger, and the third ioynt: first double 7, it maketh 14, thereto adde 5, it maketh 19, which multiplied by 5, yeeldeth 95: unto which 95, adde the number of the finger, and it maketh 100; and be-

yond 100 toward the right hand, I set downe 3 the number of the ioynt, all maketh 1003, which is the number that is to be deliuered you, from which abating 250, there resteth 753, which prefigureth vnto you the seuenth person, the fifth finger, and the third ioynt.

But note, that when you haue made your substraction, if there doe remaine a 0 in the place of tennes, that is to say, in the second place, you must then abate 1, from that figure which is in the place of hundreds, that is to wit, from the figure which is next your left hand, and that shall be worth 10 tenths, signifying the tenth figure, as if there should remain 803, you must say, that the seuenth person vpon his tenth finger, and vpon his third ioynt, hath the Ring.

And after the same manner, if a man doe cast three Dice, you may know the points of euery one of them. For if you cause him to double the pointes of one Die, and to the double to adde fiue, and the same summe to multiply by fiue, and vnto the product adde the points of one of the other Dice. And behind the number towardes the right hand to put the figure which signifieth the points of the last Die, and then to aske what number hee keepeth, from which abate 250, and there will remaine 3 figures, which doe note vnto you the pointes of euery Die.

An

Another.

If three diuerse things are to bee hidden of three diuerse persons, and you to diuine which of the three persons hath the three diuers things, doe thus: imagine the three things to be represented by A, B, C, Then secondly keepe well in your minde which of the persons you meane to be the first, second and third. Then take 24 Counters or stones, and your three things, and giue A to the party whom you imagine to bee your first man, and therewithall giue him one of your 24 Counters in his hand. And B, unto your second man, and therwithall 2 Counters. And C unto your third man, and therewithall 3 Counters. And leaue the rest which are 18 still among them, which done, seperate your selfe from them, & afterwards bid them change the things among them as they shall thinke good: which done, after they are agreede, bid him that hath such a thing, as before you haue represented by A, for euery Counter that he hath in his hand to take up as many more. And for him that hath B, for euery one in his hand to take up two. And for him that hath C, for euery one in his hand to take up 4, and the rest of them leaue still uppon the boord. These three things and the three persons being fully printed in your minde, come to the Table, and you shal euermore find one of these sixe numbers, 1. 2. 3. 5. 6. or 7. If therefore one remaine still up-

on the boord, then haue they made no exchange, but kéepe them ſtill as they were deliuered vnto them. So that the firſt man hath A, the ſecond B, and the third C. But if 2 remaine, then the firſt man hath B, your ſecond man A, and your third man C. The reſt of the worke and the order thereof, are here apparant by the Table following.

1	1	A	5	1	B
	2	B		2	C
	3	C		3	A
2	1	B	6	1	C
	2	A		2	A
	3	C		3	B
3	1	A	7	1	C
	2	C		2	B
	3	B		3	A

An

Another Diuination of a number vpon the casting of two Dice.

First let the caster cast both the Dice, and marke well the number: then let him take vp one of them, it maketh no matter which, and looke what number it hath in the bottome, and adde all together: then cast the Die againe, and keepe in his minde what altogether maketh: then let the Dice stand: bring seuen with you and thereunto adde the rest of the pits that you see vpon the vpper side of the Dice, and so many did the caster cast in all.

FINIS.

I. D. To the earnest Arithmetician.

MY louing friend to Science bent,
Some thing thou hast by this booke wonne:
But if thou wilt be excellent,
Another race thou must yet runne.

Supplies thereto but (few doe) neede,
And none but such as in our phrase,
(By Records *pen) thou maist well reade,*
Proceede therefore. Be not stunt awase.

The ground most sure, whereon this race,
With speedefull courage must be past,
Of late hath turnd his Greekish face,
By English tilth, which aye will last.

The famous Greeke of Platos *lore,*
Euclide *I meane Geometer:*
So true, so plaine, so fraught with store,
(As in our speach) is yet no where.

A treasure strange, that booke will proue,
With numbers skill, matchd in due sort,
This I thee warne of sincere loue,
And to proceede doe thee exhort.

Plus oultre.

Imprinted at London by Richard
Field, *for Iohn Harriſon*, *dwelling*
in Pater noſter Row at the
ſigne of the Greyhound.
1596.

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Mrs

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yard 23 . Crojeck yard
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James Powell

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—	10	4
—	6	6
—	10	—
—	2	6
—	8	9
	10	—
8	1	6
—	16	6
		6
—	7	6
4	10	7½
12	7	9
—	4	—
4	16	—
		3
—	10	3
—	11	
—	4	2
—	13	—

4	14	7
—	10	8
—	5	3
—	2	9
—	1	10
—	7	—
—	5	—
—	4	5
—	4	9
—	3	—
—	2	7
1	16	—
—	10	4
—	2	—
—	14	—
—	19	10
60	7	

To To Henry Burgh

The Revnd The Revnd Mr

To Henry Burg

To ~~The Revnd Mr~~

The Revnd Mr Henry Burgh

Once more Ile Tune my Vocall Shell
To Hilles and Dales my passion tell
A flame which time can never Quell
To Burns it for my [illegible]
But greater ~~Bee[illegible]~~ the Lover [illegible] lit

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